Computer algebra independent integration tests

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/6.2.1-c+d-x-^m-a+b-cosh-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [183]. This is test number [165].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.

2. Rubi 4.16.1 in Mathematica 12 on windows 10.

3. Maple 2020 (64 bit) on windows 10.

4. Maxima 5.43 on Linux. (via sagemath 8.9)

5. Fricas 1.3.6 on Linux (via sagemath 9.0)

6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.

7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (183)	% 0. (0)
Mathematica	% 98.91 (181)	% 1.09 (2)
Maple	% 59.02 (108)	% 40.98 (75)
Maxima	% 71.58 (131)	% 28.42 (52)
Fricas	% 81.97 (150)	% 18.03 (33)
Sympy	% 28.96 (53)	% 71.04 (130)
Giac	% 55.19 (101)	% 44.81 (82)

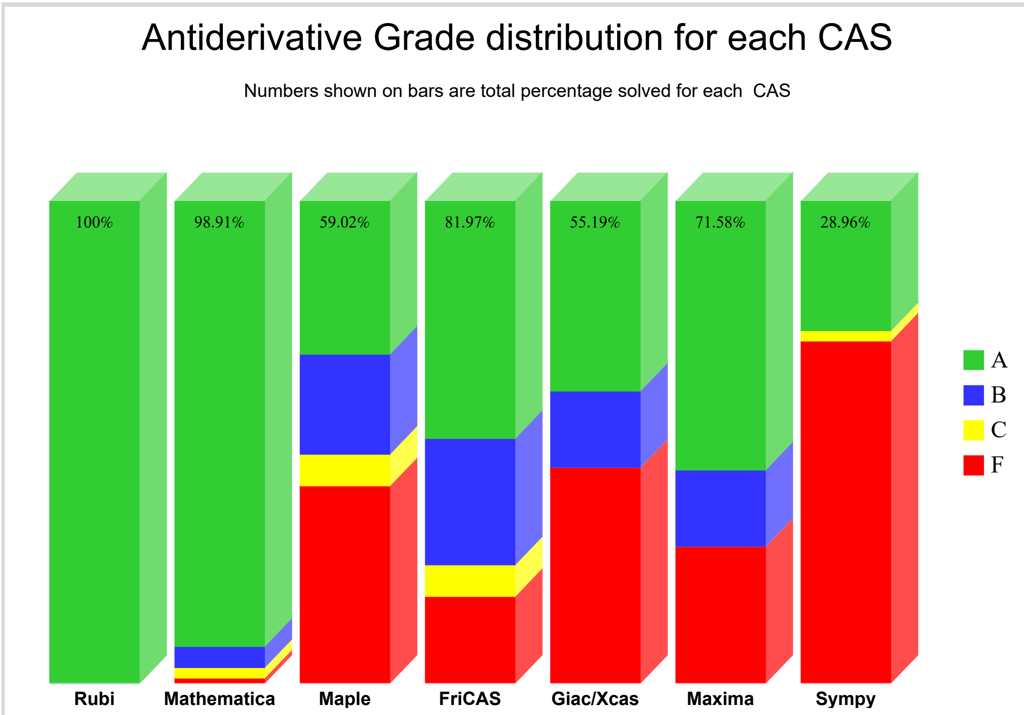
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

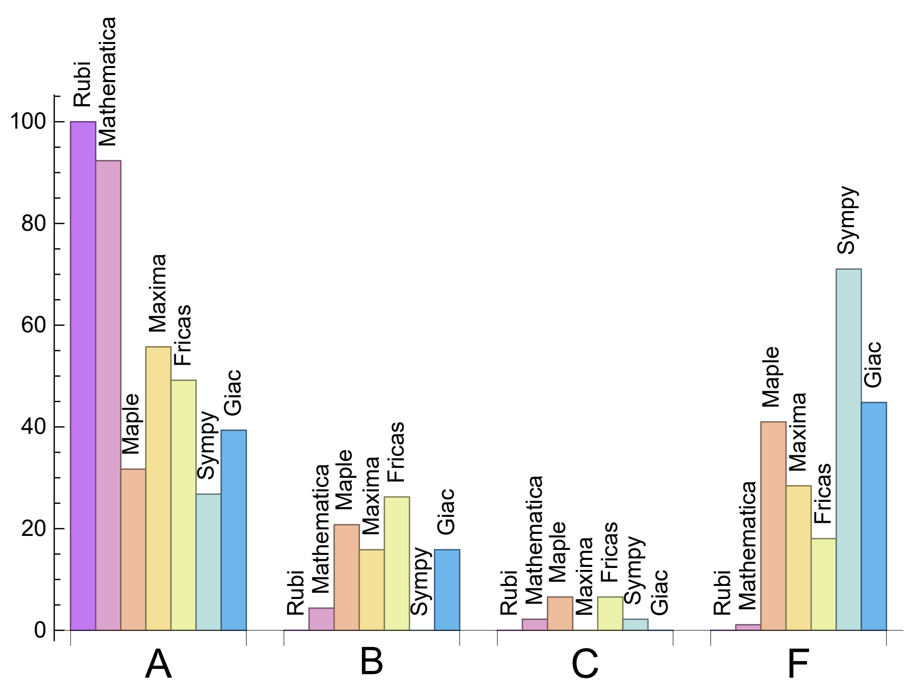
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	92.35	4.37	2.19	1.09
Maple	31.69	20.77	6.56	40.98
Maxima	55.74	15.85	0.	28.42
Fricas	49.18	26.23	6.56	18.03
Sympy	26.78	0.	2.19	71.04
Giac	39.34	15.85	0.	44.81

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	121.48	0.83	89.	1.
Mathematica	2.86	237.21	1.07	79.	0.84
Maple	0.07	196.49	1.67	108.	1.42
Maxima	0.99	177.41	1.41	134.	1.32
Fricas	1.7	1079.48	5.76	425.	4.14
Sympy	4.58	169.26	1.39	100.	1.51
Giac	0.91	199.26	1.63	132.	1.49

1.4 list of integrals that has no closed form antiderivative

{29, 30, 34, 35, 39, 40, 68, 69, 70, 75, 79, 80, 114, 115, 119, 120, 142, 143, 147, 148, 149, 150, 154, 155, 171, 172, 176, 177, 178, 182, 183}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {32, 48, 71, 73, 112, 117, 141, 145, 173, 174, 175}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command AboluteTiming[] was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command Usage was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's time.time() call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is ValueError. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception ValueError then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

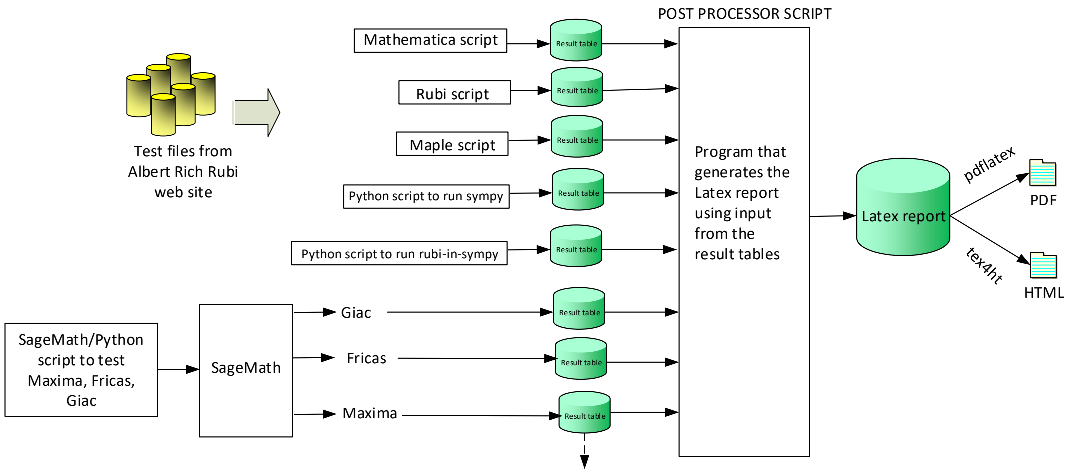
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 61, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 178, 179, 180, 181, 182, 183 }

B grade: { 28, 52, 54, 60, 62, 71, 173, 174 }

C grade: { 32, 74, 112, 117 }

F grade: { 39, 40}

2.1.3 Maple

A grade: { 4, 5, 6, 12, 13, 19, 20, 21, 25, 29, 30, 33, 34, 35, 39, 40, 68, 69, 70, 75, 79, 80, 102, 103, 107, 108, 109, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 142, 143, 147, 148, 150, 154, 155, 159, 164, 165, 166, 171, 172, 176, 177, 178, 182, 183 }

B grade: { 1, 2, 3, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 22, 23, 24, 28, 31, 32, 38, 99, 100, 101, 104, 105, 106, 110, 111, 112, 116, 156, 157, 158, 161, 163, 167, 170, 175 }

C grade: { 63, 64, 65, 66, 67, 81, 82, 83, 84, 85, 86, 87 }

F grade: { 26, 27, 36, 37, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 71, 72, 73, 74, 76, 77, 78, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 149, 151, 152, 153, 160, 162, 168, 169, 173, 174, 179, 180, 181 }

2.1.4 Maxima

A grade: { 5, 6, 7, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 29, 30, 34, 35, 39, 40, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 66, 67, 68, 69, 70, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 101, 102, 103, 104, 107, 108, 109, 110, 113, 114, 115, 119, 120, 121, 122, 123, 127, 128, 129, 133, 134, 135, 142, 143, 147, 148, 149, 150, 154, 155, 158, 159, 160, 161, 163, 164, 165, 166, 167, 171, 172, 176, 177, 178, 182, 183

B grade: { 1, 2, 3, 4, 8, 9, 16, 17, 18, 19, 31, 33, 41, 42, 43, 44, 63, 64, 65, 99, 100, 105, 106, 111, 116, 118, 156, 157, 162 }

C grade: { }

F grade: { 26, 27, 28, 32, 36, 37, 38, 71, 72, 73, 74, 77, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 112, 117, 124, 125, 126, 130, 131, 132, 136, 137, 138, 139, 140, 141, 144, 145, 146, 151, 152, 153, 168, 169, 170, 173, 174, 175, 179, 180, 181 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 9, 10, 11, 12, 18, 19, 20, 23, 24, 25, 29, 30, 34, 35, 39, 40, 44, 51, 59, 65, 68, 69, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 100, 101, 102, 103, 105, 106, 107, 108, 114, 115, 119, 120, 142, 143, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 171, 172, 176, 177, 178, 179, 180, 181, 182, 183 }

B grade: { 6, 7, 8, 13, 14, 15, 16, 17, 21, 22, 28, 33, 38, 41, 42, 43, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 72, 104, 109, 110, 112, 113, 117, 118, 161, 167, 170, 175 }

C grade: { 26, 27, 31, 32, 36, 37, 111, 116, 168, 169, 173, 174 }

F grade: { 70, 71, 73, 74, 95, 96, 97, 98, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 149 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 23, 24, 25, 29, 30, 34, 35, 39, 68, 69, 75, 79, 80, 99, 100, 101, 105, 106, 107, 113, 114, 115, 118, 119, 142, 143, 149, 154, 155, 156, 157, 158, 162, 163, 164, 182, 183 }

B grade: { }

C grade: { 63, 64, 65, 66 }

F grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 26, 27, 28, 31, 32, 33, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 67, 70, 71, 72, 73, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 102, 103, 104, 108, 109, 110, 111, 112, 116, 117, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 147, 148, 150, 151, 152, 153, 159, 160, 161, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181 }

2.1.7 Giac

A grade: { 4, 5, 10, 11, 12, 19, 20, 23, 24, 25, 29, 30, 34, 35, 39, 40, 41, 42, 43, 44, 63, 64, 65, 68, 69, 70, 75, 79, 80, 101, 102, 103, 107, 108, 113, 114, 115, 119, 120, 121, 122, 123, 124, 125, 126, 133, 134, 135, 136, 137, 138, 142, 143, 147, 148, 149, 150, 154, 155, 158, 159, 160, 164, 165, 166, 171, 172, 176, 177, 178, 182, 183 }

B grade: { 1, 2, 3, 6, 7, 8, 9, 14, 15, 16, 17, 18, 21, 22, 33, 99, 100, 104, 105, 106, 109, 110, 118, 156, 157, 161, 162, 163, 167 }

C grade: { }

F grade: { 13, 26, 27, 28, 31, 32, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 66, 67, 71, 72, 73, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 111, 112, 116, 117, 127, 128, 129, 130, 131, 132, 139, 140, 141, 144, 145, 146, 151, 152, 153, 168, 169, 170, 173, 174, 175, 179, 180, 181 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	76	547	440	351	311	437
normalized size	1	1.	0.84	6.01	4.84	3.86	3.42	4.8
time (sec)	N/A	0.119	0.303	0.013	1.241	1.72	3.25	1.456

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	308	300	232	202	275
normalized size	1	1.	0.87	4.4	4.29	3.31	2.89	3.93
time (sec)	N/A	0.079	0.193	0.006	1.289	1.979	1.564	1.39

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	147	182	140	112	151
normalized size	1	1.	0.9	3.	3.71	2.86	2.29	3.08
time (sec)	N/A	0.047	0.14	0.007	1.194	1.906	0.754	1.32

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	53	92	73	46	62
normalized size	1	1.	0.96	1.89	3.29	2.61	1.64	2.21
time (sec)	N/A	0.02	0.053	0.007	1.11	1.766	0.276	1.27

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	82	77	193	0	76
normalized size	1	1.	0.96	1.61	1.51	3.78	0.	1.49
time (sec)	N/A	0.101	0.073	0.024	1.204	1.716	0.	1.218

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	133	109	316	0	201
normalized size	1	1.	0.92	1.87	1.54	4.45	0.	2.83
time (sec)	N/A	0.119	0.243	0.033	1.359	1.769	0.	1.231

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	88	277	128	518	0	402
normalized size	1	1.	0.85	2.66	1.23	4.98	0.	3.87
time (sec)	N/A	0.163	0.525	0.039	1.215	1.849	0.	1.256

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	132	910	516	662	660	502
normalized size	1	1.	0.81	5.62	3.19	4.09	4.07	3.1
time (sec)	N/A	0.101	0.616	0.01	1.125	1.876	6.526	1.178

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	104	523	355	458	456	328
normalized size	1	1.	0.78	3.9	2.65	3.42	3.4	2.45
time (sec)	N/A	0.073	0.41	0.01	1.105	1.795	3.506	1.373

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	75	262	223	286	264	184
normalized size	1	1.	0.79	2.76	2.35	3.01	2.78	1.94
time (sec)	N/A	0.053	0.278	0.008	1.143	1.81	1.625	1.333

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	51	103	119	163	126	85
normalized size	1	1.	0.93	1.87	2.16	2.96	2.29	1.55
time (sec)	N/A	0.025	0.155	0.008	1.065	1.83	0.674	1.254

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	64	97	97	232	0	92
normalized size	1	1.	0.82	1.24	1.24	2.97	0.	1.18
time (sec)	N/A	0.154	0.114	0.071	1.345	1.783	0.	1.294

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	75	152	119	365	0	0
normalized size	1	1.	0.93	1.88	1.47	4.51	0.	0.
time (sec)	N/A	0.146	0.402	0.076	1.237	1.785	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	102	299	134	597	0	446
normalized size	1	1.	0.91	2.67	1.2	5.33	0.	3.98
time (sec)	N/A	0.187	0.88	0.081	1.288	1.85	0.	1.408

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	121	555	149	859	0	725
normalized size	1	1.	0.75	3.43	0.92	5.3	0.	4.48
time (sec)	N/A	0.181	0.834	0.092	1.298	1.892	0.	1.33

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	385	1217	869	1149	772	883
normalized size	1	1.	1.71	5.41	3.86	5.11	3.43	3.92
time (sec)	N/A	0.282	0.934	0.013	1.165	2.042	11.293	1.398

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	122	676	593	757	495	559
normalized size	1	1.	0.7	3.86	3.39	4.33	2.83	3.19
time (sec)	N/A	0.176	0.926	0.01	1.101	2.111	5.883	1.368

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	93	320	367	474	284	311
normalized size	1	1.	0.76	2.6	2.98	3.85	2.31	2.53
time (sec)	N/A	0.103	0.536	0.01	1.104	2.018	2.934	1.329

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	52	115	193	257	126	132
normalized size	1	1.	0.69	1.53	2.57	3.43	1.68	1.76
time (sec)	N/A	0.044	0.227	0.009	1.072	1.961	1.334	1.304

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	102	166	158	398	0	151
normalized size	1	1.	0.84	1.37	1.31	3.29	0.	1.25
time (sec)	N/A	0.24	0.227	0.088	1.278	1.984	0.	1.332

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	196	271	196	680	0	401
normalized size	1	1.	1.35	1.87	1.35	4.69	0.	2.77
time (sec)	N/A	0.237	0.534	0.102	1.335	1.947	0.	1.395

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	218	562	196	1122	0	813
normalized size	1	1.	1.18	3.05	1.07	6.1	0.	4.42
time (sec)	N/A	0.34	0.9	0.106	1.356	2.041	0.	1.308

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	100	432	238	486	262	203
normalized size	1	1.	0.58	2.51	1.38	2.83	1.52	1.18
time (sec)	N/A	0.146	0.405	0.011	1.043	2.05	8.106	1.237

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	90	253	178	373	209	159
normalized size	1	1.	0.67	1.89	1.33	2.78	1.56	1.19
time (sec)	N/A	0.107	0.157	0.007	1.048	1.986	4.356	1.302

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	53	120	130	306	144	116
normalized size	1	1.	0.66	1.5	1.62	3.82	1.8	1.45
time (sec)	N/A	0.044	0.175	0.007	1.08	2.021	2.379	1.274

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	343	0	0	1300	0	0
normalized size	1	1.	1.92	0.	0.	7.26	0.	0.
time (sec)	N/A	0.123	2.556	0.216	0.	2.251	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	199	0	0	845	0	0
normalized size	1	1.	1.67	0.	0.	7.1	0.	0.
time (sec)	N/A	0.081	1.459	0.128	0.	2.229	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	129	449	0	463	0	0
normalized size	1	1.	2.11	7.36	0.	7.59	0.	0.
time (sec)	N/A	0.037	0.079	0.007	0.	2.146	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	3.538	0.053	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	7.201	0.046	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	145	298	321	3131	0	0
normalized size	1	1.	1.41	2.89	3.12	30.4	0.	0.
time (sec)	N/A	0.206	2.039	0.067	1.694	2.426	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	277	159	0	1777	0	0
normalized size	1	1.	3.79	2.18	0.	24.34	0.	0.
time (sec)	N/A	0.133	6.281	0.031	0.	2.232	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	51	57	97	429	0	105
normalized size	1	1.	1.76	1.97	3.34	14.79	0.	3.62
time (sec)	N/A	0.029	0.09	0.023	1.012	2.07	0.	1.315

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	17.409	0.073	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	17.628	0.085	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	455	0	0	11732	0	0
normalized size	1	1.	1.54	0.	0.	39.64	0.	0.
time (sec)	N/A	0.217	27.244	0.193	0.	3.435	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	270	0	0	6791	0	0
normalized size	1	1.	1.54	0.	0.	38.81	0.	0.
time (sec)	N/A	0.133	5.31	0.164	0.	2.832	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	180	216	0	3492	0	0
normalized size	1	1.	1.76	2.12	0.	34.24	0.	0.
time (sec)	N/A	0.063	2.858	0.053	0.	2.521	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	180.016	0.3	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	180.015	0.422	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	107	0	416	1177	0	313
normalized size	1	1.	0.63	0.	2.43	6.88	0.	1.83
time (sec)	N/A	0.33	0.052	0.046	1.083	2.128	0.	1.479

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	107	0	362	892	0	273
normalized size	1	1.	0.73	0.	2.48	6.11	0.	1.87
time (sec)	N/A	0.243	0.101	0.045	1.086	2.166	0.	1.466

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	105	0	311	717	0	228
normalized size	1	1.	0.85	0.	2.53	5.83	0.	1.85
time (sec)	N/A	0.178	0.092	0.043	1.117	2.083	0.	1.442

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	105	0	243	271	0	123
normalized size	1	1.	1.01	0.	2.34	2.61	0.	1.18
time (sec)	N/A	0.13	0.043	0.046	1.084	2.096	0.	1.396

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	118	0	140	815	0	0
normalized size	1	1.	0.99	0.	1.18	6.85	0.	0.
time (sec)	N/A	0.183	0.35	0.044	1.051	2.101	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	150	0	155	1224	0	0
normalized size	1	1.	1.01	0.	1.04	8.21	0.	0.
time (sec)	N/A	0.248	0.709	0.043	1.206	2.157	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	191	0	155	1821	0	0
normalized size	1	1.	1.1	0.	0.89	10.47	0.	0.
time (sec)	N/A	0.312	0.395	0.046	1.187	2.293	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	189	0	379	2319	0	0
normalized size	1	1.	0.79	0.	1.59	9.7	0.	0.
time (sec)	N/A	0.4	1.156	0.085	1.59	2.256	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	163	0	323	1789	0	0
normalized size	1	1.	0.77	0.	1.53	8.48	0.	0.
time (sec)	N/A	0.303	0.583	0.08	1.57	2.218	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	129	0	255	1438	0	0
normalized size	1	1.	0.78	0.	1.54	8.66	0.	0.
time (sec)	N/A	0.269	0.44	0.08	1.549	2.215	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	141	0	144	369	0	0
normalized size	1	1.	1.02	0.	1.04	2.67	0.	0.
time (sec)	N/A	0.214	0.118	0.088	1.568	2.211	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	570	0	157	1442	0	0
normalized size	1	1.	4.01	0.	1.11	10.15	0.	0.
time (sec)	N/A	0.23	2.847	0.092	1.269	2.226	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	156	0	159	2053	0	0
normalized size	1	1.	0.9	0.	0.91	11.8	0.	0.
time (sec)	N/A	0.309	1.35	0.102	1.206	2.328	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	825	0	157	2974	0	0
normalized size	1	1.	3.75	0.	0.71	13.52	0.	0.
time (sec)	N/A	0.311	3.034	0.095	1.201	2.388	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	222	0	157	3951	0	0
normalized size	1	1.	0.88	0.	0.63	15.74	0.	0.
time (sec)	N/A	0.379	0.828	0.104	1.213	2.29	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	243	0	693	4815	0	0
normalized size	1	1.	0.64	0.	1.82	12.64	0.	0.
time (sec)	N/A	0.906	3.736	0.125	1.666	2.246	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	243	0	579	3621	0	0
normalized size	1	1.	0.75	0.	1.78	11.11	0.	0.
time (sec)	N/A	0.712	1.929	0.119	1.86	2.018	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	210	0	451	2965	0	0
normalized size	1	1.	0.76	0.	1.64	10.78	0.	0.
time (sec)	N/A	0.484	0.287	0.122	1.704	1.872	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	192	0	239	594	0	0
normalized size	1	1.	0.84	0.	1.05	2.61	0.	0.
time (sec)	N/A	0.374	0.206	0.123	1.62	2.087	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	717	0	265	3313	0	0
normalized size	1	1.	2.91	0.	1.08	13.47	0.	0.
time (sec)	N/A	0.421	2.736	0.155	1.363	2.376	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	253	0	262	4825	0	0
normalized size	1	1.	0.91	0.	0.95	17.42	0.	0.
time (sec)	N/A	0.605	2.907	0.141	1.36	2.365	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	3211	0	265	7121	0	0
normalized size	1	1.	9.7	0.	0.8	21.51	0.	0.
time (sec)	N/A	0.68	6.325	0.131	1.341	2.48	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	51	133	235	466	131	194
normalized size	1	1.	0.46	1.2	2.12	4.2	1.18	1.75
time (sec)	N/A	0.155	0.013	0.027	1.062	1.839	142.406	1.232

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	48	121	200	355	100	138
normalized size	1	1.	0.52	1.32	2.17	3.86	1.09	1.5
time (sec)	N/A	0.106	0.011	0.019	1.124	1.85	3.545	1.216

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	48	72	158	136	66	81
normalized size	1	1.	0.62	0.94	2.05	1.77	0.86	1.05
time (sec)	N/A	0.077	0.008	0.022	1.057	1.833	1.643	1.292

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	67	115	103	356	99	0
normalized size	1	1.	0.76	1.31	1.17	4.05	1.12	0.
time (sec)	N/A	0.113	0.034	0.022	1.043	1.792	7.426	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	78	126	78	452	0	0
normalized size	1	1.	0.68	1.11	0.68	3.96	0.	0.
time (sec)	N/A	0.148	0.093	0.027	1.261	1.855	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	11.162	0.068	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	9.845	0.064	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	3.849	0.026	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	20	20	46	0	0	0	0	0
normalized size	1	1.	2.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.372	0.074	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	16	0	0	374	0	0
normalized size	1	1.	0.67	0.	0.	15.58	0.	0.
time (sec)	N/A	0.05	0.074	0.079	0.	1.76	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	64	0	0	0	0	0
normalized size	1	1.	1.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.616	0.1	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	76	0	0	0	0	0
normalized size	1	1.	2.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.947	0.07	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	2.79	0.072	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	205	0	217	801	0	0
normalized size	1	1.	0.86	0.	0.92	3.38	0.	0.
time (sec)	N/A	0.284	0.188	0.135	1.28	1.948	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	132	0	0	597	0	0
normalized size	1	1.	0.92	0.	0.	4.15	0.	0.
time (sec)	N/A	0.184	0.202	0.091	0.	1.796	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	102	0	107	378	0	0
normalized size	1	1.	0.93	0.	0.97	3.44	0.	0.
time (sec)	N/A	0.091	0.056	0.06	1.244	1.905	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	5.654	0.046	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	3.295	0.052	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	74	259	0	0
normalized size	1	1.	0.92	1.24	1.25	4.39	0.	0.
time (sec)	N/A	0.074	0.038	0.033	1.26	1.818	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	74	259	0	0
normalized size	1	1.	0.92	1.24	1.25	4.39	0.	0.
time (sec)	N/A	0.072	0.019	0.055	1.144	1.898	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	74	259	0	0
normalized size	1	1.	0.92	1.24	1.25	4.39	0.	0.
time (sec)	N/A	0.071	0.032	0.048	1.153	1.748	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	74	227	0	0
normalized size	1	1.	0.92	1.24	1.25	3.85	0.	0.
time (sec)	N/A	0.069	0.018	0.039	1.153	1.928	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	67	58	238	0	0
normalized size	1	1.	1.	1.37	1.18	4.86	0.	0.
time (sec)	N/A	0.069	0.02	0.046	1.17	1.974	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	67	74	259	0	0
normalized size	1	1.	0.95	1.22	1.35	4.71	0.	0.
time (sec)	N/A	0.069	0.019	0.055	1.155	1.849	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	55	71	74	259	0	0
normalized size	1	1.	0.93	1.2	1.25	4.39	0.	0.
time (sec)	N/A	0.07	0.022	0.032	1.161	1.891	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	0	425	0	0
normalized size	1	1.	0.92	0.	0.	4.94	0.	0.
time (sec)	N/A	0.15	0.106	0.048	0.	1.878	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	0	425	0	0
normalized size	1	1.	0.92	0.	0.	5.	0.	0.
time (sec)	N/A	0.133	0.095	0.052	0.	1.803	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	0	425	0	0
normalized size	1	1.	0.92	0.	0.	4.94	0.	0.
time (sec)	N/A	0.133	0.106	0.08	0.	1.741	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	0	374	0	0
normalized size	1	1.	0.89	0.	0.	4.4	0.	0.
time (sec)	N/A	0.127	0.085	0.054	0.	1.861	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	363	0	0
normalized size	1	1.	0.89	0.	0.	5.04	0.	0.
time (sec)	N/A	0.125	0.056	0.078	0.	1.981	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	73	0	0	423	0	0
normalized size	1	1.	0.88	0.	0.	5.1	0.	0.
time (sec)	N/A	0.13	0.098	0.049	0.	2.092	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	425	0	0
normalized size	1	1.	1.	0.	0.	5.06	0.	0.
time (sec)	N/A	0.14	0.102	0.049	0.	2.198	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.087	0.079	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.139	0.077	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.104	0.082	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	0.101	0.074	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	122	482	320	365	264	351
normalized size	1	1.	1.37	5.42	3.6	4.1	2.97	3.94
time (sec)	N/A	0.132	0.541	0.016	1.121	2.012	2.418	1.255

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	80	240	190	231	151	200
normalized size	1	1.	1.19	3.58	2.84	3.45	2.25	2.99
time (sec)	N/A	0.091	0.314	0.012	1.091	1.972	0.946	1.293

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	52	91	89	128	68	89
normalized size	1	1.	1.16	2.02	1.98	2.84	1.51	1.98
time (sec)	N/A	0.045	0.218	0.012	1.06	1.983	0.402	1.253

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	54	94	95	230	0	93
normalized size	1	1.	0.84	1.47	1.48	3.59	0.	1.45
time (sec)	N/A	0.138	0.108	0.042	1.262	2.046	0.	1.294

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	68	149	117	351	0	227
normalized size	1	1.	0.78	1.71	1.34	4.03	0.	2.61
time (sec)	N/A	0.172	0.298	0.052	1.231	2.005	0.	1.271

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	90	296	132	572	0	443
normalized size	1	1.	0.73	2.41	1.07	4.65	0.	3.6
time (sec)	N/A	0.216	0.456	0.053	1.226	2.072	0.	1.197

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	217	1071	711	828	779	784
normalized size	1	1.	0.92	4.52	3.	3.49	3.29	3.31
time (sec)	N/A	0.265	1.408	0.016	1.281	2.158	6.985	1.256

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	192	541	441	491	456	450
normalized size	1	1.	1.14	3.22	2.62	2.92	2.71	2.68
time (sec)	N/A	0.188	0.521	0.014	1.249	2.118	3.247	1.237

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	81	211	225	269	219	209
normalized size	1	1.	0.69	1.79	1.91	2.28	1.86	1.77
time (sec)	N/A	0.099	0.478	0.013	1.08	2.023	1.398	1.33

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	113	191	201	470	0	188
normalized size	1	1.	0.78	1.32	1.39	3.24	0.	1.3
time (sec)	N/A	0.342	0.197	0.105	1.303	2.141	0.	1.28

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	207	308	246	767	0	485
normalized size	1	1.	1.32	1.96	1.57	4.89	0.	3.09
time (sec)	N/A	0.333	0.63	0.123	1.32	2.17	0.	2.314

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	353	618	273	1223	0	953
normalized size	1	1.	1.71	2.99	1.32	5.91	0.	4.6
time (sec)	N/A	0.505	1.082	0.134	1.41	2.196	0.	1.276

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	154	325	308	1008	0	0
normalized size	1	1.	1.32	2.78	2.63	8.62	0.	0.
time (sec)	N/A	0.273	2.059	0.119	1.615	2.06	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	472	174	0	590	0	0
normalized size	1	1.	5.36	1.98	0.	6.7	0.	0.
time (sec)	N/A	0.201	6.346	0.045	0.	2.103	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	70	63	96	251	76	96
normalized size	1	1.	1.43	1.29	1.96	5.12	1.55	1.96
time (sec)	N/A	0.068	0.258	0.039	1.036	2.108	2.011	1.31

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	8.873	0.07	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	9.314	0.083	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	462	600	824	4103	0	0
normalized size	1	1.	1.81	2.35	3.23	16.09	0.	0.
time (sec)	N/A	0.364	3.434	0.116	1.895	2.327	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	637	313	0	2267	0	0
normalized size	1	1.	3.18	1.56	0.	11.34	0.	0.
time (sec)	N/A	0.252	6.457	0.065	0.	2.194	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	114	108	323	1002	156	279
normalized size	1	1.	0.93	0.88	2.63	8.15	1.27	2.27
time (sec)	N/A	0.096	0.437	0.06	1.11	2.119	2.872	1.257

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	30.104	0.347	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	31.286	0.465	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	53	108	162	0	0	198
normalized size	1	1.	0.48	0.98	1.47	0.	0.	1.8
time (sec)	N/A	0.148	0.197	0.106	1.71	0.	0.	1.305

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	44	86	122	0	0	144
normalized size	1	1.	0.5	0.98	1.39	0.	0.	1.64
time (sec)	N/A	0.113	0.149	0.056	1.659	0.	0.	1.333

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	34	64	81	0	0	90
normalized size	1	1.	0.64	1.21	1.53	0.	0.	1.7
time (sec)	N/A	0.062	0.12	0.055	1.766	0.	0.	1.294

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	54	0	0	0	0	43
normalized size	1	1.	0.65	0.	0.	0.	0.	0.52
time (sec)	N/A	0.132	0.075	0.069	0.	0.	0.	1.3

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	75	0	0	0	0	92
normalized size	1	1.	0.68	0.	0.	0.	0.	0.84
time (sec)	N/A	0.14	0.136	0.052	0.	0.	0.	1.216

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	97	0	0	0	0	144
normalized size	1	1.	0.64	0.	0.	0.	0.	0.95
time (sec)	N/A	0.17	0.205	0.053	0.	0.	0.	1.198

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	33	62	119	0	0	0
normalized size	1	1.	0.49	0.91	1.75	0.	0.	0.
time (sec)	N/A	0.117	0.052	0.051	1.74	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	31	50	89	0	0	0
normalized size	1	1.	0.58	0.94	1.68	0.	0.	0.
time (sec)	N/A	0.097	0.044	0.038	1.695	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	22	38	59	0	0	0
normalized size	1	1.	0.69	1.19	1.84	0.	0.	0.
time (sec)	N/A	0.051	0.02	0.036	1.687	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.007	0.049	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	33	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.053	0.039	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	44	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.07	0.043	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	70	0	243	0	0	240
normalized size	1	1.	0.38	0.	1.31	0.	0.	1.3
time (sec)	N/A	0.195	0.279	0.023	1.67	0.	0.	1.188

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	54	0	184	0	0	176
normalized size	1	1.	0.37	0.	1.27	0.	0.	1.21
time (sec)	N/A	0.149	0.215	0.024	1.683	0.	0.	1.202

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	56	0	124	0	0	111
normalized size	1	1.	0.63	0.	1.39	0.	0.	1.25
time (sec)	N/A	0.075	0.084	0.023	1.743	0.	0.	1.211

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	0	0	0	0	54
normalized size	1	1.	0.65	0.	0.	0.	0.	0.98
time (sec)	N/A	0.128	0.02	0.023	0.	0.	0.	1.179

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	53	0	0	0	0	115
normalized size	1	1.	0.67	0.	0.	0.	0.	1.46
time (sec)	N/A	0.131	0.084	0.023	0.	0.	0.	1.268

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	69	0	0	0	0	180
normalized size	1	1.	0.63	0.	0.	0.	0.	1.65
time (sec)	N/A	0.173	0.065	0.023	0.	0.	0.	1.309

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	213	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	1.724	0.041	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	163	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.163	1.707	0.039	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	117	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.665	180.	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	2.664	0.043	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	1.646	0.042	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	716	0	0	0	0	0
normalized size	1	1.	1.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.257	2.923	0.024	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	248	248	214	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	0.879	0.023	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	137	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.102	0.026	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	8.373	0.026	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	10.178	0.023	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	F	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	2.513	180.	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	5.875	0.082	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	429	0	0	1665	0	0
normalized size	1	1.	1.07	0.	0.	4.14	0.	0.
time (sec)	N/A	0.553	2.289	0.15	0.	2.403	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	302	0	0	1139	0	0
normalized size	1	1.	1.15	0.	0.	4.33	0.	0.
time (sec)	N/A	0.342	1.045	0.113	0.	2.297	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	189	0	0	586	0	0
normalized size	1	1.	1.44	0.	0.	4.47	0.	0.
time (sec)	N/A	0.147	0.312	0.069	0.	2.214	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	4.925	0.055	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	8.884	0.082	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	123	482	320	365	264	351
normalized size	1	1.	1.38	5.42	3.6	4.1	2.97	3.94
time (sec)	N/A	0.132	0.461	0.014	1.203	2.121	2.231	1.281

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	83	240	190	231	151	200
normalized size	1	1.	1.24	3.58	2.84	3.45	2.25	2.99
time (sec)	N/A	0.09	0.317	0.01	1.192	2.011	1.157	1.237

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	91	89	128	68	89
normalized size	1	1.	1.02	2.02	1.98	2.84	1.51	1.98
time (sec)	N/A	0.046	0.096	0.012	1.142	2.006	0.715	1.171

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	57	94	95	230	0	93
normalized size	1	1.	0.89	1.47	1.48	3.59	0.	1.45
time (sec)	N/A	0.121	0.137	0.028	1.307	2.169	0.	1.187

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	71	0	117	351	0	227
normalized size	1	1.	0.82	0.	1.34	4.03	0.	2.61
time (sec)	N/A	0.152	0.4	180.	1.399	2.35	0.	1.202

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	95	296	132	572	0	443
normalized size	1	1.	0.77	2.41	1.07	4.65	0.	3.6
time (sec)	N/A	0.197	0.602	0.029	1.331	2.368	0.	1.149

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	232	0	706	879	779	814
normalized size	1	1.	0.93	0.	2.82	3.52	3.12	3.26
time (sec)	N/A	0.285	1.394	180.	1.252	2.114	5.54	1.224

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	252	535	437	541	456	471
normalized size	1	1.	1.38	2.94	2.4	2.97	2.51	2.59
time (sec)	N/A	0.194	0.95	0.015	1.221	2.035	2.352	1.194

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	96	208	223	294	219	221
normalized size	1	1.	0.83	1.79	1.92	2.53	1.89	1.91
time (sec)	N/A	0.101	0.846	0.013	1.179	2.043	0.929	1.202

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	133	202	200	483	0	200
normalized size	1	1.	0.85	1.29	1.28	3.1	0.	1.28
time (sec)	N/A	0.31	0.282	0.102	1.447	2.133	0.	1.254

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	233	319	244	778	0	485
normalized size	1	1.	1.27	1.74	1.33	4.25	0.	2.65
time (sec)	N/A	0.345	0.76	0.12	1.386	2.156	0.	1.924

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	394	626	271	1234	0	948
normalized size	1	1.	1.63	2.59	1.12	5.1	0.	3.92
time (sec)	N/A	0.428	1.234	0.133	1.49	2.242	0.	1.265

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	384	0	0	2418	0	0
normalized size	1	1.	0.88	0.	0.	5.55	0.	0.
time (sec)	N/A	0.821	1.469	0.226	0.	2.311	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	247	0	0	1736	0	0
normalized size	1	1.	0.77	0.	0.	5.42	0.	0.
time (sec)	N/A	0.67	0.963	0.151	0.	2.261	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	152	437	0	1127	0	0
normalized size	1	1.	0.75	2.15	0.	5.55	0.	0.
time (sec)	N/A	0.376	0.887	0.063	0.	2.129	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.893	0.05	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.961	0.05	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	823	823	11178	0	0	14923	0	0
normalized size	1	1.	13.58	0.	0.	18.13	0.	0.
time (sec)	N/A	1.353	26.463	0.289	0.	3.715	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	593	593	6018	0	0	8911	0	0
normalized size	1	1.	10.15	0.	0.	15.03	0.	0.
time (sec)	N/A	1.025	22.266	0.243	0.	3.01	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	509	585	0	4074	0	0
normalized size	1	1.	1.86	2.14	0.	14.87	0.	0.
time (sec)	N/A	0.457	4.978	0.096	0.	2.323	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	49.026	0.292	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	51.967	0.461	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	4.125	0.063	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	447	0	0	1886	0	0
normalized size	1	1.	0.82	0.	0.	3.47	0.	0.
time (sec)	N/A	0.743	1.68	0.138	0.	2.454	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	254	0	0	1191	0	0
normalized size	1	1.	0.9	0.	0.	4.22	0.	0.
time (sec)	N/A	0.363	0.721	0.112	0.	2.302	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	119	0	0	586	0	0
normalized size	1	1.	0.91	0.	0.	4.47	0.	0.
time (sec)	N/A	0.145	0.172	0.068	0.	2.159	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	1.122	0.049	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	5.085	0.076	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [144] had the largest ratio of [0.6429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	2	1.	14	0.143
2	A	4	2	1.	14	0.143
3	A	3	2	1.	14	0.143
4	A	2	2	1.	12	0.167
5	A	3	3	1.	14	0.214
6	A	4	4	1.	14	0.286
7	A	5	4	1.	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
8	A	6	4	1.	16	0.25
9	A	4	3	1.	16	0.188
10	A	4	4	1.	16	0.25
11	A	2	1	1.	14	0.071
12	A	5	4	1.	16	0.25
13	A	5	5	1.	16	0.312
14	A	7	6	1.	16	0.375
15	A	7	7	1.	16	0.438
16	A	12	4	1.	16	0.25
17	A	8	4	1.	16	0.25
18	A	6	4	1.	16	0.25
19	A	3	3	1.	14	0.214
20	A	8	4	1.	16	0.25
21	A	8	4	1.	16	0.25
22	A	12	5	1.	16	0.312
23	A	8	3	1.	12	0.25
24	A	8	4	1.	12	0.333
25	A	3	2	1.	10	0.2
26	A	9	5	1.	14	0.357
27	A	7	4	1.	14	0.286
28	A	5	3	1.	12	0.25
29	A	0	0	0.	0	0.
30	A	0	0	0.	0	0.
31	A	6	6	1.	16	0.375
32	A	5	5	1.	16	0.312
33	A	2	2	1.	14	0.143
34	A	0	0	0.	0	0.
35	A	0	0	0.	0	0.
36	A	15	8	1.	16	0.5
37	A	9	6	1.	16	0.375
38	A	6	4	1.	14	0.286
39	A	0	0	0.	0	0.
40	A	0	0	0.	0	0.
41	A	8	5	1.	16	0.312
42	A	7	5	1.	16	0.312
43	A	6	5	1.	16	0.312
44	A	5	4	1.	16	0.25
45	A	6	5	1.	16	0.312
46	A	7	5	1.	16	0.312
47	A	8	5	1.	16	0.312
48	A	10	8	1.	18	0.444
49	A	9	7	1.	18	0.389
50	A	8	6	1.	18	0.333
51	A	7	5	1.	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	7	6	1.	18	0.333
53	A	9	7	1.	18	0.389
54	A	9	8	1.	18	0.444
55	A	11	7	1.	18	0.389
56	A	23	7	1.	18	0.389
57	A	20	7	1.	18	0.389
58	A	14	6	1.	18	0.333
59	A	12	5	1.	18	0.278
60	A	12	5	1.	18	0.278
61	A	18	6	1.	18	0.333
62	A	19	7	1.	18	0.389
63	A	7	5	1.	12	0.417
64	A	6	5	1.	12	0.417
65	A	5	4	1.	12	0.333
66	A	6	5	1.	12	0.417
67	A	7	5	1.	12	0.417
68	A	0	0	0.	0	0.
69	A	0	0	0.	0	0.
70	A	0	0	0.	0	0.
71	A	2	1	1.	17	0.059
72	A	2	1	1.	20	0.05
73	A	3	1	1.	20	0.05
74	A	3	2	1.	21	0.095
75	A	0	0	0.	0	0.
76	A	8	3	1.	16	0.188
77	A	5	3	1.	16	0.188
78	A	3	2	1.	14	0.143
79	A	0	0	0.	0	0.
80	A	0	0	0.	0	0.
81	A	3	2	1.	12	0.167
82	A	3	2	1.	12	0.167
83	A	3	2	1.	12	0.167
84	A	3	2	1.	10	0.2
85	A	3	2	1.	12	0.167
86	A	3	2	1.	12	0.167
87	A	3	2	1.	12	0.167
88	A	5	3	1.	14	0.214
89	A	5	3	1.	14	0.214
90	A	5	3	1.	14	0.214
91	A	5	3	1.	12	0.25
92	A	5	3	1.	14	0.214
93	A	5	3	1.	14	0.214
94	A	5	3	1.	14	0.214
95	A	4	2	1.	20	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	2	1.	20	0.1
97	A	5	2	1.	20	0.1
98	A	7	5	1.	24	0.208
99	A	6	3	1.	18	0.167
100	A	5	3	1.	18	0.167
101	A	4	3	1.	16	0.188
102	A	5	4	1.	18	0.222
103	A	6	5	1.	18	0.278
104	A	7	5	1.	18	0.278
105	A	10	6	1.	20	0.3
106	A	9	7	1.	20	0.35
107	A	6	4	1.	18	0.222
108	A	9	5	1.	20	0.25
109	A	9	5	1.	20	0.25
110	A	15	6	1.	20	0.3
111	A	7	7	1.	20	0.35
112	A	6	6	1.	20	0.3
113	A	3	3	1.	18	0.167
114	A	0	0	0.	0	0.
115	A	0	0	0.	0	0.
116	A	10	9	1.	20	0.45
117	A	9	9	1.	20	0.45
118	A	4	4	1.	18	0.222
119	A	0	0	0.	0	0.
120	A	0	0	0.	0	0.
121	A	5	3	1.	18	0.167
122	A	4	3	1.	18	0.167
123	A	3	3	1.	16	0.188
124	A	4	4	1.	18	0.222
125	A	5	5	1.	18	0.278
126	A	6	5	1.	18	0.278
127	A	5	3	1.	14	0.214
128	A	4	3	1.	14	0.214
129	A	3	3	1.	12	0.25
130	A	2	2	1.	14	0.143
131	A	3	3	1.	14	0.214
132	A	4	3	1.	14	0.214
133	A	9	5	1.	14	0.357
134	A	7	5	1.	14	0.357
135	A	4	4	1.	12	0.333
136	A	5	3	1.	14	0.214
137	A	5	3	1.	14	0.214
138	A	7	4	1.	14	0.286
139	A	10	6	1.	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
140	A	8	5	1.	18	0.278
141	A	6	4	1.	16	0.25
142	A	0	0	0.	0	0.
143	A	0	0	0.	0	0.
144	A	16	9	1.	14	0.643
145	A	10	7	1.	14	0.5
146	A	7	5	1.	12	0.417
147	A	0	0	0.	0	0.
148	A	0	0	0.	0	0.
149	A	0	0	0.	0	0.
150	A	0	0	0.	0	0.
151	A	12	4	1.	20	0.2
152	A	9	4	1.	20	0.2
153	A	5	3	1.	18	0.167
154	A	0	0	0.	0	0.
155	A	0	0	0.	0	0.
156	A	6	3	1.	18	0.167
157	A	5	3	1.	18	0.167
158	A	4	3	1.	16	0.188
159	A	5	4	1.	18	0.222
160	A	6	5	1.	18	0.278
161	A	7	5	1.	18	0.278
162	A	10	6	1.	20	0.3
163	A	9	7	1.	20	0.35
164	A	6	4	1.	18	0.222
165	A	10	5	1.	20	0.25
166	A	11	7	1.	20	0.35
167	A	14	8	1.	20	0.4
168	A	12	7	1.	20	0.35
169	A	10	6	1.	20	0.3
170	A	8	5	1.	18	0.278
171	A	0	0	0.	0	0.
172	A	0	0	0.	0	0.
173	A	22	9	1.	20	0.45
174	A	18	10	1.	20	0.5
175	A	11	8	1.	18	0.444
176	A	0	0	0.	0	0.
177	A	0	0	0.	0	0.
178	A	0	0	0.	0	0.
179	A	18	4	1.	20	0.2
180	A	10	4	1.	20	0.2
181	A	5	3	1.	18	0.167
182	A	0	0	0.	0	0.
183	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int (c + dx)^4 \cosh(a + bx)\, dx$

Optimal. Leaf size=91

$$\frac{12d^2(c+dx)^2\sinh(a+bx)}{b^3} - \frac{24d^3(c+dx)\cosh(a+bx)}{b^4} - \frac{4d(c+dx)^3\cosh(a+bx)}{b^2} + \frac{24d^4\sinh(a+bx)}{b^5} + \frac{(c+dx)^4}{}$$

```
[Out] (-24*d^3*(c + d*x)*Cosh[a + b*x])/b^4 - (4*d*(c + d*x)^3*Cosh[a + b*x])/b^2
 + (24*d^4*Sinh[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*Sinh[a + b*x])/b^3 + ((
c + d*x)^4*Sinh[a + b*x])/b
```

Rubi [A] time = 0.119198, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2637}

$$\frac{12d^2(c+dx)^2\sinh(a+bx)}{b^3} - \frac{24d^3(c+dx)\cosh(a+bx)}{b^4} - \frac{4d(c+dx)^3\cosh(a+bx)}{b^2} + \frac{24d^4\sinh(a+bx)}{b^5} + \frac{(c+dx)^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cosh[a + b*x],x]
```

```
[Out] (-24*d^3*(c + d*x)*Cosh[a + b*x])/b^4 - (4*d*(c + d*x)^3*Cosh[a + b*x])/b^2
 + (24*d^4*Sinh[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*Sinh[a + b*x])/b^3 + ((
c + d*x)^4*Sinh[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^4 \cosh(a+bx)\,dx = \frac{(c+dx)^4\sinh(a+bx)}{b} - \frac{(4d)\int (c+dx)^3\sinh(a+bx)\,dx}{b}$$
$$= -\frac{4d(c+dx)^3\cosh(a+bx)}{b^2} + \frac{(c+dx)^4\sinh(a+bx)}{b} + \frac{\left(12d^2\right)\int (c+dx)^2\cosh(a+bx)\,dx}{b^2}$$
$$= -\frac{4d(c+dx)^3\cosh(a+bx)}{b^2} + \frac{12d^2(c+dx)^2\sinh(a+bx)}{b^3} + \frac{(c+dx)^4\sinh(a+bx)}{b} - \frac{(24d$$
$$= -\frac{24d^3(c+dx)\cosh(a+bx)}{b^4} - \frac{4d(c+dx)^3\cosh(a+bx)}{b^2} + \frac{12d^2(c+dx)^2\sinh(a+bx)}{b^3} + \frac{(}{}$$
$$= -\frac{24d^3(c+dx)\cosh(a+bx)}{b^4} - \frac{4d(c+dx)^3\cosh(a+bx)}{b^2} + \frac{24d^4\sinh(a+bx)}{b^5} + \frac{12d^2(c+d}{}$$

Mathematica [A] time = 0.302795, size = 76, normalized size = 0.84

$$\frac{\sinh(a+bx)\left(12b^2d^2(c+dx)^2+b^4(c+dx)^4+24d^4\right)-4bd(c+dx)\cosh(a+bx)\left(b^2(c+dx)^2+6d^2\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cosh[a + b*x],x]
```

```
[Out] (-4*b*d*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + (24*d^4 + 12*b^
2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Sinh[a + b*x])/b^5
```

Maple [B] time = 0.013, size = 547, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cosh(b*x+a),x)
```

```
[Out] 1/b*(-4/b^4*d^4*a*((b*x+a)^3*sinh(b*x+a)-3*(b*x+a)^2*cosh(b*x+a)+6*(b*x+a)*
sinh(b*x+a)-6*cosh(b*x+a))+4/b^3*d^3*c*((b*x+a)^3*sinh(b*x+a)-3*(b*x+a)^2*c
osh(b*x+a)+6*(b*x+a)*sinh(b*x+a)-6*cosh(b*x+a))+6/b^4*d^4*a^2*((b*x+a)^2*si
nh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))+6/b^2*d^2*c^2*((b*x+a)^2*sin
h(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))-4/b^4*d^4*a^3*((b*x+a)*sinh(b
*x+a)-cosh(b*x+a))+4/b*d*c^3*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))-4/b^3*d^3*a^
3*c*sinh(b*x+a)+1/b^4*d^4*((b*x+a)^4*sinh(b*x+a)-4*(b*x+a)^3*cosh(b*x+a)+12
*(b*x+a)^2*sinh(b*x+a)-24*(b*x+a)*cosh(b*x+a)+24*sinh(b*x+a))+1/b^4*d^4*a^4
*sinh(b*x+a)-12/b^3*d^3*a*c*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*
sinh(b*x+a))+12/b^3*d^3*a^2*c*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))-12/b^2*d^2*
a*c^2*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))+6/b^2*d^2*a^2*c^2*sinh(b*x+a)-4/b*d
*a*c^3*sinh(b*x+a)+c^4*sinh(b*x+a))
```

Maxima [B] time = 1.24141, size = 440, normalized size = 4.84

$$\frac{c^4e^{(bx+a)}}{2\,b} + \frac{2\,(bxe^a - e^a)c^3de^{(bx)}}{b^2} - \frac{c^4e^{(-bx-a)}}{2\,b} - \frac{2\,(bx+1)c^3de^{(-bx-a)}}{b^2} + \frac{3\left(b^2x^2e^a - 2\,bxe^a + 2\,e^a\right)c^2d^2e^{(bx)}}{b^3} - \frac{3\left(b^2x^2 + 2\,bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^4*e^(b*x + a)/b + 2*(b*x*e^a - e^a)*c^3*d*e^(b*x)/b^2 - 1/2*c^4*e^(-b
*x - a)/b - 2*(b*x + 1)*c^3*d*e^(-b*x - a)/b^2 + 3*(b^2*x^2*e^a - 2*b*x*e^a
 + 2*e^a)*c^2*d^2*e^(b*x)/b^3 - 3*(b^2*x^2 + 2*b*x + 2)*c^2*d^2*e^(-b*x - a
)/b^3 + 2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*c*d^3*e^(b*x)/b
^4 - 2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*c*d^3*e^(-b*x - a)/b^4 + 1/2*(b^4*
x^4*e^a - 4*b^3*x^3*e^a + 12*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*d^4*e^(b*x)
/b^5 - 1/2*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*d^4*e^(-b*x - a
)/b^5
```

Fricas [A] time = 1.71952, size = 351, normalized size = 3.86

$$-\frac{4\left(b^3d^4x^3 + 3\,b^3cd^3x^2 + b^3c^3d + 6\,bcd^3 + 3\left(b^3c^2d^2 + 2\,bd^4\right)x\right)\cosh\left(bx + a\right) - \left(b^4d^4x^4 + 4\,b^4cd^3x^3 + b^4c^4 + 12\,b^2c^2d\right.}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d + 6*b*c*d^3 + 3*(b^3*c^2*d^2
 + 2*b*d^4)*x)*cosh(b*x + a) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 + 1
2*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d + 6
*b^2*c*d^3)*x)*sinh(b*x + a))/b^5
```

Sympy [A] time = 3.25041, size = 311, normalized size = 3.42

$$\begin{cases} \frac{c^4\sinh(a+bx)}{b} + \frac{4c^3dx\sinh(a+bx)}{b} + \frac{6c^2d^2x^2\sinh(a+bx)}{b} + \frac{4cd^3x^3\sinh(a+bx)}{b} + \frac{d^4x^4\sinh(a+bx)}{b} - \frac{4c^3d\cosh(a+bx)}{b^2} - \frac{12c^2d^2x\cosh(a+bx)}{b^2} - \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\cosh(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cosh(b*x+a),x)
```

```
[Out] Piecewise((c**4*sinh(a + b*x)/b + 4*c**3*d*x*sinh(a + b*x)/b + 6*c**2*d**2*
x**2*sinh(a + b*x)/b + 4*c*d**3*x**3*sinh(a + b*x)/b + d**4*x**4*sinh(a + b
*x)/b - 4*c**3*d*cosh(a + b*x)/b**2 - 12*c**2*d**2*x*cosh(a + b*x)/b**2 - 1
2*c*d**3*x**2*cosh(a + b*x)/b**2 - 4*d**4*x**3*cosh(a + b*x)/b**2 + 12*c**2
*d**2*sinh(a + b*x)/b**3 + 24*c*d**3*x*sinh(a + b*x)/b**3 + 12*d**4*x**2*si
nh(a + b*x)/b**3 - 24*c*d**3*cosh(a + b*x)/b**4 - 24*d**4*x*cosh(a + b*x)/b
**4 + 24*d**4*sinh(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c
**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*cosh(a), True))
```

Giac [B] time = 1.45578, size = 437, normalized size = 4.8

$$\frac{\left(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 4\,b^4c^3dx - 12\,b^3cd^3x^2 + b^4c^4 - 12\,b^3c^2d^2x + 12\,b^2d^4x^2 - 4\,b^3c^3d + 24\,b^2\right.}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*
b^4*c^3*d*x - 12*b^3*c*d^3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^
2 - 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3
 + 24*d^4)*e^(b*x + a)/b^5 - 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2
*d^2*x^2 + 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*
b^3*c^2*d^2*x + 12*b^2*d^4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*
d^2 + 24*b*d^4*x + 24*b*c*d^3 + 24*d^4)*e^(-b*x - a)/b^5
```

3.2 $\int (c+dx)^3 \cosh(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{6d^2(c+dx)\sinh(a+bx)}{b^3}-\frac{3d(c+dx)^2\cosh(a+bx)}{b^2}-\frac{6d^3\cosh(a+bx)}{b^4}+\frac{(c+dx)^3\sinh(a+bx)}{b}$$

```
[Out] (-6*d^3*Cosh[a + b*x])/b^4 - (3*d*(c + d*x)^2*Cosh[a + b*x])/b^2 + (6*d^2*(
c + d*x)*Sinh[a + b*x])/b^3 + ((c + d*x)^3*Sinh[a + b*x])/b
```

Rubi [A] time = 0.0789896, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2638}

$$\frac{6d^2(c+dx)\sinh(a+bx)}{b^3}-\frac{3d(c+dx)^2\cosh(a+bx)}{b^2}-\frac{6d^3\cosh(a+bx)}{b^4}+\frac{(c+dx)^3\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cosh[a + b*x],x]
```

```
[Out] (-6*d^3*Cosh[a + b*x])/b^4 - (3*d*(c + d*x)^2*Cosh[a + b*x])/b^2 + (6*d^2*(
c + d*x)*Sinh[a + b*x])/b^3 + ((c + d*x)^3*Sinh[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\cosh(a+bx)\,dx &= \frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{(3d)\int (c+dx)^2\sinh(a+bx)\,dx}{b}\\ &= -\frac{3d(c+dx)^2\cosh(a+bx)}{b^2}+\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{\left(6d^2\right)\int (c+dx)\cosh(a+bx)\,dx}{b^2}\\ &= -\frac{3d(c+dx)^2\cosh(a+bx)}{b^2}+\frac{6d^2(c+dx)\sinh(a+bx)}{b^3}+\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{\left(6d\right.}{}\\ &= -\frac{6d^3\cosh(a+bx)}{b^4}-\frac{3d(c+dx)^2\cosh(a+bx)}{b^2}+\frac{6d^2(c+dx)\sinh(a+bx)}{b^3}+\frac{(c+dx)^3}{}\end{aligned}$$

Mathematica [A] time = 0.193341, size = 61, normalized size = 0.87

$$\frac{b(c+dx)\sinh(a+bx)\left(b^2(c+dx)^2+6d^2\right)-3d\cosh(a+bx)\left(b^2(c+dx)^2+2d^2\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cosh[a + b*x],x]
```

```
[Out] (-3*d*(2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + b*(c + d*x)*(6*d^2 + b^2*(c
 + d*x)^2)*Sinh[a + b*x])/b^4
```

Maple [B] time = 0.006, size = 308, normalized size = 4.4

$$\frac{1}{b}\left(\frac{d^3\left((bx+a)^3\sinh(bx+a)-3\,(bx+a)^2\cosh(bx+a)+6\,(bx+a)\sinh(bx+a)-6\,\cosh(bx+a)\right)}{b^3}-3\,\frac{d^3a\left((bx+a)^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cosh(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*((b*x+a)^3*sinh(b*x+a)-3*(b*x+a)^2*cosh(b*x+a)+6*(b*x+a)*sin
h(b*x+a)-6*cosh(b*x+a))-3/b^3*d^3*a*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b
*x+a)+2*sinh(b*x+a))+3/b^2*d^2*c*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+
a)+2*sinh(b*x+a))+3/b^3*d^3*a^2*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))-6/b^2*d^2
*a*c*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))+3/b*d*c^2*((b*x+a)*sinh(b*x+a)-cosh(
b*x+a))-1/b^3*d^3*a^3*sinh(b*x+a)+3/b^2*d^2*a^2*c*sinh(b*x+a)-3/b*d*a*c^2*s
inh(b*x+a)+c^3*sinh(b*x+a))
```

Maxima [B] time = 1.28895, size = 300, normalized size = 4.29

$$\frac{c^3e^{(bx+a)}}{2\,b}+\frac{3\,(bxe^a-e^a)c^2de^{(bx)}}{2\,b^2}-\frac{c^3e^{(-bx-a)}}{2\,b}-\frac{3\,(bx+1)c^2de^{(-bx-a)}}{2\,b^2}+\frac{3\left(b^2x^2e^a-2\,bxe^a+2\,e^a\right)cd^2e^{(bx)}}{2\,b^3}-\frac{3\left(b^2x^2+2\,bx+\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*e^(b*x + a)/b + 3/2*(b*x*e^a - e^a)*c^2*d*e^(b*x)/b^2 - 1/2*c^3*e^(
-b*x - a)/b - 3/2*(b*x + 1)*c^2*d*e^(-b*x - a)/b^2 + 3/2*(b^2*x^2*e^a - 2*b
*x*e^a + 2*e^a)*c*d^2*e^(b*x)/b^3 - 3/2*(b^2*x^2 + 2*b*x + 2)*c*d^2*e^(-b*x
 - a)/b^3 + 1/2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*d^3*e^(b*
x)/b^4 - 1/2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*d^3*e^(-b*x - a)/b^4
```

Fricas [A] time = 1.9787, size = 232, normalized size = 3.31

$$-\frac{3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d+2\,d^3\right)\cosh(bx+a)-\left(b^3d^3x^3+3\,b^3cd^2x^2+b^3c^3+6\,bcd^2+3\left(b^3c^2d+2\,bd^3\right)x\right)\sinh(bx}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + 2*d^3)*cosh(b*x + a) - (b^3*
d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3*c^2*d + 2*b*d^3)*x
)*sinh(b*x + a))/b^4
```

Sympy [A] time = 1.56374, size = 202, normalized size = 2.89

$$\begin{cases} \frac{c^3\sinh(a+bx)}{b} + \frac{3c^2dx\sinh(a+bx)}{b} + \frac{3cd^2x^2\sinh(a+bx)}{b} + \frac{d^3x^3\sinh(a+bx)}{b} - \frac{3c^2d\cosh(a+bx)}{b^2} - \frac{6cd^2x\cosh(a+bx)}{b^2} - \frac{3d^3x^2\cosh(a+bx)}{b^2} + 6c \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\cosh(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cosh(b*x+a),x)
```

```
[Out] Piecewise((c**3*sinh(a + b*x)/b + 3*c**2*d*x*sinh(a + b*x)/b + 3*c*d**2*x**
2*sinh(a + b*x)/b + d**3*x**3*sinh(a + b*x)/b - 3*c**2*d*cosh(a + b*x)/b**2
 - 6*c*d**2*x*cosh(a + b*x)/b**2 - 3*d**3*x**2*cosh(a + b*x)/b**2 + 6*c*d**
2*sinh(a + b*x)/b**3 + 6*d**3*x*sinh(a + b*x)/b**3 - 6*d**3*cosh(a + b*x)/b
**4, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos
h(a), True))
```

Giac [B] time = 1.39043, size = 275, normalized size = 3.93

$$\frac{\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx - 3\,b^2d^3x^2 + b^3c^3 - 6\,b^2cd^2x - 3\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 - 6\,d^3\right)e^{(bx+a)}}{2\,b^4} - \left(b^3d^3x^3 + 3\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3*b^2*d^3*x^2 + b^3*c^
3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 6*d^3)*e^(b*x + a
)/b^4 - 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*b^2*d^3*x^2
+ b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 6*d^3)*e^
(-b*x - a)/b^4
```

3.3 $\int (c+dx)^2 \cosh(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{2d(c+dx)\cosh(a+bx)}{b^2}+\frac{2d^2\sinh(a+bx)}{b^3}+\frac{(c+dx)^2\sinh(a+bx)}{b}$$

```
[Out] (-2*d*(c + d*x)*Cosh[a + b*x])/b^2 + (2*d^2*Sinh[a + b*x])/b^3 + ((c + d*x)
^2*Sinh[a + b*x])/b
```

Rubi [A] time = 0.0468619, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2637}

$$-\frac{2d(c+dx)\cosh(a+bx)}{b^2}+\frac{2d^2\sinh(a+bx)}{b^3}+\frac{(c+dx)^2\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cosh[a + b*x],x]
```

```
[Out] (-2*d*(c + d*x)*Cosh[a + b*x])/b^2 + (2*d^2*Sinh[a + b*x])/b^3 + ((c + d*x)
^2*Sinh[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cosh(a+bx)\,dx &= \frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{(2d)\int (c+dx)\sinh(a+bx)\,dx}{b}\\ &= -\frac{2d(c+dx)\cosh(a+bx)}{b^2}+\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{\left(2d^2\right)\int \cosh(a+bx)\,dx}{b^2}\\ &= -\frac{2d(c+dx)\cosh(a+bx)}{b^2}+\frac{2d^2\sinh(a+bx)}{b^3}+\frac{(c+dx)^2\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.139609, size = 44, normalized size = 0.9

$$\frac{\sinh(a+bx)\left(b^2(c+dx)^2+2d^2\right)-2bd(c+dx)\cosh(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cosh[a + b*x],x]
```

```
[Out] (-2*b*d*(c + d*x)*Cosh[a + b*x] + (2*d^2 + b^2*(c + d*x)^2)*Sinh[a + b*x])/
b^3
```

Maple [B] time = 0.007, size = 147, normalized size = 3.

$$\frac{1}{b}\left(\frac{d^2\left((bx+a)^2\sinh(bx+a)-2(bx+a)\cosh(bx+a)+2\sinh(bx+a)\right)}{b^2}-2\frac{ad^2((bx+a)\sinh(bx+a)-\cosh(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cosh(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))-
2/b^2*d^2*a*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))+2/b*d*c*((b*x+a)*sinh(b*x+a)-
cosh(b*x+a))+1/b^2*d^2*a^2*sinh(b*x+a)-2/b*d*a*c*sinh(b*x+a)+c^2*sinh(b*x+a
))
```

Maxima [B] time = 1.19404, size = 182, normalized size = 3.71

$$\frac{c^2e^{(bx+a)}}{2b}+\frac{(bxe^a-e^a)cde^{(bx)}}{b^2}-\frac{c^2e^{(-bx-a)}}{2b}-\frac{(bx+1)cde^{(-bx-a)}}{b^2}+\frac{\left(b^2x^2e^a-2bxe^a+2e^a\right)d^2e^{(bx)}}{2b^3}-\frac{\left(b^2x^2+2bx+2\right)d^2e}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*e^(b*x + a)/b + (b*x*e^a - e^a)*c*d*e^(b*x)/b^2 - 1/2*c^2*e^(-b*x -
 a)/b - (b*x + 1)*c*d*e^(-b*x - a)/b^2 + 1/2*(b^2*x^2*e^a - 2*b*x*e^a + 2*e
^a)*d^2*e^(b*x)/b^3 - 1/2*(b^2*x^2 + 2*b*x + 2)*d^2*e^(-b*x - a)/b^3
```

Fricas [A] time = 1.90595, size = 140, normalized size = 2.86

$$-\frac{2\left(bd^2x+bcd\right)\cosh(bx+a)-\left(b^2d^2x^2+2\,b^2cdx+b^2c^2+2\,d^2\right)\sinh(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*(b*d^2*x + b*c*d)*cosh(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ 2*d^2)*sinh(b*x + a))/b^3
```

Sympy [A] time = 0.753647, size = 112, normalized size = 2.29

$$\begin{cases} \frac{c^2\sinh(a+bx)}{b}+\frac{2cdx\sinh(a+bx)}{b}+\frac{d^2x^2\sinh(a+bx)}{b}-\frac{2cd\cosh(a+bx)}{b^2}-\frac{2d^2x\cosh(a+bx)}{b^2}+\frac{2d^2\sinh(a+bx)}{b^3} & \text{for } b\neq 0 \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cosh(b*x+a),x)
```

```
[Out] Piecewise((c**2*sinh(a + b*x)/b + 2*c*d*x*sinh(a + b*x)/b + d**2*x**2*sinh(
a + b*x)/b - 2*c*d*cosh(a + b*x)/b**2 - 2*d**2*x*cosh(a + b*x)/b**2 + 2*d**
2*sinh(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cosh(a)
, True))
```

Giac [B] time = 1.31982, size = 151, normalized size = 3.08

$$\frac{\left(b^{2} d^{2} x^{2}+2 \, b^{2} c d x+b^{2} c^{2}-2 \, b d^{2} x-2 \, b c d+2 \, d^{2}\right) e^{\left(b x+a\right)}}{2 \, b^{3}}-\frac{\left(b^{2} d^{2} x^{2}+2 \, b^{2} c d x+b^{2} c^{2}+2 \, b d^{2} x+2 \, b c d+2 \, d^{2}\right) e^{\left(-b x-a\right)}}{2 \, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^2*x - 2*b*c*d + 2*d^2)*e^(
b*x + a)/b^3 - 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b*d^2*x + 2*b*c
*d + 2*d^2)*e^(-b*x - a)/b^3
```

3.4 $\int (c+dx)\cosh(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}$$

```
[Out] -((d*Cosh[a + b*x])/b^2) + ((c + d*x)*Sinh[a + b*x])/b
```

Rubi [A] time = 0.0195894, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3296, 2638}

$$\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cosh[a + b*x],x]
```

```
[Out] -((d*Cosh[a + b*x])/b^2) + ((c + d*x)*Sinh[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cosh(a+bx)\,dx &= \frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int \sinh(a+bx)\,dx}{b}\\ &= -\frac{d\cosh(a+bx)}{b^2}+\frac{(c+dx)\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0527064, size = 27, normalized size = 0.96

$$\frac{b(c+dx)\sinh(a+bx)-d\cosh(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cosh[a + b*x],x]
```

```
[Out] (-(d*Cosh[a + b*x]) + b*(c + d*x)*Sinh[a + b*x])/b^2
```

Maple [A] time = 0.007, size = 53, normalized size = 1.9

$$\frac{1}{b}\left(\frac{d\left((bx+a)\sinh\left(bx+a\right)-\cosh\left(bx+a\right)\right)}{b}-\frac{da\sinh\left(bx+a\right)}{b}+c\sinh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cosh(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))-1/b*d*a*sinh(b*x+a)+c*sinh(b*x
+a))
```

Maxima [B] time = 1.10975, size = 92, normalized size = 3.29

$$\frac{ce^{(bx+a)}}{2\,b}+\frac{(bxe^{a}-e^{a})de^{(bx)}}{2\,b^{2}}-\frac{ce^{(-bx-a)}}{2\,b}-\frac{(bx+1)de^{(-bx-a)}}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*e^(b*x + a)/b + 1/2*(b*x*e^a - e^a)*d*e^(b*x)/b^2 - 1/2*c*e^(-b*x - a
)/b - 1/2*(b*x + 1)*d*e^(-b*x - a)/b^2
```

Fricas [A] time = 1.7661, size = 73, normalized size = 2.61

$$-\frac{d\cosh\left(bx+a\right)-(bdx+bc)\sinh\left(bx+a\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(d*cosh(b*x + a) - (b*d*x + b*c)*sinh(b*x + a))/b^2
```

Sympy [A] time = 0.275544, size = 46, normalized size = 1.64

$$\begin{cases}\frac{c\sinh\left(a+bx\right)}{b}+\frac{dx\sinh\left(a+bx\right)}{b}-\frac{d\cosh\left(a+bx\right)}{b^{2}} & \text{for } b\neq 0\\ \left(cx+\frac{dx^{2}}{2}\right)\cosh\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a),x)
```

```
[Out] Piecewise((c*sinh(a + b*x)/b + d*x*sinh(a + b*x)/b - d*cosh(a + b*x)/b**2,
Ne(b, 0)), ((c*x + d*x**2/2)*cosh(a), True))
```

Giac [A] time = 1.26977, size = 62, normalized size = 2.21

$$\frac{(bdx+bc-d)e^{(bx+a)}}{2\,b^{2}}-\frac{(bdx+bc+d)e^{(-bx-a)}}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x + b*c - d)*e^(b*x + a)/b^2 - 1/2*(b*d*x + b*c + d)*e^(-b*x - a)/
b^2
```

3.5 $\int \frac{\cosh(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=51

$$\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] (Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/d + (Sinh[a - (b*c)/d]*Sinh
Integral[(b*c)/d + b*x])/d
```

Rubi [A] time = 0.100972, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3303, 3298, 3301}

$$\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x),x]
```

```
[Out] (Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/d + (Sinh[a - (b*c)/d]*Sinh
Integral[(b*c)/d + b*x])/d
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)}{c+dx}\,dx &= \cosh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\\ &= \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.0733416, size = 49, normalized size = 0.96

$$\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)+\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x),x]
```

```
[Out] (Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x] + Sinh[a - (b*c)/d]*SinhInte
gral[(b*c)/d + b*x])/d
```

Maple [A] time = 0.024, size = 82, normalized size = 1.6

$$-\frac{1}{2d}e^{-\frac{da-cb}{d}}\text{Ei}\left(1,bx+a-\frac{da-cb}{d}\right)-\frac{1}{2d}e^{\frac{da-cb}{d}}\text{Ei}\left(1,-bx-a-\frac{-da+cb}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c),x)
```

```
[Out] -1/2/d*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-1/2/d*exp((a*d-b*c)/d)*Ei(
1,-b*x-a-(-a*d+b*c)/d)
```

Maxima [A] time = 1.20383, size = 77, normalized size = 1.51

$$-\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{2d}-\frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - 1/2*e^(a - b*c/d)*
exp_integral_e(1, -(d*x + c)*b/d)/d
```

Fricas [A] time = 1.71604, size = 193, normalized size = 3.78

$$\frac{\left(\text{Ei}\left(\frac{bdx+bc}{d}\right)+\text{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(\text{Ei}\left(\frac{bdx+bc}{d}\right)-\text{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((Ei((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) + (E
i((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (a+bx)}{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)/(c + d*x), x)
```

Giac [A] time = 1.21778, size = 76, normalized size = 1.49

$$\frac{\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d
))/d
```

3.6 $\int \frac{\cosh(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=71

$$\frac{b\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\cosh(a+bx)}{d(c+dx)}$$

```
[Out] -(Cosh[a + b*x]/(d*(c + d*x))) + (b*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b
*c)/d])/d^2 + (b*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d^2
```

Rubi [A] time = 0.118507, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3298, 3301}

$$\frac{b\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\cosh(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x)^2,x]
```

```
[Out] -(Cosh[a + b*x]/(d*(c + d*x))) + (b*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b
*c)/d])/d^2 + (b*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d^2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx = -\frac{\cosh(a+bx)}{d(c+dx)} + \frac{b\int \frac{\sinh(a+bx)}{c+dx}\,dx}{d}$$
$$= -\frac{\cosh(a+bx)}{d(c+dx)} + \frac{\left(b\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d} + \frac{\left(b\sinh\left(a-\frac{bc}{d}\right)\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}$$
$$= -\frac{\cosh(a+bx)}{d(c+dx)} + \frac{b\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d^2} + \frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}$$

Mathematica [A] time = 0.243469, size = 65, normalized size = 0.92

$$\frac{b\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)+b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)-\frac{d\cosh(a+bx)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x)^2,x]
```

```
[Out] (-((d*Cosh[a + b*x])/(c + d*x)) + b*CoshIntegral[b*(c/d + x)]*Sinh[a - (b*c
)/d] + b*Cosh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)])/d^2
```

Maple [A] time = 0.033, size = 133, normalized size = 1.9

$$-\frac{be^{-bx-a}}{2d(bdx+cb)} + \frac{b}{2d^2}e^{-\frac{da-cb}{d}}\text{Ei}\left(1,bx+a-\frac{da-cb}{d}\right) - \frac{be^{bx+a}}{2d^2}\left(\frac{cb}{d}+bx\right)^{-1} - \frac{b}{2d^2}e^{\frac{da-cb}{d}}\text{Ei}\left(1,-bx-a-\frac{-da+cb}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c)^2,x)
```

```
[Out] -1/2*b*exp(-b*x-a)/d/(b*d*x+b*c)+1/2*b/d^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*
d-b*c)/d)-1/2*b/d^2*exp(b*x+a)/(b*c/d+b*x)-1/2*b/d^2*exp((a*d-b*c)/d)*Ei(1,
-b*x-a-(-a*d+b*c)/d)
```

Maxima [A] time = 1.35875, size = 109, normalized size = 1.54

$$\frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{d} - \frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{d}\right)}{2d} - \frac{\cosh(bx+a)}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - e^(a - b*c/d)*ex
p_integral_e(1, -(d*x + c)*b/d)/d)/d - cosh(b*x + a)/((d*x + c)*d)
```

Fricas [B] time = 1.76942, size = 316, normalized size = 4.45

$$-\frac{2\,d\cosh\left(bx+a\right)-\left(\left(bdx+bc\right)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\left(bdx+bc\right)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left(\left(bdx+bc\right)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\left(bdx\right.\right.}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*cosh(b*x + a) - ((b*d*x + b*c)*Ei((b*d*x + b*c)/d) - (b*d*x + b*c
)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - ((b*d*x + b*c)*Ei((b*d*x + b
*c)/d) + (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^3*x +
 c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23058, size = 201, normalized size = 2.83

$$\frac{bdx\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}-bdx\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+bc\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}-bc\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}-de^{(bx+a)}-de^{(-bx-a)}}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) - b*d*x*Ei(-(b*d*x + b*c)/d)*e
^(-a + b*c/d) + b*c*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) - b*c*Ei(-(b*d*x + b*
c)/d)*e^(-a + b*c/d) - d*e^(b*x + a) - d*e^(-b*x - a))/(d^3*x + c*d^2)
```

3.7 $\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=104

$$\frac{b^2\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b^2\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b\sinh(a+bx)}{2d^2(c+dx)}-\frac{\cosh(a+bx)}{2d(c+dx)^2}$$

```
[Out] -Cosh[a + b*x]/(2*d*(c + d*x)^2) + (b^2*Cosh[a - (b*c)/d]*CoshIntegral[(b*c
)/d + b*x])/(2*d^3) - (b*Sinh[a + b*x])/(2*d^2*(c + d*x)) + (b^2*Sinh[a - (
b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(2*d^3)
```

Rubi [A] time = 0.162841, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3298, 3301}

$$\frac{b^2\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b^2\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b\sinh(a+bx)}{2d^2(c+dx)}-\frac{\cosh(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -Cosh[a + b*x]/(2*d*(c + d*x)^2) + (b^2*Cosh[a - (b*c)/d]*CoshIntegral[(b*c
)/d + b*x])/(2*d^3) - (b*Sinh[a + b*x])/(2*d^2*(c + d*x)) + (b^2*Sinh[a - (
b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(2*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx = -\frac{\cosh(a+bx)}{2d(c+dx)^2} + \frac{b\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx}{2d}$$
$$= -\frac{\cosh(a+bx)}{2d(c+dx)^2} - \frac{b\sinh(a+bx)}{2d^2(c+dx)} + \frac{b^2\int \frac{\cosh(a+bx)}{c+dx}\,dx}{2d^2}$$
$$= -\frac{\cosh(a+bx)}{2d(c+dx)^2} - \frac{b\sinh(a+bx)}{2d^2(c+dx)} + \frac{\left(b^2\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{2d^2} + \frac{\left(b^2\sinh\left(a-\frac{bc}{d}\right)\right)\int \frac{\text{si}}{-}}{2d^2}$$
$$= -\frac{\cosh(a+bx)}{2d(c+dx)^2} + \frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{2d^3} - \frac{b\sinh(a+bx)}{2d^2(c+dx)} + \frac{b^2\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+\right.}{2d^3}$$

Mathematica [A] time = 0.525075, size = 88, normalized size = 0.85

$$\frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)+b^2\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)-\frac{d(b(c+dx)\sinh(a+bx)+d\cosh(a+bx))}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x)^3,x]
```

```
[Out] (b^2*Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] - (d*(d*Cosh[a + b*x] + b*
(c + d*x)*Sinh[a + b*x]))/(c + d*x)^2 + b^2*Sinh[a - (b*c)/d]*SinhIntegral[
b*(c/d + x)])/(2*d^3)
```

Maple [B] time = 0.039, size = 277, normalized size = 2.7

$$\frac{b^3 e^{-bx-a}x}{4d\left(b^2d^2x^2+2b^2cdx+c^2b^2\right)} + \frac{b^3 e^{-bx-a}c}{4d^2\left(b^2d^2x^2+2b^2cdx+c^2b^2\right)} - \frac{b^2 e^{-bx-a}}{4d\left(b^2d^2x^2+2b^2cdx+c^2b^2\right)} - \frac{b^2}{4d^3}e^{-\frac{da-cb}{d}}\text{Ei}\left(1, bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c)^3,x)
```

```
[Out] 1/4*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x+1/4*b^3*exp(-b*x-
a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/4*b^2*exp(-b*x-a)/d/(b^2*d^2*x
^2+2*b^2*c*d*x+b^2*c^2)-1/4*b^2/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/
d)-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)^2-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)-
1/4*b^2/d^3*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)
```

Maxima [A] time = 1.21465, size = 128, normalized size = 1.23

$$\frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{(dx+c)d} - \frac{e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{(dx+c)d}\right)}{4d} - \frac{\cosh(bx+a)}{2(dx+c)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*(e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) - e^(a
 - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d))/d - 1/2*cosh(b*x
 + a)/((d*x + c)^2*d)
```

Fricas [B] time = 1.84861, size = 518, normalized size = 4.98

$$-\frac{2\,d^2\cosh\left(bx+a\right)-\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)+2}{4\left(d^5x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*cosh(b*x + a) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*
x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*c
osh(-(b*c - a*d)/d) + 2*(b*d^2*x + b*c*d)*sinh(b*x + a) - ((b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c
^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25575, size = 402, normalized size = 3.87

$$\frac{b^2d^2x^2\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+b^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+2\,b^2cdx\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+2\,b^2cdx\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+b^2c^2}{4\left(d^5x^2+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*d^2*x^2*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + b^2*d^2*x^2*Ei(-(b*d*x
 + b*c)/d)*e^(-a + b*c/d) + 2*b^2*c*d*x*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) +
 2*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + b^2*c^2*Ei((b*d*x + b*c)
/d)*e^(a - b*c/d) + b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - b*d^2*x*e
^(b*x + a) + b*d^2*x*e^(-b*x - a) - b*c*d*e^(b*x + a) + b*c*d*e^(-b*x - a)
- d^2*e^(b*x + a) - d^2*e^(-b*x - a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.8 $\int (c+dx)^4 \cosh^2(a+bx)\,dx$

Optimal. Leaf size=162

$$-\frac{3d^3(c+dx)\cosh^2(a+bx)}{2b^4}+\frac{3d^2(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b^3}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{3d^4\sinh(a+bx)}{4b^5}$$

```
[Out] (3*d^4*x)/(4*b^4) + (d*(c + d*x)^3)/(2*b^2) + (c + d*x)^5/(10*d) - (3*d^3*(
c + d*x)*Cosh[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Cosh[a + b*x]^2)/b^2 + (
3*d^4*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^5) + (3*d^2*(c + d*x)^2*Cosh[a + b*
x]*Sinh[a + b*x])/(2*b^3) + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rubi [A] time = 0.10147, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 32, 2635, 8}

$$-\frac{3d^3(c+dx)\cosh^2(a+bx)}{2b^4}+\frac{3d^2(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b^3}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{3d^4\sinh(a+bx)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cosh[a + b*x]^2,x]
```


```
[Out] (3*d^4*x)/(4*b^4) + (d*(c + d*x)^3)/(2*b^2) + (c + d*x)^5/(10*d) - (3*d^3*(
c + d*x)*Cosh[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Cosh[a + b*x]^2)/b^2 + (
3*d^4*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^5) + (3*d^2*(c + d*x)^2*Cosh[a + b*
x]*Sinh[a + b*x])/(2*b^3) + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^4\cosh^2(a+bx)\,dx = -\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{(c+dx)^4\cosh(a+bx)\sinh(a+bx)}{2b}+\frac{1}{2}\int (c+dx)^4\,dx+$$

$$=\frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cosh^2(a+bx)}{2b^4}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{3d^2(c+dx)^2\cosh(a}{2b}$$

$$=\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cosh^2(a+bx)}{2b^4}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+3d^4\text{co}$$

$$=\frac{3d^4x}{4b^4}+\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cosh^2(a+bx)}{2b^4}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}$$

Mathematica [A] time = 0.615663, size = 132, normalized size = 0.81

$$\frac{10\sinh(2(a+bx))\left(6b^2d^2(c+dx)^2+2b^4(c+dx)^4+3d^4\right)-20bd(c+dx)\cosh(2(a+bx))\left(2b^2(c+dx)^2+3d^2\right)+8b^5x\,(10}{80b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cosh[a + b*x]^2,x]
```

```
[Out] (8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) - 20
*b*d*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + 10*(3*d^4 +
6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sinh[2*(a + b*x)])/(80*b^5)
```

Maple [B] time = 0.01, size = 910, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cosh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/2*(b*x+a)^4*cosh(b*x+a)*sinh(b*x+a)+1/10*(b*x+a)^5-(b*x+a
)^3*cosh(b*x+a)^2+3/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)+1/2*(b*x+a)^3-3/2*(
b*x+a)*cosh(b*x+a)^2+3/4*cosh(b*x+a)*sinh(b*x+a)+3/4*b*x+3/4*a)+1/b^4*d^4*a
^4*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)+12/b^3*d^3*c*a^2*(1/2*(b*x+a
)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)-12/b^2*d^2*c^2*a
*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)-12/b
^3*d^3*c*a*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)+1/6*(b*x+a)^3-1/2*(b*x+a)
*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)+4/b*d*c^3*(1/2*(b
*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)-4/b^3*d^3*a^
3*c*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)+6/b^2*d^2*a^2*c^2*(1/2*cosh
(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)-4/b*d*a*c^3*(1/2*cosh(b*x+a)*sinh(b*x+a)
+1/2*b*x+1/2*a)-4/b^4*d^4*a^3*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x
+a)^2-1/4*cosh(b*x+a)^2)+4/b^3*d^3*c*(1/2*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)
+1/8*(b*x+a)^4-3/4*(b*x+a)^2*cosh(b*x+a)^2+3/4*(b*x+a)*cosh(b*x+a)*sinh(b*x
+a)+3/8*(b*x+a)^2-3/8*cosh(b*x+a)^2)+6/b^2*d^2*c^2*(1/2*(b*x+a)^2*cosh(b*x+
a)*sinh(b*x+a)+1/6*(b*x+a)^3-1/2*(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh
(b*x+a)+1/4*b*x+1/4*a)+6/b^4*d^4*a^2*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)
+1/6*(b*x+a)^3-1/2*(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*
x+1/4*a)-4/b^4*d^4*a*(1/2*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)+1/8*(b*x+a)^4-3
/4*(b*x+a)^2*cosh(b*x+a)^2+3/4*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+3/8*(b*x+a)^
2-3/8*cosh(b*x+a)^2)+c^4*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [B] time = 1.12478, size = 516, normalized size = 3.19

$$\frac{1}{4}\left(4\,x^2+\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{b^2}\right)c^3d+\frac{1}{8}\left(8\,x^3+\frac{3\left(2\,b^2x^2e^{(2\,a)}-2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}}{b^3}-\frac{3\,(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*x^2 + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 - (2*b*x + 1)*e^(-2*b*
x - 2*a)/b^2)*c^3*d + 1/8*(8*x^3 + 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e
^(2*a))*e^(2*b*x)/b^3 - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*c^2
*d^2 + 1/8*(4*x^4 + (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a)
- 3*e^(2*a))*e^(2*b*x)/b^4 - (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x
- 2*a)/b^4)*c*d^3 + 1/80*(8*x^5 + 5*(2*b^4*x^4*e^(2*a) - 4*b^3*x^3*e^(2*a)
+ 6*b^2*x^2*e^(2*a) - 6*b*x*e^(2*a) + 3*e^(2*a))*e^(2*b*x)/b^5 - 5*(2*b^4*x
^4 + 4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^5)*d^4 + 1/8*c^4
*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)
```

Fricas [B] time = 1.87598, size = 662, normalized size = 4.09

$$\frac{2\,b^5d^4x^5+10\,b^5cd^3x^4+20\,b^5c^2d^2x^3+20\,b^5c^3dx^2+10\,b^5c^4x-5\left(2\,b^3d^4x^3+6\,b^3cd^3x^2+2\,b^3c^3d+3\,bcd^3+3\left(2\,b^3c^2d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 20*b^5*c^2*d^2*x^3 + 20*b^5*c^3*d*
x^2 + 10*b^5*c^4*x - 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*b
*c*d^3 + 3*(2*b^3*c^2*d^2 + b*d^4)*x)*cosh(b*x + a)^2 + 5*(2*b^4*d^4*x^4 +
8*b^4*c*d^3*x^3 + 2*b^4*c^4 + 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 + b^
2*d^4)*x^2 + 4*(2*b^4*c^3*d + 3*b^2*c*d^3)*x)*cosh(b*x + a)*sinh(b*x + a) -
 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*b*c*d^3 + 3*(2*b^3*c^
2*d^2 + b*d^4)*x)*sinh(b*x + a)^2)/b^5
```

Sympy [A] time = 6.52616, size = 660, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-c**4*x*sinh(a + b*x)**2/2 + c**4*x*cosh(a + b*x)**2/2 - c**3*d*
x**2*sinh(a + b*x)**2 + c**3*d*x**2*cosh(a + b*x)**2 - c**2*d**2*x**3*sinh(
a + b*x)**2 + c**2*d**2*x**3*cosh(a + b*x)**2 - c*d**3*x**4*sinh(a + b*x)**
2/2 + c*d**3*x**4*cosh(a + b*x)**2/2 - d**4*x**5*sinh(a + b*x)**2/10 + d**4
*x**5*cosh(a + b*x)**2/10 + c**4*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 2*c**3
*d*x*sinh(a + b*x)*cosh(a + b*x)/b + 3*c**2*d**2*x**2*sinh(a + b*x)*cosh(a
+ b*x)/b + 2*c*d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/b + d**4*x**4*sinh(a +
 b*x)*cosh(a + b*x)/(2*b) - c**3*d*sinh(a + b*x)**2/b**2 - 3*c**2*d**2*x*si
```

```
nh(a + b*x)**2/(2*b**2) - 3*c**2*d**2*x*cosh(a + b*x)**2/(2*b**2) - 3*c*d**
3*x**2*sinh(a + b*x)**2/(2*b**2) - 3*c*d**3*x**2*cosh(a + b*x)**2/(2*b**2)
- d**4*x**3*sinh(a + b*x)**2/(2*b**2) - d**4*x**3*cosh(a + b*x)**2/(2*b**2)
 + 3*c**2*d**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) + 3*c*d**3*x*sinh(a + b
*x)*cosh(a + b*x)/b**3 + 3*d**4*x**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) -
 3*c*d**3*sinh(a + b*x)**2/(2*b**4) - 3*d**4*x*sinh(a + b*x)**2/(4*b**4) -
3*d**4*x*cosh(a + b*x)**2/(4*b**4) + 3*d**4*sinh(a + b*x)*cosh(a + b*x)/(4*
b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4
 + d**4*x**5/5)*cosh(a)**2, True))
```

Giac [B] time = 1.17811, size = 502, normalized size = 3.1

$$\frac{1}{10}\,d^4x^5 + \frac{1}{2}\,cd^3x^4 + c^2d^2x^3 + c^3dx^2 + \frac{1}{2}\,c^4x + \frac{\left(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 8\,b^4c^3dx - 12\,b^3cd^3x^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*d^4*x^5 + 1/2*c*d^3*x^4 + c^2*d^2*x^3 + c^3*d*x^2 + 1/2*c^4*x + 1/16*(
2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 8*b^
4*c^3*d*x - 12*b^3*c*d^3*x^2 + 2*b^4*c^4 - 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^2
 - 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*c^2*d^2 - 6*b*d^4*x - 6*b*c*d^3 + 3
*d^4)*e^(2*b*x + 2*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*
c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 8*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + 2*b^4*c^4
+ 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^2 + 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*c
^2*d^2 + 6*b*d^4*x + 6*b*c*d^3 + 3*d^4)*e^(-2*b*x - 2*a)/b^5
```

3.9 $\int (c+dx)^3 \cosh^2(a+bx)\,dx$

Optimal. Leaf size=134

$$\frac{3d^2(c+dx)\sinh(a+bx)\cosh(a+bx)}{4b^3}-\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2}-\frac{3d^3\cosh^2(a+bx)}{8b^4}+\frac{(c+dx)^3\sinh(a+bx)\cos}{2b}$$

```
[Out] (3*c*d^2*x)/(4*b^2) + (3*d^3*x^2)/(8*b^2) + (c + d*x)^4/(8*d) - (3*d^3*Cosh
[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Cosh[a + b*x]^2)/(4*b^2) + (3*d^2*(
c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^3*Cosh[a + b*x]*
Sinh[a + b*x])/(2*b)
```

Rubi [A] time = 0.0727015, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3311, 32, 3310}

$$\frac{3d^2(c+dx)\sinh(a+bx)\cosh(a+bx)}{4b^3}-\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2}-\frac{3d^3\cosh^2(a+bx)}{8b^4}+\frac{(c+dx)^3\sinh(a+bx)\cos}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cosh[a + b*x]^2,x]
```


```
[Out] (3*c*d^2*x)/(4*b^2) + (3*d^3*x^2)/(8*b^2) + (c + d*x)^4/(8*d) - (3*d^3*Cosh
[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Cosh[a + b*x]^2)/(4*b^2) + (3*d^2*(
c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^3*Cosh[a + b*x]*
Sinh[a + b*x])/(2*b)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^3\cosh^2(a+bx)\,dx = -\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2} + \frac{(c+dx)^3\cosh(a+bx)\sinh(a+bx)}{2b} + \frac{1}{2}\int (c+dx)^3\,dx +$$
$$= \frac{(c+dx)^4}{8d} - \frac{3d^3\cosh^2(a+bx)}{8b^4} - \frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2} + \frac{3d^2(c+dx)\cosh(a+bx)\,\mathrm{s}}{4b^3}$$
$$= \frac{3cd^2x}{4b^2} + \frac{3d^3x^2}{8b^2} + \frac{(c+dx)^4}{8d} - \frac{3d^3\cosh^2(a+bx)}{8b^4} - \frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2} + \frac{3d^2(c+}{}$$

Mathematica [A] time = 0.410089, size = 104, normalized size = 0.78

$$\frac{2b(c+dx)\sinh(2(a+bx))\left(2b^2(c+dx)^2+3d^2\right) - 3d\cosh(2(a+bx))\left(2b^2(c+dx)^2+d^2\right) + 2b^4x\left(6c^2dx+4c^3+4cd^2x^2+\right.}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cosh[a + b*x]^2,x]
```

```
[Out] (2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(d^2 + 2*b^2*(c
+ d*x)^2)*Cosh[2*(a + b*x)] + 2*b*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Sin
h[2*(a + b*x)])/(16*b^4)
```

Maple [B] time = 0.01, size = 523, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cosh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/2*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)+1/8*(b*x+a)^4-3/4*(b*
x+a)^2*cosh(b*x+a)^2+3/4*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+3/8*(b*x+a)^2-3/8*
cosh(b*x+a)^2)-3/b^3*d^3*a*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)+1/6*(b*x+
a)^3-1/2*(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)+3
/b^3*d^3*a^2*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*
x+a)^2)-1/b^3*d^3*a^3*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)+3/b^2*c*d
^2*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)+1/6*(b*x+a)^3-1/2*(b*x+a)*cosh(b*
x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)-6/b^2*c*d^2*a*(1/2*(b*x+a
)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+3/b^2*c*d^2*a^2*
(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)+3/b*c^2*d*(1/2*(b*x+a)*cosh(b*x
+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)-3/b*c^2*d*a*(1/2*cosh(b*x+
a)*sinh(b*x+a)+1/2*b*x+1/2*a)+c^3*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*
a))
```

Maxima [B] time = 1.10515, size = 355, normalized size = 2.65

$$\frac{3}{16}\left(4x^2 + \frac{\left(2bxe^{(2a)} - e^{(2a)}\right)e^{(2bx)}}{b^2} - \frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)c^2d + \frac{1}{16}\left(8x^3 + \frac{3\left(2b^2x^2e^{(2a)} - 2bxe^{(2a)} + e^{(2a)}\right)e^{(2bx)}}{b^3} - \frac{3\left(2b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/16*(4*x^2 + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 - (2*b*x + 1)*e^(-2*b
*x - 2*a)/b^2)*c^2*d + 1/16*(8*x^3 + 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) +
 e^(2*a))*e^(2*b*x)/b^3 - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*c
*d^2 + 1/32*(4*x^4 + (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a)
 - 3*e^(2*a))*e^(2*b*x)/b^4 - (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x
 - 2*a)/b^4)*d^3 + 1/8*c^3*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)
```

Fricas [A] time = 1.79494, size = 458, normalized size = 3.42

$$\frac{2\,b^4d^3x^4 + 8\,b^4cd^2x^3 + 12\,b^4c^2dx^2 + 8\,b^4c^3x - 3\left(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d + d^3\right)\cosh\left(bx+a\right)^2 + 4\left(2\,b^3d^3x^3 + 6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*b^4*d^3*x^4 + 8*b^4*c*d^2*x^3 + 12*b^4*c^2*d*x^2 + 8*b^4*c^3*x - 3*
(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x + a)^2 + 4*(2*
b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*
d^3)*x)*cosh(b*x + a)*sinh(b*x + a) - 3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*
b^2*c^2*d + d^3)*sinh(b*x + a)^2)/b^4
```

Sympy [A] time = 3.50572, size = 456, normalized size = 3.4

$$\begin{cases} -\frac{c^3x\sinh^2(a+bx)}{2} + \frac{c^3x\cosh^2(a+bx)}{2} - \frac{3c^2dx^2\sinh^2(a+bx)}{4} + \frac{3c^2dx^2\cosh^2(a+bx)}{4} - \frac{cd^2x^3\sinh^2(a+bx)}{2} + \frac{cd^2x^3\cosh^2(a+bx)}{2} - \frac{d^3x^4\sinh^2(a-}{8} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\cosh^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-c**3*x*sinh(a + b*x)**2/2 + c**3*x*cosh(a + b*x)**2/2 - 3*c**2*
d*x**2*sinh(a + b*x)**2/4 + 3*c**2*d*x**2*cosh(a + b*x)**2/4 - c*d**2*x**3*
sinh(a + b*x)**2/2 + c*d**2*x**3*cosh(a + b*x)**2/2 - d**3*x**4*sinh(a + b*
x)**2/8 + d**3*x**4*cosh(a + b*x)**2/8 + c**3*sinh(a + b*x)*cosh(a + b*x)/(
2*b) + 3*c**2*d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 3*c*d**2*x**2*sinh(a
+ b*x)*cosh(a + b*x)/(2*b) + d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/(2*b) -
3*c**2*d*sinh(a + b*x)**2/(4*b**2) - 3*c*d**2*x*sinh(a + b*x)**2/(4*b**2) -
 3*c*d**2*x*cosh(a + b*x)**2/(4*b**2) - 3*d**3*x**2*sinh(a + b*x)**2/(8*b**
2) - 3*d**3*x**2*cosh(a + b*x)**2/(8*b**2) + 3*c*d**2*sinh(a + b*x)*cosh(a
+ b*x)/(4*b**3) + 3*d**3*x*sinh(a + b*x)*cosh(a + b*x)/(4*b**3) - 3*d**3*si
nh(a + b*x)**2/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**
3 + d**3*x**4/4)*cosh(a)**2, True))
```

Giac [B] time = 1.37301, size = 328, normalized size = 2.45

$$\frac{1}{8}\,d^3x^4 + \frac{1}{2}\,cd^2x^3 + \frac{3}{4}\,c^2dx^2 + \frac{1}{2}\,c^3x + \frac{\left(4\,b^3d^3x^3 + 12\,b^3cd^2x^2 + 12\,b^3c^2dx - 6\,b^2d^3x^2 + 4\,b^3c^3 - 12\,b^2cd^2x - 6\,b^2c^2d +\right.}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*d^3*x^4 + 1/2*c*d^2*x^3 + 3/4*c^2*d*x^2 + 1/2*c^3*x + 1/32*(4*b^3*d^3*x
^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 6*b^2*d^3*x^2 + 4*b^3*c^3 - 12*b^2
*c*d^2*x - 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 3*d^3)*e^(2*b*x + 2*a)/b^4
 - 1/32*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 6*b^2*d^3*x^2
+ 4*b^3*c^3 + 12*b^2*c*d^2*x + 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 3*d^3)
*e^(-2*b*x - 2*a)/b^4
```

3.10 $\int (c+dx)^2 \cosh^2(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{d^2\sinh(a+bx)\cosh(a+bx)}{4b^3}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}$$

```
[Out] (d^2*x)/(4*b^2) + (c + d*x)^3/(6*d) - (d*(c + d*x)*Cosh[a + b*x]^2)/(2*b^2)
 + (d^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^2*Cosh[a + b*x]*S
inh[a + b*x])/(2*b)
```

Rubi [A] time = 0.0527716, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 32, 2635, 8}

$$-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{d^2\sinh(a+bx)\cosh(a+bx)}{4b^3}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cosh[a + b*x]^2,x]
```

```
[Out] (d^2*x)/(4*b^2) + (c + d*x)^3/(6*d) - (d*(c + d*x)*Cosh[a + b*x]^2)/(2*b^2)
 + (d^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^2*Cosh[a + b*x]*S
inh[a + b*x])/(2*b)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^2\cosh^2(a+bx)\,dx = -\frac{d(c+dx)\cosh^2(a+bx)}{2b^2} + \frac{(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b} + \frac{1}{2}\int (c+dx)^2\,dx + \frac{d^2}{}$$
$$= \frac{(c+dx)^3}{6d} - \frac{d(c+dx)\cosh^2(a+bx)}{2b^2} + \frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^2\cosh(a}{2}$$
$$= \frac{d^2x}{4b^2} + \frac{(c+dx)^3}{6d} - \frac{d(c+dx)\cosh^2(a+bx)}{2b^2} + \frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^2 c}{}$$

Mathematica [A] time = 0.278099, size = 75, normalized size = 0.79

$$\frac{3\sinh(2(a+bx))\left(2b^2(c+dx)^2+d^2\right)-6bd(c+dx)\cosh(2(a+bx))+4b^3x\left(3c^2+3cdx+d^2x^2\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cosh[a + b*x]^2,x]
```

```
[Out] (4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 6*b*d*(c + d*x)*Cosh[2*(a + b*x)] +
3*(d^2 + 2*b^2*(c + d*x)^2)*Sinh[2*(a + b*x)])/(24*b^3)
```

Maple [B] time = 0.008, size = 262, normalized size = 2.8

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left(\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{2}+\frac{(bx+a)^3}{6}-\frac{(bx+a)(\cosh(bx+a))^2}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{4}+\frac{bx}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cosh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)+1/6*(b*x+a)^3-1/2*(b*
x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)-2/b^2*d^2*a*(
1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+1/b^2*
d^2*a^2*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)+2/b*c*d*(1/2*(b*x+a)*co
sh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)-2/b*c*d*a*(1/2*cosh(
b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)+c^2*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+
1/2*a))
```

Maxima [A] time = 1.14336, size = 223, normalized size = 2.35

$$\frac{1}{8}\left(4x^2+\frac{\left(2bxe^{(2a)}-e^{(2a)}\right)e^{(2bx)}}{b^2}-\frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)cd+\frac{1}{48}\left(8x^3+\frac{3\left(2b^2x^2e^{(2a)}-2bxe^{(2a)}+e^{(2a)}\right)e^{(2bx)}}{b^3}-\frac{3\left(2b^2x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(4*x^2 + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 - (2*b*x + 1)*e^(-2*b*
x - 2*a)/b^2)*c*d + 1/48*(8*x^3 + 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^
(2*a))*e^(2*b*x)/b^3 - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*d^2
+ 1/8*c^2*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)
```

Fricas [A] time = 1.81044, size = 286, normalized size = 3.01

$$\frac{2\,b^3d^2x^3+6\,b^3cdx^2+6\,b^3c^2x-3\left(bd^2x+bcd\right)\cosh\left(bx+a\right)^2+3\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2+d^2\right)\cosh\left(bx+a\right)\sin}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x - 3*(b*d^2*x + b*c*d)*cos
h(b*x + a)^2 + 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cosh(b*x +
 a)*sinh(b*x + a) - 3*(b*d^2*x + b*c*d)*sinh(b*x + a)^2)/b^3
```

Sympy [A] time = 1.62463, size = 264, normalized size = 2.78

$$\begin{cases} -\frac{c^2x\sinh^2(a+bx)}{2}+\frac{c^2x\cosh^2(a+bx)}{2}-\frac{cdx^2\sinh^2(a+bx)}{2}+\frac{cdx^2\cosh^2(a+bx)}{2}-\frac{d^2x^3\sinh^2(a+bx)}{6}+\frac{d^2x^3\cosh^2(a+bx)}{6}+\frac{c^2\sinh(a+bx)\cosh}{2b} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\cosh^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-c**2*x*sinh(a + b*x)**2/2 + c**2*x*cosh(a + b*x)**2/2 - c*d*x**
2*sinh(a + b*x)**2/2 + c*d*x**2*cosh(a + b*x)**2/2 - d**2*x**3*sinh(a + b*x
)**2/6 + d**2*x**3*cosh(a + b*x)**2/6 + c**2*sinh(a + b*x)*cosh(a + b*x)/(2
*b) + c*d*x*sinh(a + b*x)*cosh(a + b*x)/b + d**2*x**2*sinh(a + b*x)*cosh(a
+ b*x)/(2*b) - c*d*sinh(a + b*x)**2/(2*b**2) - d**2*x*sinh(a + b*x)**2/(4*b
**2) - d**2*x*cosh(a + b*x)**2/(4*b**2) + d**2*sinh(a + b*x)*cosh(a + b*x)/
(4*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cosh(a)**2, True))
```

Giac [A] time = 1.3331, size = 184, normalized size = 1.94

$$\frac{1}{6}\,d^2x^3+\frac{1}{2}\,cdx^2+\frac{1}{2}\,c^2x+\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-2\,bd^2x-2\,bcd+d^2\right)e^{(2\,bx+2\,a)}}{16\,b^3}-\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*d^2*x^3 + 1/2*c*d*x^2 + 1/2*c^2*x + 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x +
 2*b^2*c^2 - 2*b*d^2*x - 2*b*c*d + d^2)*e^(2*b*x + 2*a)/b^3 - 1/16*(2*b^2*d
^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*b*d^2*x + 2*b*c*d + d^2)*e^(-2*b*x - 2
*a)/b^3
```

3.11 $\int (c+dx)\cosh^2(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{cx}{2}+\frac{dx^2}{4}$$

```
[Out] (c*x)/2 + (d*x^2)/4 - (d*Cosh[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cosh[a + b*x
]*Sinh[a + b*x])/(2*b)
```

Rubi [A] time = 0.0249827, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3310}

$$-\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{cx}{2}+\frac{dx^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cosh[a + b*x]^2,x]
```

```
[Out] (c*x)/2 + (d*x^2)/4 - (d*Cosh[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cosh[a + b*x
]*Sinh[a + b*x])/(2*b)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cosh^2(a+bx)\,dx &= -\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b}+\frac{1}{2}\int (c+dx)\,dx\\ &= \frac{cx}{2}+\frac{dx^2}{4}-\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.15492, size = 51, normalized size = 0.93

$$\frac{2b((c+dx)\sinh(2(a+bx))+2ac+bx(2c+dx))-d\cosh(2(a+bx))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cosh[a + b*x]^2,x]
```

```
[Out] (-(d*Cosh[2*(a + b*x)]) + 2*b*(2*a*c + b*x*(2*c + d*x) + (c + d*x)*Sinh[2*(
a + b*x)]))/(8*b^2)
```

Maple [B] time = 0.008, size = 103, normalized size = 1.9

$$\frac{1}{b}\left(\frac{d}{b}\left(\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{2}+\frac{(bx+a)^2}{4}-\frac{(\cosh(bx+a))^2}{4}\right)-\frac{da}{b}\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cosh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b*d*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+
a)^2)-1/b*d*a*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)+c*(1/2*cosh(b*x+a
)*sinh(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [A] time = 1.06472, size = 119, normalized size = 2.16

$$\frac{1}{16}\left(4x^2+\frac{\left(2bxe^{(2a)}-e^{(2a)}\right)e^{(2bx)}}{b^2}-\frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)d+\frac{1}{8}c\left(4x+\frac{e^{(2bx+2a)}}{b}-\frac{e^{(-2bx-2a)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*x^2 + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 - (2*b*x + 1)*e^(-2*b
*x - 2*a)/b^2)*d + 1/8*c*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)
```

Fricas [A] time = 1.82963, size = 163, normalized size = 2.96

$$\frac{2b^2dx^2+4b^2cx-d\cosh(bx+a)^2+4(bdx+bc)\cosh(bx+a)\sinh(bx+a)-d\sinh(bx+a)^2}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*b^2*d*x^2 + 4*b^2*c*x - d*cosh(b*x + a)^2 + 4*(b*d*x + b*c)*cosh(b*x
 + a)*sinh(b*x + a) - d*sinh(b*x + a)^2)/b^2
```

Sympy [A] time = 0.673685, size = 126, normalized size = 2.29

$$\begin{cases}-\frac{cx\sinh^2(a+bx)}{2}+\frac{cx\cosh^2(a+bx)}{2}-\frac{dx^2\sinh^2(a+bx)}{4}+\frac{dx^2\cosh^2(a+bx)}{4}+\frac{c\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{dx\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{d\sinh^2}{4b} \\ \left(cx+\frac{dx^2}{2}\right)\cosh^2(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-c*x*sinh(a + b*x)**2/2 + c*x*cosh(a + b*x)**2/2 - d*x**2*sinh(a
 + b*x)**2/4 + d*x**2*cosh(a + b*x)**2/4 + c*sinh(a + b*x)*cosh(a + b*x)/(2
*b) + d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) - d*sinh(a + b*x)**2/(4*b**2),
```

```
Ne(b, 0)), ((c*x + d*x**2/2)*cosh(a)**2, True))
```

Giac [A] time = 1.25421, size = 85, normalized size = 1.55

$$\frac{1}{4}dx^2 + \frac{1}{2}cx + \frac{(2\,bdx + 2\,bc - d)e^{(2\,bx+2\,a)}}{16\,b^2} - \frac{(2\,bdx + 2\,bc + d)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*d*x^2 + 1/2*c*x + 1/16*(2*b*d*x + 2*b*c - d)*e^(2*b*x + 2*a)/b^2 - 1/16
*(2*b*d*x + 2*b*c + d)*e^(-2*b*x - 2*a)/b^2
```

3.12 $\int \frac{\cosh^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=78

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}$$

```
[Out] (Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*x
]/(2*d) + (Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rubi [A] time = 0.154454, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3303, 3298, 3301}

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x),x]
```

```
[Out] (Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*x
]/(2*d) + (Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{1}{2(c+dx)}+\frac{\cosh(2a+2bx)}{2(c+dx)}\right)dx \\
&= \frac{\log(c+dx)}{2d}+\frac{1}{2}\int \frac{\cosh(2a+2bx)}{c+dx}\,dx \\
&= \frac{\log(c+dx)}{2d}+\frac{1}{2}\cosh\left(2a-\frac{2bc}{d}\right)\int \frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\frac{1}{2}\sinh\left(2a-\frac{2bc}{d}\right)\int \frac{\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx} \\
&= \frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.114101, size = 64, normalized size = 0.82

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2b(c+dx)}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)+\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x),x]
```

```
[Out] (Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] + Sin
h[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d)
```

Maple [A] time = 0.071, size = 97, normalized size = 1.2

$$\frac{\ln(dx+c)}{2d}-\frac{1}{4d}e^{-2\frac{da-cb}{d}}\text{Ei}\left(1,2bx+2a-2\frac{da-cb}{d}\right)-\frac{1}{4d}e^{2\frac{da-cb}{d}}\text{Ei}\left(1,-2bx-2a-2\frac{-da+cb}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/2*ln(d*x+c)/d-1/4/d*exp(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/4
/d*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Maxima [A] time = 1.34529, size = 97, normalized size = 1.24

$$-\frac{e^{\left(-2a+\frac{2bc}{d}\right)}E_1\left(\frac{2(dx+c)b}{d}\right)}{4d}-\frac{e^{\left(2a-\frac{2bc}{d}\right)}E_1\left(-\frac{2(dx+c)b}{d}\right)}{4d}+\frac{\log(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(1, 2*(d*x + c)*b/d)/d - 1/4*e^(2*a -
 2*b*c/d)*exp_integral_e(1, -2*(d*x + c)*b/d)/d + 1/2*log(d*x + c)/d
```

Fricas [A] time = 1.78312, size = 232, normalized size = 2.97

$$\frac{\left(\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)+\left(\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-ad)}{d}\right)+2\,\log\left(dx+\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*((Ei(2*(b*d*x + b*c)/d) + Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/d
) + (Ei(2*(b*d*x + b*c)/d) - Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d)
 + 2*log(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 1.29424, size = 92, normalized size = 1.18

$$\frac{\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}+2\,\log\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + Ei(-2*(b*d*x + b*c)/d)*e^(-2
*a + 2*b*c/d) + 2*log(d*x + c))/d
```

3.13 $\int \frac{\cosh^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=81

$$\frac{b\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\cosh^2(a+bx)}{d(c+dx)}$$

```
[Out] -(Cosh[a + b*x]^2/(d*(c + d*x))) + (b*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[
2*a - (2*b*c)/d])/d^2 + (b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2
*b*x])/d^2
```

Rubi [A] time = 0.145647, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3313, 12, 3303, 3298, 3301}

$$\frac{b\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\cosh^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^2,x]
```


```
[Out] -(Cosh[a + b*x]^2/(d*(c + d*x))) + (b*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[
2*a - (2*b*c)/d])/d^2 + (b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2
*b*x])/d^2
```


Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx = -\frac{\cosh^2(a+bx)}{d(c+dx)} + \frac{(2ib)\int -\frac{i\sinh(2a+2bx)}{2(c+dx)}\,dx}{d}$$
$$= -\frac{\cosh^2(a+bx)}{d(c+dx)} + \frac{b\int \frac{\sinh(2a+2bx)}{c+dx}\,dx}{d}$$
$$= -\frac{\cosh^2(a+bx)}{d(c+dx)} + \frac{\left(b\cosh\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} + \frac{\left(b\sinh\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,d}{d}$$
$$= -\frac{\cosh^2(a+bx)}{d(c+dx)} + \frac{b\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{d^2} + \frac{b\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

Mathematica [A] time = 0.401756, size = 75, normalized size = 0.93

$$\frac{b\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2b(c+dx)}{d}\right)+b\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2b(c+dx)}{d}\right)-\frac{d\cosh^2(a+bx)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] (-((d*Cosh[a + b*x]^2)/(c + d*x)) + b*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[
2*a - (2*b*c)/d] + b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])
/d^2
```

Maple [A] time = 0.076, size = 152, normalized size = 1.9

$$-\frac{1}{2d(dx+c)} - \frac{be^{-2bx-2a}}{(4bdx+4cb)d} + \frac{b}{2d^2}e^{-2\frac{da-cb}{d}}\mathrm{Ei}\left(1, 2bx+2a-2\frac{da-cb}{d}\right) - \frac{be^{2bx+2a}}{4d^2}\left(\frac{cb}{d}+bx\right)^{-1} - \frac{b}{2d^2}e^{2\frac{da-cb}{d}}\mathrm{Ei}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] -1/2/d/(d*x+c)-1/4*b*exp(-2*b*x-2*a)/(b*d*x+b*c)/d+1/2*b/d^2*exp(-2*(a*d-b*
c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/4*b/d^2*exp(2*b*x+2*a)/(b*c/d+b*x)-1/
2*b/d^2*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Maxima [A] time = 1.23692, size = 119, normalized size = 1.47

$$-\frac{e^{\left(-2a+\frac{2bc}{d}\right)}E_2\left(\frac{2(dx+c)b}{d}\right)}{4(dx+c)d} - \frac{e^{\left(2a-\frac{2bc}{d}\right)}E_2\left(-\frac{2(dx+c)b}{d}\right)}{4(dx+c)d} - \frac{1}{2\left(d^2x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(2, 2*(d*x + c)*b/d)/((d*x + c)*d) -
1/4*e^(2*a - 2*b*c/d)*exp_integral_e(2, -2*(d*x + c)*b/d)/((d*x + c)*d) - 1
/2/(d^2*x + c*d)
```

Fricas [B] time = 1.78497, size = 365, normalized size = 4.51

$$-\frac{d\cosh(bx+a)^2 + d\sinh(bx+a)^2 - \Big(\big((bdx+bc)\mathrm{Ei}\big(\tfrac{2(bdx+bc)}{d}\big) - (bdx+bc)\mathrm{Ei}\big(-\tfrac{2(bdx+bc)}{d}\big)\big)\cosh\big(-\tfrac{2(bc-ad)}{d}\big) - \big((bdx+b}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d*cosh(b*x + a)^2 + d*sinh(b*x + a)^2 - ((b*d*x + b*c)*Ei(2*(b*d*x +
b*c)/d) - (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/d) - ((
b*d*x + b*c)*Ei(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d))*
sinh(-2*(b*c - a*d)/d) + d)/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)**2/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2/(d*x + c)^2, x)
```

3.14 $\int \frac{\cosh^2(a+bx)}{(c+dx)^3} dx$

Optimal. Leaf size=112

$$\frac{b^2 \cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} + \frac{b^2 \sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b \sinh(a+bx) \cosh(a+bx)}{d^2(c+dx)} - \frac{\cosh^2(a+b}{2d(c+dx)}$$

```
[Out] -Cosh[a + b*x]^2/(2*d*(c + d*x)^2) + (b^2*Cosh[2*a - (2*b*c)/d]*CoshIntegra
l[(2*b*c)/d + 2*b*x])/d^3 - (b*Cosh[a + b*x]*Sinh[a + b*x])/(d^2*(c + d*x))
 + (b^2*Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rubi [A] time = 0.187065, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3314, 31, 3312, 3303, 3298, 3301}

$$\frac{b^2 \cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} + \frac{b^2 \sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b \sinh(a+bx) \cosh(a+bx)}{d^2(c+dx)} - \frac{\cosh^2(a+b}{2d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] -Cosh[a + b*x]^2/(2*d*(c + d*x)^2) + (b^2*Cosh[2*a - (2*b*c)/d]*CoshIntegra
l[(2*b*c)/d + 2*b*x])/d^3 - (b*Cosh[a + b*x]*Sinh[a + b*x])/(d^2*(c + d*x))
 + (b^2*Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\,dx = -\frac{\cosh^2(a+bx)}{2d(c+dx)^2} - \frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} - \frac{b^2\int \frac{1}{c+dx}\,dx}{d^2} + \frac{\left(2b^2\right)\int \frac{\cosh^2(a+bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{\cosh^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\log(c+dx)}{d^3} - \frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} + \frac{\left(2b^2\right)\int \left(\frac{1}{2(c+dx)} + \frac{\cosh(2a+2bx)}{2(c+dx)}\right)$$

$$= -\frac{\cosh^2(a+bx)}{2d(c+dx)^2} - \frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} + \frac{b^2\int \frac{\cosh(2a+2bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{\cosh^2(a+bx)}{2d(c+dx)^2} - \frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} + \frac{\left(b^2\cosh\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d^2} + \frac{\left(b^2\sin\right.$$

$$= -\frac{\cosh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\cosh\left(2a-\frac{2bc}{d}\right)\operatorname{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^3} - \frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} + \frac{b^2\sinh\left(2\right.$$

Mathematica [A] time = 0.879828, size = 102, normalized size = 0.91

$$\frac{2b^2\cosh\left(2a-\frac{2bc}{d}\right)\operatorname{Chi}\left(\frac{2b(c+dx)}{d}\right) + 2b^2\sinh\left(2a-\frac{2bc}{d}\right)\operatorname{Shi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d\left(b(c+dx)\sinh(2(a+bx))+d\cosh^2(a+bx)\right)}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] (2*b^2*Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] - (d*(d*Cosh[a
 + b*x]^2 + b*(c + d*x)*Sinh[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sinh[2*a -
(2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

Maple [B] time = 0.081, size = 299, normalized size = 2.7

$$-\frac{1}{4d(dx+c)^2} + \frac{b^3e^{-2bx-2a}x}{4d\left(b^2d^2x^2+2b^2cdx+c^2b^2\right)} + \frac{b^3e^{-2bx-2a}c}{4d^2\left(b^2d^2x^2+2b^2cdx+c^2b^2\right)} - \frac{b^2e^{-2bx-2a}}{8d\left(b^2d^2x^2+2b^2cdx+c^2b^2\right)} - \frac{b^2}{2d^3}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] -1/4/d/(d*x+c)^2+1/4*b^3*exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2
)*x+1/4*b^3*exp(-2*b*x-2*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/8*b^2
```

```
*exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-1/2*b^2/d^3*exp(-2*(a*
d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/8*b^2/d^3*exp(2*b*x+2*a)/(b*c/d+b
*x)^2-1/4*b^2/d^3*exp(2*b*x+2*a)/(b*c/d+b*x)-1/2*b^2/d^3*exp(2*(a*d-b*c)/d)
*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Maxima [A] time = 1.28828, size = 134, normalized size = 1.2

$$-\frac{1}{4\left(d^3x^2+2\,cd^2x+c^2d\right)}-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_3\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^2d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_3\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(d^3*x^2 + 2*c*d^2*x + c^2*d) - 1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(
3, 2*(d*x + c)*b/d)/((d*x + c)^2*d) - 1/4*e^(2*a - 2*b*c/d)*exp_integral_e(
3, -2*(d*x + c)*b/d)/((d*x + c)^2*d)
```

Fricas [B] time = 1.8496, size = 597, normalized size = 5.33

$$-\frac{d^2\cosh\left(bx+a\right)^2+d^2\sinh\left(bx+a\right)^2+4\left(bd^2x+bcd\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+d^2-2\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(d^2*cosh(b*x + a)^2 + d^2*sinh(b*x + a)^2 + 4*(b*d^2*x + b*c*d)*cosh(
b*x + a)*sinh(b*x + a) + d^2 - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(
2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x + b
*c)/d))*cosh(-2*(b*c - a*d)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*E
i(2*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x +
 b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh^2\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(cosh(a + b*x)**2/(c + d*x)**3, x)
```

Giac [B] time = 1.40771, size = 446, normalized size = 3.98

$$\frac{4\,b^2d^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+4\,b^2d^2x^2\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}+8\,b^2cdx\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+8\,b^2cdx\mathrm{Ei}\left(-\frac{2\,(\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*d^2*x^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*d^2*x^2*
Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 8*b^2*c*d*x*Ei(2*(b*d*x + b*c)/
d)*e^(2*a - 2*b*c/d) + 8*b^2*c*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d
) + 4*b^2*c^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*c^2*Ei(-2*(b*
d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b*d^2*x*e^(2*b*x + 2*a) + 2*b*d^2*x*e^
(-2*b*x - 2*a) - 2*b*c*d*e^(2*b*x + 2*a) + 2*b*c*d*e^(-2*b*x - 2*a) - d^2*e
^(2*b*x + 2*a) - d^2*e^(-2*b*x - 2*a) - 2*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d
^3)
```

3.15 $\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=162

$$\frac{2b^3\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)}-\frac{b\sinh(a+bx)\cosh}{3d^2(c+dx)}$$

```
[Out] b^2/(3*d^3*(c + d*x)) - Cosh[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Cosh[a +
 b*x]^2)/(3*d^3*(c + d*x)) + (2*b^3*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*
a - (2*b*c)/d])/(3*d^4) - (b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*(c + d*x)^
2) + (2*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rubi [A] time = 0.180605, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3314, 32, 3313, 12, 3303, 3298, 3301}

$$\frac{2b^3\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)}-\frac{b\sinh(a+bx)\cosh}{3d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^4,x]
```

```
[Out] b^2/(3*d^3*(c + d*x)) - Cosh[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Cosh[a +
 b*x]^2)/(3*d^3*(c + d*x)) + (2*b^3*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*
a - (2*b*c)/d])/(3*d^4) - (b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*(c + d*x)^
2) + (2*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx = -\frac{\cosh^2(a+bx)}{3d(c+dx)^3} - \frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2} - \frac{b^2\int\frac{1}{(c+dx)^2}\,dx}{3d^2} + \frac{(2b^2)\int\frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx}{3d^2}$$

$$= \frac{b^2}{3d^3(c+dx)} - \frac{\cosh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)} - \frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2} + \frac{(4ib^3)\int -\frac{i\sin\ldots}{\ldots}}{3d\ldots}$$

$$= \frac{b^2}{3d^3(c+dx)} - \frac{\cosh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)} - \frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2} + \frac{(2b^3)\int\frac{\sinh(2\ldots}{c+\ldots}}{3d^3}$$

$$= \frac{b^2}{3d^3(c+dx)} - \frac{\cosh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)} - \frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2} + \frac{\left(2b^3\cosh\left(2a\ldots\right.\right.}{\ldots}$$

$$= \frac{b^2}{3d^3(c+dx)} - \frac{\cosh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)} + \frac{2b^3\text{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{3d^4} - \frac{b\cosh\ldots}{\ldots}$$

Mathematica [A] time = 0.833647, size = 121, normalized size = 0.75

$$\frac{4b^3\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d\left(\cosh(2(a+bx))\left(2b^2(c+dx)^2+d^2\right)+d(b(c+dx)\sinh(2(a+bx))+d)\right)}{(c+dx)^3} + 4b^3\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^4,x]
```

```
[Out] (4*b^3*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[2*a - (2*b*c)/d] - (d*((d^2 + 2
*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + d*(d + b*(c + d*x)*Sinh[2*(a + b*x)])
))/(c + d*x)^3 + 4*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d
])/(6*d^4)
```

Maple [B] time = 0.092, size = 555, normalized size = 3.4

$$-\frac{1}{6d(dx+c)^3} - \frac{b^5e^{-2bx-2a}x^2}{6d\left(b^3d^3x^3+3b^3cd^2x^2+3b^3c^2dx+c^3b^3\right)} - \frac{b^5e^{-2bx-2a}cx}{3d^2\left(b^3d^3x^3+3b^3cd^2x^2+3b^3c^2dx+c^3b^3\right)} - \frac{\ldots}{6d^3\left(b^3d^3x^3\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^4,x)
```

```
[Out] -1/6/d/(d*x+c)^3-1/6*b^5*exp(-2*b*x-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b
^3*c^2*d*x+b^3*c^3)*x^2-1/3*b^5*exp(-2*b*x-2*a)/d^2/(b^3*d^3*x^3+3*b^3*c*d^
2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c*x-1/6*b^5*exp(-2*b*x-2*a)/d^3/(b^3*d^3*x^3+3
*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c^2+1/12*b^4*exp(-2*b*x-2*a)/d/(b^3*d
^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*x+1/12*b^4*exp(-2*b*x-2*a)/d^
2/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c-1/12*b^3*exp(-2*b*x
-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)+1/3*b^3/d^4*exp
(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/12*b^3/d^4*exp(2*b*x+2*a)/
(b*c/d+b*x)^3-1/12*b^3/d^4*exp(2*b*x+2*a)/(b*c/d+b*x)^2-1/6*b^3/d^4*exp(2*b
*x+2*a)/(b*c/d+b*x)-1/3*b^3/d^4*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+
b*c)/d)
```

Maxima [A] time = 1.29803, size = 149, normalized size = 0.92

$$-\frac{1}{6\left(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d\right)}-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_4\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_4\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/6/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d) - 1/4*e^(-2*a + 2*b*c/d)
*exp_integral_e(4, 2*(d*x + c)*b/d)/((d*x + c)^3*d) - 1/4*e^(2*a - 2*b*c/d)
*exp_integral_e(4, -2*(d*x + c)*b/d)/((d*x + c)^3*d)
```

Fricas [B] time = 1.89245, size = 859, normalized size = 5.3

$$-\frac{d^3+\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d+d^3\right)\cosh\left(bx+a\right)^2+2\left(bd^3x+bcd^2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\left(2\,b^2d^3x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(d^3 + (2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x +
a)^2 + 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) + (2*b^2*d^3*x^2 +
 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*sinh(b*x + a)^2 - 2*((b^3*d^3*x^3 + 3*b
^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(2*(b*d*x + b*c)/d) - (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*cosh
(-2*(b*c - a*d)/d) - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^
3*c^3)*Ei(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*
c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(cosh(a + b*x)**2/(c + d*x)**4, x)
```

Giac [B] time = 1.32984, size = 725, normalized size = 4.48

$$\frac{4\,b^3d^3x^3\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 4\,b^3d^3x^3\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} + 12\,b^3cd^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 12\,b^3cd^2x^2\mathrm{Ei}\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/12*(4*b^3*d^3*x^3*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) - 4*b^3*d^3*x^3
*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 12*b^3*c*d^2*x^2*Ei(2*(b*d*x +
 b*c)/d)*e^(2*a - 2*b*c/d) - 12*b^3*c*d^2*x^2*Ei(-2*(b*d*x + b*c)/d)*e^(-2*
a + 2*b*c/d) + 12*b^3*c^2*d*x*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) - 12*
b^3*c^2*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b^2*d^3*x^2*e^(2*
b*x + 2*a) - 2*b^2*d^3*x^2*e^(-2*b*x - 2*a) + 4*b^3*c^3*Ei(2*(b*d*x + b*c)/
d)*e^(2*a - 2*b*c/d) - 4*b^3*c^3*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d)
- 4*b^2*c*d^2*x*e^(2*b*x + 2*a) - 4*b^2*c*d^2*x*e^(-2*b*x - 2*a) - 2*b^2*c^
2*d*e^(2*b*x + 2*a) - b*d^3*x*e^(2*b*x + 2*a) - 2*b^2*c^2*d*e^(-2*b*x - 2*a
) + b*d^3*x*e^(-2*b*x - 2*a) - b*c*d^2*e^(2*b*x + 2*a) + b*c*d^2*e^(-2*b*x
- 2*a) - d^3*e^(2*b*x + 2*a) - d^3*e^(-2*b*x - 2*a) - 2*d^3)/(d^7*x^3 + 3*c
*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

3.16 $\int (c+dx)^4 \cosh^3(a+bx)\,dx$

Optimal. Leaf size=225

$$\frac{80d^2(c+dx)^2\sinh(a+bx)}{9b^3}-\frac{8d^3(c+dx)\cosh^3(a+bx)}{27b^4}-\frac{160d^3(c+dx)\cosh(a+bx)}{9b^4}+\frac{4d^2(c+dx)^2\sinh(a+bx)\mathrm{co}}{9b^3}$$

```
[Out] (-160*d^3*(c + d*x)*Cosh[a + b*x])/(9*b^4) - (8*d*(c + d*x)^3*Cosh[a + b*x]
)/(3*b^2) - (8*d^3*(c + d*x)*Cosh[a + b*x]^3)/(27*b^4) - (4*d*(c + d*x)^3*C
osh[a + b*x]^3)/(9*b^2) + (488*d^4*Sinh[a + b*x])/(27*b^5) + (80*d^2*(c + d
*x)^2*Sinh[a + b*x])/(9*b^3) + (2*(c + d*x)^4*Sinh[a + b*x])/(3*b) + (4*d^2
*(c + d*x)^2*Cosh[a + b*x]^2*Sinh[a + b*x])/(9*b^3) + ((c + d*x)^4*Cosh[a +
 b*x]^2*Sinh[a + b*x])/(3*b) + (8*d^4*Sinh[a + b*x]^3)/(81*b^5)
```

Rubi [A] time = 0.281545, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 3296, 2637, 2633}

$$\frac{80d^2(c+dx)^2\sinh(a+bx)}{9b^3}-\frac{8d^3(c+dx)\cosh^3(a+bx)}{27b^4}-\frac{160d^3(c+dx)\cosh(a+bx)}{9b^4}+\frac{4d^2(c+dx)^2\sinh(a+bx)\mathrm{co}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cosh[a + b*x]^3,x]
```


```
[Out] (-160*d^3*(c + d*x)*Cosh[a + b*x])/(9*b^4) - (8*d*(c + d*x)^3*Cosh[a + b*x]
)/(3*b^2) - (8*d^3*(c + d*x)*Cosh[a + b*x]^3)/(27*b^4) - (4*d*(c + d*x)^3*C
osh[a + b*x]^3)/(9*b^2) + (488*d^4*Sinh[a + b*x])/(27*b^5) + (80*d^2*(c + d
*x)^2*Sinh[a + b*x])/(9*b^3) + (2*(c + d*x)^4*Sinh[a + b*x])/(3*b) + (4*d^2
*(c + d*x)^2*Cosh[a + b*x]^2*Sinh[a + b*x])/(9*b^3) + ((c + d*x)^4*Cosh[a +
 b*x]^2*Sinh[a + b*x])/(3*b) + (8*d^4*Sinh[a + b*x]^3)/(81*b^5)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (c+dx)^4\cosh^3(a+bx)\,dx = -\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\cosh^2(a+bx)\sinh(a+bx)}{3b}+\frac{2}{3}\int (c+dx)^4\cosh$$
$$= -\frac{8d^3(c+dx)\cosh^3(a+bx)}{27b^4}-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{2(c+dx)^4\sinh(a+bx)}{3b}+\frac{4d}{}$$
$$= -\frac{8d(c+dx)^3\cosh(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cosh^3(a+bx)}{27b^4}-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{8}{}$$
$$= -\frac{16d^3(c+dx)\cosh(a+bx)}{9b^4}-\frac{8d(c+dx)^3\cosh(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cosh^3(a+bx)}{27b^4}-\frac{4}{}$$
$$= -\frac{160d^3(c+dx)\cosh(a+bx)}{9b^4}-\frac{8d(c+dx)^3\cosh(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cosh^3(a+bx)}{27b^4}-$$
$$= -\frac{160d^3(c+dx)\cosh(a+bx)}{9b^4}-\frac{8d(c+dx)^3\cosh(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cosh^3(a+bx)}{27b^4}-$$

Mathematica [A] time = 0.934196, size = 385, normalized size = 1.71

$$\frac{1458b^4c^2d^2x^2\sinh(a+bx)+162b^4c^2d^2x^2\sinh(3(a+bx))+2916b^2c^2d^2\sinh(a+bx)+36b^2c^2d^2\sinh(3(a+bx))+972b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cosh[a + b*x]^3,x]
```

```
[Out] (-972*b*d*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 12*b*d*(c + d
*x)*(2*d^2 + 3*b^2*(c + d*x)^2)*Cosh[3*(a + b*x)] + 243*b^4*c^4*Sinh[a + b*
x] + 2916*b^2*c^2*d^2*Sinh[a + b*x] + 5832*d^4*Sinh[a + b*x] + 972*b^4*c^3*
d*x*Sinh[a + b*x] + 5832*b^2*c*d^3*x*Sinh[a + b*x] + 1458*b^4*c^2*d^2*x^2*S
inh[a + b*x] + 2916*b^2*d^4*x^2*Sinh[a + b*x] + 972*b^4*c*d^3*x^3*Sinh[a +
b*x] + 243*b^4*d^4*x^4*Sinh[a + b*x] + 27*b^4*c^4*Sinh[3*(a + b*x)] + 36*b^
2*c^2*d^2*Sinh[3*(a + b*x)] + 8*d^4*Sinh[3*(a + b*x)] + 108*b^4*c^3*d*x*Sin
h[3*(a + b*x)] + 72*b^2*c*d^3*x*Sinh[3*(a + b*x)] + 162*b^4*c^2*d^2*x^2*Sin
h[3*(a + b*x)] + 36*b^2*d^4*x^2*Sinh[3*(a + b*x)] + 108*b^4*c*d^3*x^3*Sinh[
3*(a + b*x)] + 27*b^4*d^4*x^4*Sinh[3*(a + b*x)])/(324*b^5)
```

Maple [B] time = 0.013, size = 1217, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cosh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(2/3*(b*x+a)^4*sinh(b*x+a)+1/3*(b*x+a)^4*sinh(b*x+a)*cosh(b*
x+a)^2-28/9*(b*x+a)^3*cosh(b*x+a)+80/9*(b*x+a)^2*sinh(b*x+a)-488/27*(b*x+a)
*cosh(b*x+a)+1456/81*sinh(b*x+a)-4/9*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)^2+4/
9*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2-8/27*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a
)+8/81*sinh(b*x+a)*cosh(b*x+a)^2)-4/b^4*d^4*a*(2/3*(b*x+a)^3*sinh(b*x+a)+1/
3*(b*x+a)^3*sinh(b*x+a)*cosh(b*x+a)^2-7/3*(b*x+a)^2*cosh(b*x+a)+40/9*(b*x+a
)*sinh(b*x+a)-122/27*cosh(b*x+a)-1/3*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)+2/
```

```
9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-2/27*sinh(b*x+a)^2*cosh(b*x+a))+6/b^4*d
^4*a^2*(1/3*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2+2/3*(b*x+a)^2*sinh(b*x+a)-2
/9*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)-14/9*(b*x+a)*cosh(b*x+a)+2/27*sinh(b*x
+a)*cosh(b*x+a)^2+40/27*sinh(b*x+a))-4/b^4*d^4*a^3*(2/3*(b*x+a)*sinh(b*x+a)
+1/3*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-7/9*cosh(b*x+a)-1/9*sinh(b*x+a)^2*co
sh(b*x+a))+1/b^4*d^4*a^4*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+4/b^3*c*d^3*(2
/3*(b*x+a)^3*sinh(b*x+a)+1/3*(b*x+a)^3*sinh(b*x+a)*cosh(b*x+a)^2-7/3*(b*x+a
)^2*cosh(b*x+a)+40/9*(b*x+a)*sinh(b*x+a)-122/27*cosh(b*x+a)-1/3*(b*x+a)^2*s
inh(b*x+a)^2*cosh(b*x+a)+2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-2/27*sinh(b*
x+a)^2*cosh(b*x+a))-12/b^3*c*d^3*a*(1/3*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2
+2/3*(b*x+a)^2*sinh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)-14/9*(b*x+
a)*cosh(b*x+a)+2/27*sinh(b*x+a)*cosh(b*x+a)^2+40/27*sinh(b*x+a))+12/b^3*c*d
^3*a^2*(2/3*(b*x+a)*sinh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-7/9*c
osh(b*x+a)-1/9*sinh(b*x+a)^2*cosh(b*x+a))-4/b^3*c*d^3*a^3*(2/3+1/3*cosh(b*x
+a)^2)*sinh(b*x+a)+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2+2
/3*(b*x+a)^2*sinh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)-14/9*(b*x+a)
*cosh(b*x+a)+2/27*sinh(b*x+a)*cosh(b*x+a)^2+40/27*sinh(b*x+a))-12/b^2*c^2*d
^2*a*(2/3*(b*x+a)*sinh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-7/9*cos
h(b*x+a)-1/9*sinh(b*x+a)^2*cosh(b*x+a))+6/b^2*c^2*d^2*a^2*(2/3+1/3*cosh(b*x
+a)^2)*sinh(b*x+a)+4/b*c^3*d*(2/3*(b*x+a)*sinh(b*x+a)+1/3*(b*x+a)*sinh(b*x+
a)*cosh(b*x+a)^2-7/9*cosh(b*x+a)-1/9*sinh(b*x+a)^2*cosh(b*x+a))-4/b*c^3*d*a
*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+c^4*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a
))
```

Maxima [B] time = 1.16462, size = 869, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*c^3*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 27*(b*x*e^a - e^a)*e^
(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 - (3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2) + 1/24*c^4*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(
-3*b*x - 3*a)/b) + 1/36*c^2*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(
3*a))*e^(3*b*x)/b^3 + 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81
*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x
 - 3*a)/b^3) + 1/54*c*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e
^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 + 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x
*e^a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x -
a)/b^4 - (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4) + 1/648*
d^4*((27*b^4*x^4*e^(3*a) - 36*b^3*x^3*e^(3*a) + 36*b^2*x^2*e^(3*a) - 24*b*x
*e^(3*a) + 8*e^(3*a))*e^(3*b*x)/b^5 + 243*(b^4*x^4*e^a - 4*b^3*x^3*e^a + 12
*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*e^(b*x)/b^5 - 243*(b^4*x^4 + 4*b^3*x^3
+ 12*b^2*x^2 + 24*b*x + 24)*e^(-b*x - a)/b^5 - (27*b^4*x^4 + 36*b^3*x^3 + 3
6*b^2*x^2 + 24*b*x + 8)*e^(-3*b*x - 3*a)/b^5)
```

Fricas [B] time = 2.04164, size = 1149, normalized size = 5.11

$$-\frac{12\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b^3c^3d+2\,bcd^3+\left(9\,b^3c^2d^2+2\,bd^4\right)x\right)\cosh\left(bx+a\right)^3+36\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b^3c^3\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/324*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9*
b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a)^3 + 36*(3*b^3*d^4*x^3 + 9*b^3*c*d^3
*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9*b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a)
*sinh(b*x + a)^2 - (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 + 36*b^
2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d +
2*b^2*c*d^3)*x)*sinh(b*x + a)^3 + 972*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*
c^3*d + 6*b*c*d^3 + 3*(b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a) - 3*(81*b^4*
d^4*x^4 + 324*b^4*c*d^3*x^3 + 81*b^4*c^4 + 972*b^2*c^2*d^2 + 1944*d^4 + 486
*(b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b
^4*c^4 + 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(
3*b^4*c^3*d + 2*b^2*c*d^3)*x)*cosh(b*x + a)^2 + 324*(b^4*c^3*d + 6*b^2*c*d^
3)*x)*sinh(b*x + a))/b^5
```

Sympy [A] time = 11.2935, size = 772, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((-2*c**4*sinh(a + b*x)**3/(3*b) + c**4*sinh(a + b*x)*cosh(a + b*x
)**2/b - 8*c**3*d*x*sinh(a + b*x)**3/(3*b) + 4*c**3*d*x*sinh(a + b*x)*cosh(
a + b*x)**2/b - 4*c**2*d**2*x**2*sinh(a + b*x)**3/b + 6*c**2*d**2*x**2*sinh
(a + b*x)*cosh(a + b*x)**2/b - 8*c*d**3*x**3*sinh(a + b*x)**3/(3*b) + 4*c*d
**3*x**3*sinh(a + b*x)*cosh(a + b*x)**2/b - 2*d**4*x**4*sinh(a + b*x)**3/(3
*b) + d**4*x**4*sinh(a + b*x)*cosh(a + b*x)**2/b + 8*c**3*d*sinh(a + b*x)**
2*cosh(a + b*x)/(3*b**2) - 28*c**3*d*cosh(a + b*x)**3/(9*b**2) + 8*c**2*d**
2*x*sinh(a + b*x)**2*cosh(a + b*x)/b**2 - 28*c**2*d**2*x*cosh(a + b*x)**3/(
3*b**2) + 8*c*d**3*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b**2 - 28*c*d**3*x**
2*cosh(a + b*x)**3/(3*b**2) + 8*d**4*x**3*sinh(a + b*x)**2*cosh(a + b*x)/(3
*b**2) - 28*d**4*x**3*cosh(a + b*x)**3/(9*b**2) - 80*c**2*d**2*sinh(a + b*x
)**3/(9*b**3) + 28*c**2*d**2*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**3) - 160*
c*d**3*x*sinh(a + b*x)**3/(9*b**3) + 56*c*d**3*x*sinh(a + b*x)*cosh(a + b*x
)**2/(3*b**3) - 80*d**4*x**2*sinh(a + b*x)**3/(9*b**3) + 28*d**4*x**2*sinh(
a + b*x)*cosh(a + b*x)**2/(3*b**3) + 160*c*d**3*sinh(a + b*x)**2*cosh(a + b
*x)/(9*b**4) - 488*c*d**3*cosh(a + b*x)**3/(27*b**4) + 160*d**4*x*sinh(a +
b*x)**2*cosh(a + b*x)/(9*b**4) - 488*d**4*x*cosh(a + b*x)**3/(27*b**4) - 14
56*d**4*sinh(a + b*x)**3/(81*b**5) + 488*d**4*sinh(a + b*x)*cosh(a + b*x)**
2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**
3*x**4 + d**4*x**5/5)*cosh(a)**3, True))
```

Giac [B] time = 1.39823, size = 883, normalized size = 3.92

$$\frac{\left(27\, b^4 d^4 x^4 + 108\, b^4 c d^3 x^3 + 162\, b^4 c^2 d^2 x^2 - 36\, b^3 d^4 x^3 + 108\, b^4 c^3 d x - 108\, b^3 c d^3 x^2 + 27\, b^4 c^4 - 108\, b^3 c^2 d^2 x + 36\, b^2 d^4 x^2 - 3}{648\, b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 - 36*b^3*d^
4*x^3 + 108*b^4*c^3*d*x - 108*b^3*c*d^3*x^2 + 27*b^4*c^4 - 108*b^3*c^2*d^2*
```

```
x + 36*b^2*d^4*x^2 - 36*b^3*c^3*d + 72*b^2*c*d^3*x + 36*b^2*c^2*d^2 - 24*b*
d^4*x - 24*b*c*d^3 + 8*d^4)*e^(3*b*x + 3*a)/b^5 + 3/8*(b^4*d^4*x^4 + 4*b^4*
c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x - 12*b^3*c*d^
3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 - 4*b^3*c^3*d + 24*b^2*
c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3 + 24*d^4)*e^(b*x + a)/b^
5 - 3/8*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^3*d^4*x^3
+ 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*b^3*c^2*d^2*x + 12*b^2*d^
4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c
*d^3 + 24*d^4)*e^(-b*x - a)/b^5 - 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3
 + 162*b^4*c^2*d^2*x^2 + 36*b^3*d^4*x^3 + 108*b^4*c^3*d*x + 108*b^3*c*d^3*x
^2 + 27*b^4*c^4 + 108*b^3*c^2*d^2*x + 36*b^2*d^4*x^2 + 36*b^3*c^3*d + 72*b^
2*c*d^3*x + 36*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c*d^3 + 8*d^4)*e^(-3*b*x - 3
*a)/b^5
```

3.17 $\int (c+dx)^3 \cosh^3(a+bx)\,dx$

Optimal. Leaf size=175

$$\frac{40d^2(c+dx)\sinh(a+bx)}{9b^3}+\frac{2d^2(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{9b^3}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}-\frac{2d(c+dx)^2\cosh(a+}{b^2}$$

```
[Out] (-40*d^3*Cosh[a + b*x])/(9*b^4) - (2*d*(c + d*x)^2*Cosh[a + b*x])/b^2 - (2*
d^3*Cosh[a + b*x]^3)/(27*b^4) - (d*(c + d*x)^2*Cosh[a + b*x]^3)/(3*b^2) + (
40*d^2*(c + d*x)*Sinh[a + b*x])/(9*b^3) + (2*(c + d*x)^3*Sinh[a + b*x])/(3*
b) + (2*d^2*(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x])/(9*b^3) + ((c + d*x)^3
*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b)
```

Rubi [A] time = 0.175649, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 3296, 2638, 3310}

$$\frac{40d^2(c+dx)\sinh(a+bx)}{9b^3}+\frac{2d^2(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{9b^3}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}-\frac{2d(c+dx)^2\cosh(a+}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cosh[a + b*x]^3,x]
```


```
[Out] (-40*d^3*Cosh[a + b*x])/(9*b^4) - (2*d*(c + d*x)^2*Cosh[a + b*x])/b^2 - (2*
d^3*Cosh[a + b*x]^3)/(27*b^4) - (d*(c + d*x)^2*Cosh[a + b*x]^3)/(3*b^2) + (
40*d^2*(c + d*x)*Sinh[a + b*x])/(9*b^3) + (2*(c + d*x)^3*Sinh[a + b*x])/(3*
b) + (2*d^2*(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x])/(9*b^3) + ((c + d*x)^3
*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

]

Rubi steps

$$\int (c+dx)^3\cosh^3(a+bx)\,dx = -\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\cosh^2(a+bx)\sinh(a+bx)}{3b}+\frac{2}{3}\int (c+dx)^3 \text{co}$$

$$= -\frac{2d^3\cosh^3(a+bx)}{27b^4}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{2(c+dx)^3\sinh(a+bx)}{3b}+\frac{2d^2(c+}{}$$

$$= -\frac{2d(c+dx)^2\cosh(a+bx)}{b^2}-\frac{2d^3\cosh^3(a+bx)}{27b^4}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{4d^2(c+}{}$$

$$= -\frac{4d^3\cosh(a+bx)}{9b^4}-\frac{2d(c+dx)^2\cosh(a+bx)}{b^2}-\frac{2d^3\cosh^3(a+bx)}{27b^4}-\frac{d(c+dx)^2\cosh}{3b^2}$$

$$= -\frac{40d^3\cosh(a+bx)}{9b^4}-\frac{2d(c+dx)^2\cosh(a+bx)}{b^2}-\frac{2d^3\cosh^3(a+bx)}{27b^4}-\frac{d(c+dx)^2\cos}{3b^2}$$

Mathematica [A] time = 0.926061, size = 122, normalized size = 0.7

$$\frac{-486d\cosh(a+bx)\left(b^2(c+dx)^2+2d^2\right)-2d\cosh(3(a+bx))\left(9b^2(c+dx)^2+2d^2\right)+12b(c+dx)\sinh(a+bx)\left(\cosh(2\right.}{216b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cosh[a + b*x]^3,x]
```

```
[Out] (-486*d*(2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 2*d*(2*d^2 + 9*b^2*(c + d
*x)^2)*Cosh[3*(a + b*x)] + 12*b*(c + d*x)*(82*d^2 + 15*b^2*(c + d*x)^2 + (2
*d^2 + 3*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(216*b^4)
```

Maple [B] time = 0.01, size = 676, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cosh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(2/3*(b*x+a)^3*sinh(b*x+a)+1/3*(b*x+a)^3*sinh(b*x+a)*cosh(b*
x+a)^2-7/3*(b*x+a)^2*cosh(b*x+a)+40/9*(b*x+a)*sinh(b*x+a)-122/27*cosh(b*x+a
)-1/3*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)+2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+
a)^2-2/27*sinh(b*x+a)^2*cosh(b*x+a))-3/b^3*d^3*a*(1/3*(b*x+a)^2*sinh(b*x+a)
*cosh(b*x+a)^2+2/3*(b*x+a)^2*sinh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)^2*cosh(b*x
+a)-14/9*(b*x+a)*cosh(b*x+a)+2/27*sinh(b*x+a)*cosh(b*x+a)^2+40/27*sinh(b*x+
a))+3/b^3*d^3*a^2*(2/3*(b*x+a)*sinh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)*cosh(b*x
+a)^2-7/9*cosh(b*x+a)-1/9*sinh(b*x+a)^2*cosh(b*x+a))-1/b^3*d^3*a^3*(2/3+1/3
*cosh(b*x+a)^2)*sinh(b*x+a)+3/b^2*c*d^2*(1/3*(b*x+a)^2*sinh(b*x+a)*cosh(b*x
+a)^2+2/3*(b*x+a)^2*sinh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)-14/9*
(b*x+a)*cosh(b*x+a)+2/27*sinh(b*x+a)*cosh(b*x+a)^2+40/27*sinh(b*x+a))-6/b^2
*c*d^2*a*(2/3*(b*x+a)*sinh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-7/9
*cosh(b*x+a)-1/9*sinh(b*x+a)^2*cosh(b*x+a))+3/b^2*c*d^2*a^2*(2/3+1/3*cosh(b
*x+a)^2)*sinh(b*x+a)+3/b*c^2*d*(2/3*(b*x+a)*sinh(b*x+a)+1/3*(b*x+a)*sinh(b*
x+a)*cosh(b*x+a)^2-7/9*cosh(b*x+a)-1/9*sinh(b*x+a)^2*cosh(b*x+a))-3/b*c^2*d
*a*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+c^3*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x
```

```
+a))
```

Maxima [B] time = 1.10077, size = 593, normalized size = 3.39

$$\frac{1}{24}c^2d\left(\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{b^2}+\frac{27\,(bxe^a-e^a)e^{(bx)}}{b^2}-\frac{27\,(bx+1)e^{(-bx-a)}}{b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right)+\frac{1}{24}c^3\left(\frac{e^{(3\,bx+3\,a)}}{b}+\frac{9\,e^{(}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*c^2*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 27*(b*x*e^a - e^a)*e^
(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 - (3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2) + 1/24*c^3*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(
-3*b*x - 3*a)/b) + 1/72*c*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*
a))*e^(3*b*x)/b^3 + 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81*(
b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x -
 3*a)/b^3) + 1/216*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e^(3
*a) - 2*e^(3*a))*e^(3*b*x)/b^4 + 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^
a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/
b^4 - (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4)
```

Fricas [B] time = 2.11137, size = 757, normalized size = 4.33

$$-\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d+2\,d^3\right)\cosh\left(bx+a\right)^3+3\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d+2\,d^3\right)\cosh\left(bx+a\right)\sinh\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*cosh(b*x + a
)^3 + 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*cosh(b*x + a
)*sinh(b*x + a)^2 - 3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 + 2*b*c*
d^2 + (9*b^3*c^2*d + 2*b*d^3)*x)*sinh(b*x + a)^3 + 243*(b^2*d^3*x^2 + 2*b^2
*c*d^2*x + b^2*c^2*d + 2*d^3)*cosh(b*x + a) - 9*(9*b^3*d^3*x^3 + 27*b^3*c*d
^2*x^2 + 9*b^3*c^3 + 54*b*c*d^2 + (3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*
c^3 + 2*b*c*d^2 + (9*b^3*c^2*d + 2*b*d^3)*x)*cosh(b*x + a)^2 + 27*(b^3*c^2*
d + 2*b*d^3)*x)*sinh(b*x + a))/b^4
```

Sympy [A] time = 5.88348, size = 495, normalized size = 2.83

$$\begin{cases} -\frac{2c^3\sinh^3(a+bx)}{3b}+\frac{c^3\sinh(a+bx)\cosh^2(a+bx)}{b}-\frac{2c^2dx\sinh^3(a+bx)}{b}+\frac{3c^2dx\sinh(a+bx)\cosh^2(a+bx)}{b}-\frac{2cd^2x^2\sinh^3(a+bx)}{b}+\frac{3cd^2x^2\sinh(a+bx)}{b} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\cosh^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((-2*c**3*sinh(a + b*x)**3/(3*b) + c**3*sinh(a + b*x)*cosh(a + b*x
)**2/b - 2*c**2*d*x*sinh(a + b*x)**3/b + 3*c**2*d*x*sinh(a + b*x)*cosh(a +
```

```
b*x)**2/b - 2*c*d**2*x**2*sinh(a + b*x)**3/b + 3*c*d**2*x**2*sinh(a + b*x)*
cosh(a + b*x)**2/b - 2*d**3*x**3*sinh(a + b*x)**3/(3*b) + d**3*x**3*sinh(a
+ b*x)*cosh(a + b*x)**2/b + 2*c**2*d*sinh(a + b*x)**2*cosh(a + b*x)/b**2 -
7*c**2*d*cosh(a + b*x)**3/(3*b**2) + 4*c*d**2*x*sinh(a + b*x)**2*cosh(a + b
*x)/b**2 - 14*c*d**2*x*cosh(a + b*x)**3/(3*b**2) + 2*d**3*x**2*sinh(a + b*x
)**2*cosh(a + b*x)/b**2 - 7*d**3*x**2*cosh(a + b*x)**3/(3*b**2) - 40*c*d**2
*sinh(a + b*x)**3/(9*b**3) + 14*c*d**2*sinh(a + b*x)*cosh(a + b*x)**2/(3*b*
*3) - 40*d**3*x*sinh(a + b*x)**3/(9*b**3) + 14*d**3*x*sinh(a + b*x)*cosh(a
+ b*x)**2/(3*b**3) + 40*d**3*sinh(a + b*x)**2*cosh(a + b*x)/(9*b**4) - 122*
d**3*cosh(a + b*x)**3/(27*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*
d**2*x**3 + d**3*x**4/4)*cosh(a)**3, True))
```

Giac [B] time = 1.36789, size = 559, normalized size = 3.19

$$\frac{\left(9\,b^3d^3x^3+27\,b^3cd^2x^2+27\,b^3c^2dx-9\,b^2d^3x^2+9\,b^3c^3-18\,b^2cd^2x-9\,b^2c^2d+6\,bd^3x+6\,bcd^2-2\,d^3\right)e^{(3\,bx+3\,a)}}{216\,b^4}+\frac{3\,(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 27*b^3*c^2*d*x - 9*b^2*d^3*x^2 +
9*b^3*c^3 - 18*b^2*c*d^2*x - 9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 2*d^3)*e
^(3*b*x + 3*a)/b^4 + 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3
*b^2*d^3*x^2 + b^3*c^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^
2 - 6*d^3)*e^(b*x + a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2
*d*x + 3*b^2*d^3*x^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x +
6*b*c*d^2 + 6*d^3)*e^(-b*x - a)/b^4 - 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x
^2 + 27*b^3*c^2*d*x + 9*b^2*d^3*x^2 + 9*b^3*c^3 + 18*b^2*c*d^2*x + 9*b^2*c^
2*d + 6*b*d^3*x + 6*b*c*d^2 + 2*d^3)*e^(-3*b*x - 3*a)/b^4
```

3.18 $\int (c+dx)^2 \cosh^3(a+bx)\,dx$

Optimal. Leaf size=123

$$-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}-\frac{4d(c+dx)\cosh(a+bx)}{3b^2}+\frac{2d^2\sinh^3(a+bx)}{27b^3}+\frac{14d^2\sinh(a+bx)}{9b^3}+\frac{2(c+dx)^2\sinh(a+bx}{3b}$$

```
[Out] (-4*d*(c + d*x)*Cosh[a + b*x])/(3*b^2) - (2*d*(c + d*x)*Cosh[a + b*x]^3)/(9
*b^2) + (14*d^2*Sinh[a + b*x])/(9*b^3) + (2*(c + d*x)^2*Sinh[a + b*x])/(3*b
) + ((c + d*x)^2*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b) + (2*d^2*Sinh[a + b*x
]^3)/(27*b^3)
```

Rubi [A] time = 0.103424, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 3296, 2637, 2633}

$$-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}-\frac{4d(c+dx)\cosh(a+bx)}{3b^2}+\frac{2d^2\sinh^3(a+bx)}{27b^3}+\frac{14d^2\sinh(a+bx)}{9b^3}+\frac{2(c+dx)^2\sinh(a+bx}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cosh[a + b*x]^3,x]
```

```
[Out] (-4*d*(c + d*x)*Cosh[a + b*x])/(3*b^2) - (2*d*(c + d*x)*Cosh[a + b*x]^3)/(9
*b^2) + (14*d^2*Sinh[a + b*x])/(9*b^3) + (2*(c + d*x)^2*Sinh[a + b*x])/(3*b
) + ((c + d*x)^2*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b) + (2*d^2*Sinh[a + b*x
]^3)/(27*b^3)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (c+dx)^2 \cosh^3(a+bx)\,dx = -\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2} + \frac{(c+dx)^2\cosh^2(a+bx)\sinh(a+bx)}{3b} + \frac{2}{3}\int (c+dx)^2 \text{co}$$
$$= -\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2} + \frac{2(c+dx)^2\sinh(a+bx)}{3b} + \frac{(c+dx)^2\cosh^2(a+bx)\sinh(}{3b}$$
$$= -\frac{4d(c+dx)\cosh(a+bx)}{3b^2} - \frac{2d(c+dx)\cosh^3(a+bx)}{9b^2} + \frac{2d^2\sinh(a+bx)}{9b^3} + \frac{2(c+dx)}{}$$
$$= -\frac{4d(c+dx)\cosh(a+bx)}{3b^2} - \frac{2d(c+dx)\cosh^3(a+bx)}{9b^2} + \frac{14d^2\sinh(a+bx)}{9b^3} + \frac{2(c+dx}{}$$

Mathematica [A] time = 0.536308, size = 93, normalized size = 0.76

$$\frac{2\sinh(a+bx)\left(\cosh(2(a+bx))\left(9b^2(c+dx)^2+2d^2\right)+45b^2(c+dx)^2+82d^2\right)-162bd(c+dx)\cosh(a+bx)-6bd(c+}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cosh[a + b*x]^3,x]
```

```
[Out] (-162*b*d*(c + d*x)*Cosh[a + b*x] - 6*b*d*(c + d*x)*Cosh[3*(a + b*x)] + 2*(
82*d^2 + 45*b^2*(c + d*x)^2 + (2*d^2 + 9*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)]
)*Sinh[a + b*x])/(108*b^3)
```

Maple [B] time = 0.01, size = 320, normalized size = 2.6

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left(\frac{(bx+a)^2\sinh(bx+a)(\cosh(bx+a))^2}{3} + \frac{2\,(bx+a)^2\sinh(bx+a)}{3} - \frac{(2\,bx+2\,a)(\sinh(bx+a))^2\cosh(bx+}{9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cosh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/3*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2+2/3*(b*x+a)^2*sinh(
b*x+a)-2/9*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)-14/9*(b*x+a)*cosh(b*x+a)+2/27*
sinh(b*x+a)*cosh(b*x+a)^2+40/27*sinh(b*x+a))-2/b^2*d^2*a*(2/3*(b*x+a)*sinh(
b*x+a)+1/3*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-7/9*cosh(b*x+a)-1/9*sinh(b*x+a
)^2*cosh(b*x+a))+1/b^2*d^2*a^2*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+2/b*c*d*
(2/3*(b*x+a)*sinh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-7/9*cosh(b*x
+a)-1/9*sinh(b*x+a)^2*cosh(b*x+a))-2/b*c*d*a*(2/3+1/3*cosh(b*x+a)^2)*sinh(b
*x+a)+c^2*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a))
```

Maxima [B] time = 1.10391, size = 367, normalized size = 2.98

$$\frac{1}{36}cd\left(\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{b^2} + \frac{27\,(bxe^a-e^a)e^{(bx)}}{b^2} - \frac{27\,(bx+1)e^{(-bx-a)}}{b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right) + \frac{1}{24}c^2\left(\frac{e^{(3\,bx+3\,a)}}{b} + \frac{9}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*c*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 27*(b*x*e^a - e^a)*e^(b
*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 - (3*b*x + 1)*e^(-3*b*x - 3*a)/b^2)
 + 1/24*c^2*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3
*b*x - 3*a)/b) + 1/216*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))
*e^(3*b*x)/b^3 + 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81*(b^2
*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*
a)/b^3)
```

Fricas [A] time = 2.01771, size = 474, normalized size = 3.85

$$-\frac{6\left(bd^2x + bcd\right)\cosh\left(bx + a\right)^3 + 18\left(bd^2x + bcd\right)\cosh\left(bx + a\right)\sinh\left(bx + a\right)^2 - \left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 + 2\,d^2\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(6*(b*d^2*x + b*c*d)*cosh(b*x + a)^3 + 18*(b*d^2*x + b*c*d)*cosh(b*x
 + a)*sinh(b*x + a)^2 - (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2)*
sinh(b*x + a)^3 + 162*(b*d^2*x + b*c*d)*cosh(b*x + a) - 3*(27*b^2*d^2*x^2 +
 54*b^2*c*d*x + 27*b^2*c^2 + (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*
d^2)*cosh(b*x + a)^2 + 54*d^2)*sinh(b*x + a))/b^3
```

Sympy [A] time = 2.93358, size = 284, normalized size = 2.31

$$\begin{cases} -\frac{2c^2\sinh^3(a+bx)}{3b} + \frac{c^2\sinh(a+bx)\cosh^2(a+bx)}{b} - \frac{4cdx\sinh^3(a+bx)}{3b} + \frac{2cdx\sinh(a+bx)\cosh^2(a+bx)}{b} - \frac{2d^2x^2\sinh^3(a+bx)}{3b} + \frac{d^2x^2\sinh(a+bx)\cosh}{b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\cosh^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((-2*c**2*sinh(a + b*x)**3/(3*b) + c**2*sinh(a + b*x)*cosh(a + b*x
)**2/b - 4*c*d*x*sinh(a + b*x)**3/(3*b) + 2*c*d*x*sinh(a + b*x)*cosh(a + b*
x)**2/b - 2*d**2*x**2*sinh(a + b*x)**3/(3*b) + d**2*x**2*sinh(a + b*x)*cosh
(a + b*x)**2/b + 4*c*d*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**2) - 14*c*d*cos
h(a + b*x)**3/(9*b**2) + 4*d**2*x*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**2) -
 14*d**2*x*cosh(a + b*x)**3/(9*b**2) - 40*d**2*sinh(a + b*x)**3/(27*b**3) +
 14*d**2*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*x + c*d
*x**2 + d**2*x**3/3)*cosh(a)**3, True))
```

Giac [B] time = 1.32913, size = 311, normalized size = 2.53

$$\frac{\left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 - 6\,bd^2x - 6\,bcd + 2\,d^2\right)e^{(3\,bx+3\,a)}}{216\,b^3} + \frac{3\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,bd^2x - 2\,bcd + 2\,d^2\right)e^{(bx+}}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 6*b*d^2*x - 6*b*c*d + 2*d
^2)*e^(3*b*x + 3*a)/b^3 + 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^
2*x - 2*b*c*d + 2*d^2)*e^(b*x + a)/b^3 - 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2 + 2*b*d^2*x + 2*b*c*d + 2*d^2)*e^(-b*x - a)/b^3 - 1/216*(9*b^2*d^2*x
^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 6*b*d^2*x + 6*b*c*d + 2*d^2)*e^(-3*b*x - 3*
a)/b^3
```

3.19 $\int (c + dx) \cosh^3(a + bx) \, dx$

Optimal. Leaf size=75

$$-\frac{d \cosh^3(a+bx)}{9b^2} - \frac{2d \cosh(a+bx)}{3b^2} + \frac{2(c+dx)\sinh(a+bx)}{3b} + \frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

```
[Out] (-2*d*Cosh[a + b*x])/(3*b^2) - (d*Cosh[a + b*x]^3)/(9*b^2) + (2*(c + d*x)*S
inh[a + b*x])/(3*b) + ((c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b)
```

Rubi [A] time = 0.0442555, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3310, 3296, 2638}

$$-\frac{d \cosh^3(a+bx)}{9b^2} - \frac{2d \cosh(a+bx)}{3b^2} + \frac{2(c+dx)\sinh(a+bx)}{3b} + \frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cosh[a + b*x]^3,x]
```

```
[Out] (-2*d*Cosh[a + b*x])/(3*b^2) - (d*Cosh[a + b*x]^3)/(9*b^2) + (2*(c + d*x)*S
inh[a + b*x])/(3*b) + ((c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)\cosh^3(a+bx)\,dx = -\frac{d\cosh^3(a+bx)}{9b^2} + \frac{(c+dx)\cosh^2(a+bx)\sinh(a+bx)}{3b} + \frac{2}{3}\int (c+dx)\cosh(a+bx)\,dx$$
$$= -\frac{d\cosh^3(a+bx)}{9b^2} + \frac{2(c+dx)\sinh(a+bx)}{3b} + \frac{(c+dx)\cosh^2(a+bx)\sinh(a+bx)}{3b} - \frac{(2d)}{\ldots}$$
$$= -\frac{2d\cosh(a+bx)}{3b^2} - \frac{d\cosh^3(a+bx)}{9b^2} + \frac{2(c+dx)\sinh(a+bx)}{3b} + \frac{(c+dx)\cosh^2(a+bx)\,\mathrm{si}}{3b}$$

Mathematica [A] time = 0.227353, size = 52, normalized size = 0.69

$$-\frac{-3b(c+dx)(9\sinh(a+bx)+\sinh(3(a+bx)))+27d\cosh(a+bx)+d\cosh(3(a+bx))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cosh[a + b*x]^3,x]
```

```
[Out] -(27*d*Cosh[a + b*x] + d*Cosh[3*(a + b*x)] - 3*b*(c + d*x)*(9*Sinh[a + b*x]
 + Sinh[3*(a + b*x)]))/(36*b^2)
```

Maple [A] time = 0.009, size = 115, normalized size = 1.5

$$\frac{1}{b}\left(\frac{d}{b}\left(\frac{(2\,bx+2\,a)\sinh(bx+a)}{3}+\frac{(bx+a)\sinh(bx+a)\,(\cosh(bx+a))^{2}}{3}-\frac{7\,\cosh(bx+a)}{9}-\frac{(\sinh(bx+a))^{2}\cosh(b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cosh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b*d*(2/3*(b*x+a)*sinh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-7
/9*cosh(b*x+a)-1/9*sinh(b*x+a)^2*cosh(b*x+a))-1/b*d*a*(2/3+1/3*cosh(b*x+a)^
2)*sinh(b*x+a)+c*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a))
```

Maple [B] time = 1.07191, size = 193, normalized size = 2.57

$$\frac{1}{72}\,d\left(\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{b^{2}}+\frac{27\,(bxe^{a}-e^{a})e^{(bx)}}{b^{2}}-\frac{27\,(bx+1)e^{(-bx-a)}}{b^{2}}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^{2}}\right)+\frac{1}{24}\,c\left(\frac{e^{(3\,bx+3\,a)}}{b}+\frac{9\,e^{(}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/72*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 27*(b*x*e^a - e^a)*e^(b*x
)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 - (3*b*x + 1)*e^(-3*b*x - 3*a)/b^2) +
 1/24*c*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3*b*x
 - 3*a)/b)
```

Fricas [A] time = 1.96059, size = 257, normalized size = 3.43

$$-\frac{d\cosh(bx+a)^{3}+3\,d\cosh(bx+a)\sinh(bx+a)^{2}-3\,(bdx+bc)\sinh(bx+a)^{3}+27\,d\cosh(bx+a)-9\left(3\,bdx+(bc\right.}{36\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/36*(d*cosh(b*x + a)^3 + 3*d*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b*d*x + b
*c)*sinh(b*x + a)^3 + 27*d*cosh(b*x + a) - 9*(3*b*d*x + (b*d*x + b*c)*cosh(
b*x + a)^2 + 3*b*c)*sinh(b*x + a))/b^2
```

Sympy [A] time = 1.33362, size = 126, normalized size = 1.68

$$\begin{cases} -\frac{2c\sinh^3(a+bx)}{3b} + \frac{c\sinh(a+bx)\cosh^2(a+bx)}{b} - \frac{2dx\sinh^3(a+bx)}{3b} + \frac{dx\sinh(a+bx)\cosh^2(a+bx)}{b} + \frac{2d\sinh^2(a+bx)\cosh(a+bx)}{3b^2} - \frac{7d\cosh^3(a+bx)}{9b^2} \\ \left(cx + \frac{dx^2}{2}\right)\cosh^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((-2*c*sinh(a + b*x)**3/(3*b) + c*sinh(a + b*x)*cosh(a + b*x)**2/b
 - 2*d*x*sinh(a + b*x)**3/(3*b) + d*x*sinh(a + b*x)*cosh(a + b*x)**2/b + 2*
d*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**2) - 7*d*cosh(a + b*x)**3/(9*b**2),
Ne(b, 0)), ((c*x + d*x**2/2)*cosh(a)**3, True))
```

Giac [A] time = 1.30403, size = 132, normalized size = 1.76

$$\frac{(3\,bdx + 3\,bc - d)e^{(3\,bx+3\,a)}}{72\,b^2} + \frac{3\,(bdx + bc - d)e^{(bx+a)}}{8\,b^2} - \frac{3\,(bdx + bc + d)e^{(-bx-a)}}{8\,b^2} - \frac{(3\,bdx + 3\,bc + d)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*b*d*x + 3*b*c - d)*e^(3*b*x + 3*a)/b^2 + 3/8*(b*d*x + b*c - d)*e^(b
*x + a)/b^2 - 3/8*(b*d*x + b*c + d)*e^(-b*x - a)/b^2 - 1/72*(3*b*d*x + 3*b*
c + d)*e^(-3*b*x - 3*a)/b^2
```

3.20 $\int \frac{\cosh^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$\frac{3\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cosh\left(3a-\frac{3bc}{d}\right)\mathrm{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sinh\left(3a-\frac{3bc}{d}\right)\mathrm{S}}{4d}$$

```
[Out] (3*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/(4*d) + (Cosh[3*a - (3*b*
c)/d]*CoshIntegral[(3*b*c)/d + 3*b*x])/(4*d) + (3*Sinh[a - (b*c)/d]*SinhInt
egral[(b*c)/d + b*x])/(4*d) + (Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d
 + 3*b*x])/(4*d)
```

Rubi [A] time = 0.23984, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3303, 3298, 3301}

$$\frac{3\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cosh\left(3a-\frac{3bc}{d}\right)\mathrm{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sinh\left(3a-\frac{3bc}{d}\right)\mathrm{S}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/(c + d*x),x]
```

```
[Out] (3*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/(4*d) + (Cosh[3*a - (3*b*
c)/d]*CoshIntegral[(3*b*c)/d + 3*b*x])/(4*d) + (3*Sinh[a - (b*c)/d]*SinhInt
egral[(b*c)/d + b*x])/(4*d) + (Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d
 + 3*b*x])/(4*d)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)}{c+dx}\,dx = \int \left(\frac{3\cosh(a+bx)}{4(c+dx)} + \frac{\cosh(3a+3bx)}{4(c+dx)}\right) dx$$
$$= \frac{1}{4}\int \frac{\cosh(3a+3bx)}{c+dx}\,dx + \frac{3}{4}\int \frac{\cosh(a+bx)}{c+dx}\,dx$$
$$= \frac{1}{4}\cosh\left(3a-\frac{3bc}{d}\right)\int \frac{\cosh\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx + \frac{1}{4}\left(3\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx + \frac{1}{4}\sinh\left(\right.$$
$$= \frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d} + \frac{3\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d}$$

Mathematica [A] time = 0.227433, size = 102, normalized size = 0.84

$$\frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)+\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3b(c+dx)}{d}\right)+3\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)+\sinh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/(c + d*x),x]
```

```
[Out] (3*Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] + Cosh[3*a - (3*b*c)/d]*Cosh
Integral[(3*b*(c + d*x))/d] + 3*Sinh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)]
 + Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Maple [A] time = 0.088, size = 166, normalized size = 1.4

$$-\frac{1}{8d}e^{-3\frac{da-cb}{d}}\text{Ei}\left(1,3bx+3a-3\frac{da-cb}{d}\right)-\frac{3}{8d}e^{-\frac{da-cb}{d}}\text{Ei}\left(1,bx+a-\frac{da-cb}{d}\right)-\frac{3}{8d}e^{\frac{da-cb}{d}}\text{Ei}\left(1,-bx-a-\frac{-da+cb}{d}\right)-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c),x)
```

```
[Out] -1/8/d*exp(-3*(a*d-b*c)/d)*Ei(1,3*b*x+3*a-3*(a*d-b*c)/d)-3/8/d*exp(-(a*d-b*
c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-3/8/d*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)
/d)-1/8/d*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

Maxima [A] time = 1.27836, size = 158, normalized size = 1.31

$$-\frac{e^{\left(-3a+\frac{3bc}{d}\right)}E_1\left(\frac{3(dx+c)b}{d}\right)}{8d}-\frac{3e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{8d}-\frac{3e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{8d}-\frac{e^{\left(3a-\frac{3bc}{d}\right)}E_1\left(-\frac{3(dx+c)b}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(1, 3*(d*x + c)*b/d)/d - 3/8*e^(-a +
b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - 3/8*e^(a - b*c/d)*exp_integral_
e(1, -(d*x + c)*b/d)/d - 1/8*e^(3*a - 3*b*c/d)*exp_integral_e(1, -3*(d*x +
```

```
c)*b/d)/d
```

Fricas [A] time = 1.98404, size = 398, normalized size = 3.29

$$\frac{3\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{3\,(bc-ad)}{d}\right)+3\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\mathrm{I}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(Ei((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) +
(Ei(3*(b*d*x + b*c)/d) + Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d) + 3
*(Ei((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) + (Ei(3*
(b*d*x + b*c)/d) - Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3{(a+bx)}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)**3/(c + d*x), x)
```

Giac [A] time = 1.33214, size = 151, normalized size = 1.25

$$\frac{\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}+3\,\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+3\,\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 3*Ei((b*d*x + b*c)/d)*e^(a -
 b*c/d) + 3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + Ei(-3*(b*d*x + b*c)/d)*e^
(-3*a + 3*b*c/d))/d
```

3.21 $\int \frac{\cosh^3(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=145

$$\frac{3b\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}+\frac{3b\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cosh\left(3a-\right.}{}$$

```
[Out] -(Cosh[a + b*x]^3/(d*(c + d*x))) + (3*b*CoshIntegral[(3*b*c)/d + 3*b*x]*Sin
h[3*a - (3*b*c)/d])/(4*d^2) + (3*b*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*
c)/d])/(4*d^2) + (3*b*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(4*d^2
) + (3*b*Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```

Rubi [A] time = 0.23707, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3313, 3303, 3298, 3301}

$$\frac{3b\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}+\frac{3b\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cosh\left(3a-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] -(Cosh[a + b*x]^3/(d*(c + d*x))) + (3*b*CoshIntegral[(3*b*c)/d + 3*b*x]*Sin
h[3*a - (3*b*c)/d])/(4*d^2) + (3*b*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*
c)/d])/(4*d^2) + (3*b*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(4*d^2
) + (3*b*Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx = -\frac{\cosh^3(a+bx)}{d(c+dx)} + \frac{(3ib)\int\left(-\frac{i\sinh(a+bx)}{4(c+dx)} - \frac{i\sinh(3a+3bx)}{4(c+dx)}\right)dx}{d}$$
$$= -\frac{\cosh^3(a+bx)}{d(c+dx)} + \frac{(3b)\int\frac{\sinh(a+bx)}{c+dx}\,dx}{4d} + \frac{(3b)\int\frac{\sinh(3a+3bx)}{c+dx}\,dx}{4d}$$
$$= -\frac{\cosh^3(a+bx)}{d(c+dx)} + \frac{\left(3b\cosh\left(3a-\frac{3bc}{d}\right)\right)\int\frac{\sinh\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d} + \frac{\left(3b\cosh\left(a-\frac{bc}{d}\right)\right)\int\frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{4d}$$
$$= -\frac{\cosh^3(a+bx)}{d(c+dx)} + \frac{3b\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{4d^2} + \frac{3b\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{4d^2} + \ldots$$

Mathematica [A] time = 0.533832, size = 196, normalized size = 1.35

$$-\frac{3b\left(-2\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)-2\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)-2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)-2\cosh\left(3a\ldots\right.\right.}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] (-3*Cosh[a]*Cosh[b*x])/(4*d*(c + d*x)) - (Cosh[3*a]*Cosh[3*b*x])/(4*d*(c +
d*x)) - (3*Sinh[a]*Sinh[b*x])/(4*d*(c + d*x)) - (Sinh[3*a]*Sinh[3*b*x])/(4*
d*(c + d*x)) - (3*b*(-2*CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*c)/
d] - 2*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d] - 2*Cosh[a - (b*c)/d]*
SinhIntegral[(b*c)/d + b*x] - 2*Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/
d + 3*b*x]))/(8*d^2)
```

Maple [A] time = 0.102, size = 271, normalized size = 1.9

$$-\frac{b\mathrm{e}^{-3bx-3a}}{(8bdx+8cb)d} + \frac{3b}{8d^2}\mathrm{e}^{-3\frac{da-cb}{d}}\mathrm{Ei}\left(1,3bx+3a-3\frac{da-cb}{d}\right) - \frac{3b\mathrm{e}^{-bx-a}}{8d(bdx+cb)} + \frac{3b}{8d^2}\mathrm{e}^{-\frac{da-cb}{d}}\mathrm{Ei}\left(1,bx+a-\frac{da-cb}{d}\right) - \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] -1/8*b*exp(-3*b*x-3*a)/(b*d*x+b*c)/d+3/8*b/d^2*exp(-3*(a*d-b*c)/d)*Ei(1,3*b
*x+3*a-3*(a*d-b*c)/d)-3/8*b*exp(-b*x-a)/d/(b*d*x+b*c)+3/8*b/d^2*exp(-(a*d-b
*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-3/8*b/d^2*exp(b*x+a)/(b*c/d+b*x)-3/8*b/d^2*e
xp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)-1/8*b/d^2*exp(3*b*x+3*a)/(b*c/d+b
*x)-3/8*b/d^2*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

Maxima [A] time = 1.33496, size = 196, normalized size = 1.35

$$-\frac{e^{\left(-3a+\frac{3bc}{d}\right)}E_2\left(\frac{3(dx+c)b}{d}\right)}{8(dx+c)d} - \frac{3e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{8(dx+c)d} - \frac{3e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{8(dx+c)d} - \frac{e^{\left(3a-\frac{3bc}{d}\right)}E_2\left(-\frac{3(dx+c)b}{d}\right)}{8(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(2, 3*(d*x + c)*b/d)/((d*x + c)*d) -
3/8*e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) - 3/8*e^(
a - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d) - 1/8*e^(3*a - 3
*b*c/d)*exp_integral_e(2, -3*(d*x + c)*b/d)/((d*x + c)*d)
```

Fricas [B] time = 1.94727, size = 680, normalized size = 4.69

$$-\frac{2\,d\cosh(bx+a)^3+6\,d\cosh(bx+a)\sinh(bx+a)^2+6\,d\cosh(bx+a)-3\left((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-(bdx+bc)\mathrm{Ei}\left(-\frac{bdx}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*d*cosh(b*x + a)^3 + 6*d*cosh(b*x + a)*sinh(b*x + a)^2 + 6*d*cosh(b*
x + a) - 3*((b*d*x + b*c)*Ei((b*d*x + b*c)/d) - (b*d*x + b*c)*Ei(-(b*d*x +
b*c)/d))*cosh(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*Ei(3*(b*d*x + b*c)/d) - (b
*d*x + b*c)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d) - 3*((b*d*x + b*
c)*Ei((b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a
*d)/d) - 3*((b*d*x + b*c)*Ei(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-3*(b*d*
x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)**3/(c + d*x)**2, x)
```

Giac [B] time = 1.39483, size = 401, normalized size = 2.77

$$\frac{3\,bdx\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}+3\,bdx\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}-3\,bdx\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}-3\,bdx\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}+3\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*d*x*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 3*b*d*x*Ei((b*d*x +
b*c)/d)*e^(a - b*c/d) - 3*b*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 3*b*d
*x*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 3*b*c*Ei(3*(b*d*x + b*c)/d)*
```

```
e^(3*a - 3*b*c/d) + 3*b*c*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) - 3*b*c*Ei(-(b*
d*x + b*c)/d)*e^(-a + b*c/d) - 3*b*c*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c
/d) - d*e^(3*b*x + 3*a) - 3*d*e^(b*x + a) - 3*d*e^(-b*x - a) - d*e^(-3*b*x
- 3*a))/(d^3*x + c*d^2)
```

3.22 $\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=184

$$\frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{3b^2\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\sinh\left(3a\right.}{}$$

```
[Out] -Cosh[a + b*x]^3/(2*d*(c + d*x)^2) + (3*b^2*Cosh[a - (b*c)/d]*CoshIntegral[
(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cosh[3*a - (3*b*c)/d]*CoshIntegral[(3*b*c)
/d + 3*b*x])/(8*d^3) - (3*b*Cosh[a + b*x]^2*Sinh[a + b*x])/(2*d^2*(c + d*x)
) + (3*b^2*Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*
Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rubi [A] time = 0.340069, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3314, 3303, 3298, 3301, 3312}

$$\frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{3b^2\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\sinh\left(3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/(c + d*x)^3,x]
```


```
[Out] -Cosh[a + b*x]^3/(2*d*(c + d*x)^2) + (3*b^2*Cosh[a - (b*c)/d]*CoshIntegral[
(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cosh[3*a - (3*b*c)/d]*CoshIntegral[(3*b*c)
/d + 3*b*x])/(8*d^3) - (3*b*Cosh[a + b*x]^2*Sinh[a + b*x])/(2*d^2*(c + d*x)
) + (3*b^2*Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*
Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx = -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} - \frac{3b\cosh^2(a+bx)\sinh(a+bx)}{2d^2(c+dx)} - \frac{\left(3b^2\right)\int \frac{\cosh(a+bx)}{c+dx}\,dx}{d^2} + \frac{\left(9b^2\right)\int \frac{\cosh^3(a+bx)}{c+dx}$$

$$= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} - \frac{3b\cosh^2(a+bx)\sinh(a+bx)}{2d^2(c+dx)} + \frac{\left(9b^2\right)\int \left(\frac{3\cosh(a+bx)}{4(c+dx)} + \frac{\cosh(3a+3bx)}{4(c+dx)}\right)dx}{2d^2} - \Big($$

$$= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} - \frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d^3} - \frac{3b\cosh^2(a+bx)\sinh(a+bx)}{2d^2(c+dx)} - \frac{3b^2\sinh}{}$$

$$= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} - \frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d^3} - \frac{3b\cosh^2(a+bx)\sinh(a+bx)}{2d^2(c+dx)} - \frac{3b^2\sinh}{}$$

$$= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{8d^3} + \frac{9b^2\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{8d^3} - 3$$

Mathematica [A] time = 0.899937, size = 218, normalized size = 1.18

$$-\frac{-6b^2(c+dx)^2\left(\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)+3\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3b(c+dx)}{d}\right)+\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)+3\,\text{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] -(6*d*Cosh[b*x]*(d*Cosh[a] + b*(c + d*x)*Sinh[a]) + 2*d*Cosh[3*b*x]*(d*Cosh
[3*a] + 3*b*(c + d*x)*Sinh[3*a]) + 6*d*(b*(c + d*x)*Cosh[a] + d*Sinh[a])*Si
nh[b*x] + 2*d*(3*b*(c + d*x)*Cosh[3*a] + d*Sinh[3*a])*Sinh[3*b*x] - 6*b^2*(
c + d*x)^2*(Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] + 3*Cosh[3*a - (3*b
*c)/d]*CoshIntegral[(3*b*(c + d*x))/d] + Sinh[a - (b*c)/d]*SinhIntegral[b*(
c/d + x)] + 3*Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d]))/(16*d
^3*(c + d*x)^2)
```

Maple [B] time = 0.106, size = 562, normalized size = 3.1

$$\frac{3\,b^3 e^{-3\,bx-3\,a}x}{16\,d\left(b^2d^2x^2+2\,b^2cdx+c^2b^2\right)} + \frac{3\,b^3 e^{-3\,bx-3\,a}c}{16\,d^2\left(b^2d^2x^2+2\,b^2cdx+c^2b^2\right)} - \frac{b^2 e^{-3\,bx-3\,a}}{16\,d\left(b^2d^2x^2+2\,b^2cdx+c^2b^2\right)} - \frac{9\,b^2}{16\,d^3}e^{-3\,\frac{da-cb}{d}}\text{Ei}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 3/16*b^3*exp(-3*b*x-3*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x+3/16*b^3*exp
(-3*b*x-3*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/16*b^2*exp(-3*b*x-3*
a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-9/16*b^2/d^3*exp(-3*(a*d-b*c)/d)*Ei(
1,3*b*x+3*a-3*(a*d-b*c)/d)+3/16*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+
b^2*c^2)*x+3/16*b^3*exp(-b*x-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-3/1
6*b^2*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-3/16*b^2/d^3*exp(-(a*
d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-3/16*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)^2-3/1
6*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)-3/16*b^2/d^3*exp((a*d-b*c)/d)*Ei(1,-b*x-a-
(-a*d+b*c)/d)-1/16*b^2/d^3*exp(3*b*x+3*a)/(b*c/d+b*x)^2-3/16*b^2/d^3*exp(3*
b*x+3*a)/(b*c/d+b*x)-9/16*b^2/d^3*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*
d+b*c)/d)
```

Maxima [A] time = 1.35596, size = 196, normalized size = 1.07

$$-\frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_3\left(\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)^2d}-\frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_3\left(\frac{(dx+c)b}{d}\right)}{8\,(dx+c)^2d}-\frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_3\left(-\frac{(dx+c)b}{d}\right)}{8\,(dx+c)^2d}-\frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_3\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(3, 3*(d*x + c)*b/d)/((d*x + c)^2*d)
- 3/8*e^(-a + b*c/d)*exp_integral_e(3, (d*x + c)*b/d)/((d*x + c)^2*d) - 3/8
*e^(a - b*c/d)*exp_integral_e(3, -(d*x + c)*b/d)/((d*x + c)^2*d) - 1/8*e^(3
*a - 3*b*c/d)*exp_integral_e(3, -3*(d*x + c)*b/d)/((d*x + c)^2*d)
```

Fricas [B] time = 2.04085, size = 1122, normalized size = 6.1

$$-\frac{2\,d^2\cosh\left(bx+a\right)^3+6\,d^2\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+6\left(bd^2x+bcd\right)\sinh\left(bx+a\right)^3+6\,d^2\cosh\left(bx+a\right)-3\left(\left(b^2d^2x^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*d^2*cosh(b*x + a)^3 + 6*d^2*cosh(b*x + a)*sinh(b*x + a)^2 + 6*(b*d
^2*x + b*c*d)*sinh(b*x + a)^3 + 6*d^2*cosh(b*x + a) - 3*((b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d) + 6*(b*d^2*x + b*c*d + 3*
(b*d^2*x + b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a) - 3*((b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*
d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh^3\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(cosh(a + b*x)**3/(c + d*x)**3, x)
```

Giac [B] time = 1.30806, size = 813, normalized size = 4.42

$$\frac{9\,b^2d^2x^2\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}+3\,b^2d^2x^2\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+3\,b^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+9\,b^2d^2x^2\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*(9*b^2*d^2*x^2*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 3*b^2*d^2*x^2
*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + 3*b^2*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e^(
-a + b*c/d) + 9*b^2*d^2*x^2*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 18*
b^2*c*d*x*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 6*b^2*c*d*x*Ei((b*d*x +
 b*c)/d)*e^(a - b*c/d) + 6*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) +
18*b^2*c*d*x*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 9*b^2*c^2*Ei(3*(b*
d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 3*b^2*c^2*Ei((b*d*x + b*c)/d)*e^(a - b*c/
d) + 3*b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 9*b^2*c^2*Ei(-3*(b*d*x
 + b*c)/d)*e^(-3*a + 3*b*c/d) - 3*b*d^2*x*e^(3*b*x + 3*a) - 3*b*d^2*x*e^(b*
x + a) + 3*b*d^2*x*e^(-b*x - a) + 3*b*d^2*x*e^(-3*b*x - 3*a) - 3*b*c*d*e^(3
*b*x + 3*a) - 3*b*c*d*e^(b*x + a) + 3*b*c*d*e^(-b*x - a) + 3*b*c*d*e^(-3*b*
x - 3*a) - d^2*e^(3*b*x + 3*a) - 3*d^2*e^(b*x + a) - 3*d^2*e^(-b*x - a) - d
^2*e^(-3*b*x - 3*a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.23 $\int x^3 \cosh^4(a + bx) \, dx$

Optimal. Leaf size=172

$$-\frac{3x^2\cosh^4(a+bx)}{16b^2}-\frac{9x^2\cosh^2(a+bx)}{16b^2}-\frac{3\cosh^4(a+bx)}{128b^4}-\frac{45\cosh^2(a+bx)}{128b^4}+\frac{3x\sinh(a+bx)\cosh^3(a+bx)}{32b^3}+\frac{45x\,\mathrm{s}}{}$$

```
[Out] (45*x^2)/(128*b^2) + (3*x^4)/32 - (45*Cosh[a + b*x]^2)/(128*b^4) - (9*x^2*C
osh[a + b*x]^2)/(16*b^2) - (3*Cosh[a + b*x]^4)/(128*b^4) - (3*x^2*Cosh[a +
b*x]^4)/(16*b^2) + (45*x*Cosh[a + b*x]*Sinh[a + b*x])/(64*b^3) + (3*x^3*Cos
h[a + b*x]*Sinh[a + b*x])/(8*b) + (3*x*Cosh[a + b*x]^3*Sinh[a + b*x])/(32*b
^3) + (x^3*Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b)
```

Rubi [A] time = 0.145787, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 30, 3310}

$$-\frac{3x^2\cosh^4(a+bx)}{16b^2}-\frac{9x^2\cosh^2(a+bx)}{16b^2}-\frac{3\cosh^4(a+bx)}{128b^4}-\frac{45\cosh^2(a+bx)}{128b^4}+\frac{3x\sinh(a+bx)\cosh^3(a+bx)}{32b^3}+\frac{45x\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^4,x]
```


```
[Out] (45*x^2)/(128*b^2) + (3*x^4)/32 - (45*Cosh[a + b*x]^2)/(128*b^4) - (9*x^2*C
osh[a + b*x]^2)/(16*b^2) - (3*Cosh[a + b*x]^4)/(128*b^4) - (3*x^2*Cosh[a +
b*x]^4)/(16*b^2) + (45*x*Cosh[a + b*x]*Sinh[a + b*x])/(64*b^3) + (3*x^3*Cos
h[a + b*x]*Sinh[a + b*x])/(8*b) + (3*x*Cosh[a + b*x]^3*Sinh[a + b*x])/(32*b
^3) + (x^3*Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int x^3\cosh^4(a+bx)\,dx = -\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{x^3\cosh^3(a+bx)\sinh(a+bx)}{4b}+\frac{3}{4}\int x^3\cosh^2(a+bx)\,dx+\frac{3\int x\cos$$

$$= -\frac{9x^2\cosh^2(a+bx)}{16b^2}-\frac{3\cosh^4(a+bx)}{128b^4}-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{3x^3\cosh(a+bx)\sinh(a+bx}{8b}$$

$$= \frac{3x^4}{32}-\frac{45\cosh^2(a+bx)}{128b^4}-\frac{9x^2\cosh^2(a+bx)}{16b^2}-\frac{3\cosh^4(a+bx)}{128b^4}-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{45x}{}$$

$$= \frac{45x^2}{128b^2}+\frac{3x^4}{32}-\frac{45\cosh^2(a+bx)}{128b^4}-\frac{9x^2\cosh^2(a+bx)}{16b^2}-\frac{3\cosh^4(a+bx)}{128b^4}-\frac{3x^2\cosh^4(a+b}{16b^2}$$

Mathematica [A] time = 0.404904, size = 100, normalized size = 0.58

$$\frac{4bx\left(32\left(2b^2x^2+3\right)\sinh(2(a+bx))+\left(8b^2x^2+3\right)\sinh(4(a+bx))+24b^3x^3\right)-192\left(2b^2x^2+1\right)\cosh(2(a+bx))-3\,(8}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^4,x]
```

```
[Out] (-192*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] - 3*(1 + 8*b^2*x^2)*Cosh[4*(a + b*x
)] + 4*b*x*(24*b^3*x^3 + 32*(3 + 2*b^2*x^2)*Sinh[2*(a + b*x)] + (3 + 8*b^2*
x^2)*Sinh[4*(a + b*x)]))/(1024*b^4)
```

Maple [B] time = 0.011, size = 432, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^4,x)
```

```
[Out] 1/b^4*(1/4*(b*x+a)^3*sinh(b*x+a)*cosh(b*x+a)^3+3/8*(b*x+a)^3*cosh(b*x+a)*si
nh(b*x+a)+3/32*(b*x+a)^4-3/16*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^2-3/4*(b*
x+a)^2*cosh(b*x+a)^2+3/32*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3+45/64*(b*x+a)*c
osh(b*x+a)*sinh(b*x+a)+45/128*(b*x+a)^2-3/128*sinh(b*x+a)^2*cosh(b*x+a)^2-3
/8*cosh(b*x+a)^2-3*a*(1/4*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^3+3/8*(b*x+a)^2
*cosh(b*x+a)*sinh(b*x+a)+1/8*(b*x+a)^3-1/8*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a
)^2-1/2*(b*x+a)*cosh(b*x+a)^2+1/32*sinh(b*x+a)*cosh(b*x+a)^3+15/64*cosh(b*x
+a)*sinh(b*x+a)+15/64*b*x+15/64*a)+3*a^2*(1/4*(b*x+a)*sinh(b*x+a)*cosh(b*x+
a)^3+3/8*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+3/16*(b*x+a)^2-1/16*sinh(b*x+a)^2*
cosh(b*x+a)^2-1/4*cosh(b*x+a)^2)-a^3*((1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*s
inh(b*x+a)+3/8*b*x+3/8*a))
```

Maxima [A] time = 1.04323, size = 238, normalized size = 1.38

$$\frac{3}{32}x^4+\frac{\left(32\,b^3x^3e^{(4\,a)}-24\,b^2x^2e^{(4\,a)}+12\,bxe^{(4\,a)}-3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4}+\frac{\left(4\,b^3x^3e^{(2\,a)}-6\,b^2x^2e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 3/32*x^4 + 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a)
 - 3*e^(4*a))*e^(4*b*x)/b^4 + 1/32*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) +
 6*b*x*e^(2*a) - 3*e^(2*a))*e^(2*b*x)/b^4 - 1/32*(4*b^3*x^3 + 6*b^2*x^2 + 6
*b*x + 3)*e^(-2*b*x - 2*a)/b^4 - 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x +
 3)*e^(-4*b*x - 4*a)/b^4
```

Fricas [A] time = 2.05032, size = 486, normalized size = 2.83

$$\frac{96\,b^4x^4 - 3\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)^4 + 16\left(8\,b^3x^3+3\,bx\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3 - 3\left(8\,b^2x^2+1\right)\sinh\left(bx+a\right)^4 - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/1024*(96*b^4*x^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^4 + 16*(8*b^3*x^3 + 3*
b*x)*cosh(b*x + a)*sinh(b*x + a)^3 - 3*(8*b^2*x^2 + 1)*sinh(b*x + a)^4 - 19
2*(2*b^2*x^2 + 1)*cosh(b*x + a)^2 - 6*(64*b^2*x^2 + 3*(8*b^2*x^2 + 1)*cosh(
b*x + a)^2 + 32)*sinh(b*x + a)^2 + 16*((8*b^3*x^3 + 3*b*x)*cosh(b*x + a)^3
+ 16*(2*b^3*x^3 + 3*b*x)*cosh(b*x + a))*sinh(b*x + a))/b^4
```

Sympy [A] time = 8.10609, size = 262, normalized size = 1.52

$$\begin{cases} \frac{3x^4\sinh^4(a+bx)}{32} - \frac{3x^4\sinh^2(a+bx)\cosh^2(a+bx)}{16} + \frac{3x^4\cosh^4(a+bx)}{32} - \frac{3x^3\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{5x^3\sinh(a+bx)\cosh^3(a+bx)}{8b} + \frac{45x^2\sinh^4(a \cdots}{128b^2} \cdots \\ \frac{x^4\cosh^4(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x**4*sinh(a + b*x)**4/32 - 3*x**4*sinh(a + b*x)**2*cosh(a + b*
x)**2/16 + 3*x**4*cosh(a + b*x)**4/32 - 3*x**3*sinh(a + b*x)**3*cosh(a + b*
x)/(8*b) + 5*x**3*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) + 45*x**2*sinh(a + b
*x)**4/(128*b**2) - 9*x**2*sinh(a + b*x)**2*cosh(a + b*x)**2/(64*b**2) - 51
*x**2*cosh(a + b*x)**4/(128*b**2) - 45*x*sinh(a + b*x)**3*cosh(a + b*x)/(64
*b**3) + 51*x*sinh(a + b*x)*cosh(a + b*x)**3/(64*b**3) + 3*sinh(a + b*x)**4
/(8*b**4) - 51*sinh(a + b*x)**2*cosh(a + b*x)**2/(128*b**4), Ne(b, 0)), (x*
*4*cosh(a)**4/4, True))
```

Giac [A] time = 1.23744, size = 203, normalized size = 1.18

$$\frac{3}{32}\,x^4 + \frac{\left(32\,b^3x^3 - 24\,b^2x^2 + 12\,bx - 3\right)e^{\left(4\,bx+4\,a\right)}}{2048\,b^4} + \frac{\left(4\,b^3x^3 - 6\,b^2x^2 + 6\,bx - 3\right)e^{\left(2\,bx+2\,a\right)}}{32\,b^4} - \frac{\left(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3\right)e^{(- \cdots}}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/32*x^4 + 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b^
4 + 1/32*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 - 1/32*(4*
b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 - 1/2048*(32*b^3*x^3
+ 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

3.24 $\int x^2 \cosh^4(a + bx)\, dx$

Optimal. Leaf size=134

$$-\frac{x\cosh^4(a+bx)}{8b^2}-\frac{3x\cosh^2(a+bx)}{8b^2}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{32b^3}+\frac{15\sinh(a+bx)\cosh(a+bx)}{64b^3}+\frac{x^2\sinh(a+bx}{4}$$

```
[Out] (15*x)/(64*b^2) + x^3/8 - (3*x*Cosh[a + b*x]^2)/(8*b^2) - (x*Cosh[a + b*x]^
4)/(8*b^2) + (15*Cosh[a + b*x]*Sinh[a + b*x])/(64*b^3) + (3*x^2*Cosh[a + b*
x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x])/(32*b^3) + (x^2*C
osh[a + b*x]^3*Sinh[a + b*x])/(4*b)
```

Rubi [A] time = 0.106692, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3311, 30, 2635, 8}

$$-\frac{x\cosh^4(a+bx)}{8b^2}-\frac{3x\cosh^2(a+bx)}{8b^2}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{32b^3}+\frac{15\sinh(a+bx)\cosh(a+bx)}{64b^3}+\frac{x^2\sinh(a+bx}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^4,x]
```


```
[Out] (15*x)/(64*b^2) + x^3/8 - (3*x*Cosh[a + b*x]^2)/(8*b^2) - (x*Cosh[a + b*x]^
4)/(8*b^2) + (15*Cosh[a + b*x]*Sinh[a + b*x])/(64*b^3) + (3*x^2*Cosh[a + b*
x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x])/(32*b^3) + (x^2*C
osh[a + b*x]^3*Sinh[a + b*x])/(4*b)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x^2\cosh^4(a+bx)\,dx = -\frac{x\cosh^4(a+bx)}{8b^2} + \frac{x^2\cosh^3(a+bx)\sinh(a+bx)}{4b} + \frac{3}{4}\int x^2\cosh^2(a+bx)\,dx + \frac{\int\cosh^4(a+b}{8b^2}$$
$$= -\frac{3x\cosh^2(a+bx)}{8b^2} - \frac{x\cosh^4(a+bx)}{8b^2} + \frac{3x^2\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a}{32b^3}$$
$$= \frac{x^3}{8} - \frac{3x\cosh^2(a+bx)}{8b^2} - \frac{x\cosh^4(a+bx)}{8b^2} + \frac{15\cosh(a+bx)\sinh(a+bx)}{64b^3} + \frac{3x^2\cosh(a+bx)\mathrm{s}}{8b}$$
$$= \frac{15x}{64b^2} + \frac{x^3}{8} - \frac{3x\cosh^2(a+bx)}{8b^2} - \frac{x\cosh^4(a+bx)}{8b^2} + \frac{15\cosh(a+bx)\sinh(a+bx)}{64b^3} + \frac{3x^2\cosh(}{}$$

Mathematica [A] time = 0.157266, size = 90, normalized size = 0.67

$$\frac{64b^2x^2\sinh(2(a+bx)) + 8b^2x^2\sinh(4(a+bx)) + 32\sinh(2(a+bx)) + \sinh(4(a+bx)) - 64bx\cosh(2(a+bx)) - 4bx\cos}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^4,x]
```

```
[Out] (32*b^3*x^3 - 64*b*x*Cosh[2*(a + b*x)] - 4*b*x*Cosh[4*(a + b*x)] + 32*Sinh[
2*(a + b*x)] + 64*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 8*b^2*x^2
*Sinh[4*(a + b*x)])/(256*b^3)
```

Maple [B] time = 0.007, size = 253, normalized size = 1.9

$$\frac{1}{b^3}\left(\frac{(bx+a)^2\sinh(bx+a)(\cosh(bx+a))^3}{4} + \frac{3\,(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{8} + \frac{(bx+a)^3}{8} - \frac{(bx+a)(\sinh(bx+}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^4,x)
```

```
[Out] 1/b^3*(1/4*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^3+3/8*(b*x+a)^2*cosh(b*x+a)*si
nh(b*x+a)+1/8*(b*x+a)^3-1/8*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^2-1/2*(b*x+a)
*cosh(b*x+a)^2+1/32*sinh(b*x+a)*cosh(b*x+a)^3+15/64*cosh(b*x+a)*sinh(b*x+a)
+15/64*b*x+15/64*a-2*a*(1/4*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3+3/8*(b*x+a)*c
osh(b*x+a)*sinh(b*x+a)+3/16*(b*x+a)^2-1/16*sinh(b*x+a)^2*cosh(b*x+a)^2-1/4*
cosh(b*x+a)^2)+a^2*((1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)+3/8*b*x
+3/8*a))
```

Maxima [A] time = 1.04815, size = 178, normalized size = 1.33

$$\frac{1}{8}x^3 + \frac{\left(8\,b^2x^2e^{(4\,a)} - 4\,bxe^{(4\,a)} + e^{(4\,a)}\right)e^{(4\,bx)}}{512\,b^3} + \frac{\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^3} - \frac{\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx-2\,a)}}{16\,b^3} - \frac{(8\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/8*x^3 + 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b^3
 + 1/16*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 - 1/16*
```

```
(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 - 1/512*(8*b^2*x^2 + 4*b*x + 1
)*e^(-4*b*x - 4*a)/b^3
```

Fricas [A] time = 1.98625, size = 373, normalized size = 2.78

$$\frac{8\,b^3x^3 - bx\cosh\left(bx+a\right)^4 - bx\sinh\left(bx+a\right)^4 + \left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3 - 16\,bx\cosh\left(bx+a\right)^2 - 2\left(3\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/64*(8*b^3*x^3 - b*x*cosh(b*x + a)^4 - b*x*sinh(b*x + a)^4 + (8*b^2*x^2 +
1)*cosh(b*x + a)*sinh(b*x + a)^3 - 16*b*x*cosh(b*x + a)^2 - 2*(3*b*x*cosh(b
*x + a)^2 + 8*b*x)*sinh(b*x + a)^2 + ((8*b^2*x^2 + 1)*cosh(b*x + a)^3 + 16*
(2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a))/b^3
```

Sympy [A] time = 4.3555, size = 209, normalized size = 1.56

$$\begin{cases} \frac{x^3\sinh^4\left(a+bx\right)}{8} - \frac{x^3\sinh^2\left(a+bx\right)\cosh^2\left(a+bx\right)}{4} + \frac{x^3\cosh^4\left(a+bx\right)}{8} - \frac{3x^2\sinh^3\left(a+bx\right)\cosh\left(a+bx\right)}{8b} + \frac{5x^2\sinh\left(a+bx\right)\cosh^3\left(a+bx\right)}{8b} + \frac{15x\sinh^4\left(a\right.}{64b^2} \\ \frac{x^3\cosh^4\left(a\right)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((x**3*sinh(a + b*x)**4/8 - x**3*sinh(a + b*x)**2*cosh(a + b*x)**2
/4 + x**3*cosh(a + b*x)**4/8 - 3*x**2*sinh(a + b*x)**3*cosh(a + b*x)/(8*b)
+ 5*x**2*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) + 15*x*sinh(a + b*x)**4/(64*b
**2) - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/(32*b**2) - 17*x*cosh(a + b*x)
**4/(64*b**2) - 15*sinh(a + b*x)**3*cosh(a + b*x)/(64*b**3) + 17*sinh(a + b
*x)*cosh(a + b*x)**3/(64*b**3), Ne(b, 0)), (x**3*cosh(a)**4/3, True))
```

Giac [A] time = 1.30187, size = 159, normalized size = 1.19

$$\frac{1}{8}\,x^3 + \frac{\left(8\,b^2x^2 - 4\,bx + 1\right)e^{\left(4\,bx+4\,a\right)}}{512\,b^3} + \frac{\left(2\,b^2x^2 - 2\,bx + 1\right)e^{\left(2\,bx+2\,a\right)}}{16\,b^3} - \frac{\left(2\,b^2x^2 + 2\,bx + 1\right)e^{\left(-2\,bx-2\,a\right)}}{16\,b^3} - \frac{\left(8\,b^2x^2 + 4\,bx + 1\right.}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/8*x^3 + 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 + 1/16*(2*b^2*x
^2 - 2*b*x + 1)*e^(2*b*x + 2*a)/b^3 - 1/16*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*
x - 2*a)/b^3 - 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

3.25 $\int x \cosh^4(a+bx)\,dx$

Optimal. Leaf size=80

$$-\frac{\cosh^4(a+bx)}{16b^2}-\frac{3\cosh^2(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3x\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x^2}{16}$$

```
[Out] (3*x^2)/16 - (3*Cosh[a + b*x]^2)/(16*b^2) - Cosh[a + b*x]^4/(16*b^2) + (3*x
*Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (x*Cosh[a + b*x]^3*Sinh[a + b*x])/(4*
b)
```

Rubi [A] time = 0.0436869, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3310, 30}

$$-\frac{\cosh^4(a+bx)}{16b^2}-\frac{3\cosh^2(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3x\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^4,x]
```

```
[Out] (3*x^2)/16 - (3*Cosh[a + b*x]^2)/(16*b^2) - Cosh[a + b*x]^4/(16*b^2) + (3*x
*Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (x*Cosh[a + b*x]^3*Sinh[a + b*x])/(4*
b)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\cosh^4(a+bx)\,dx &= -\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\cosh^3(a+bx)\sinh(a+bx)}{4b}+\frac{3}{4}\int x\cosh^2(a+bx)\,dx\\ &= -\frac{3\cosh^2(a+bx)}{16b^2}-\frac{\cosh^4(a+bx)}{16b^2}+\frac{3x\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{x\cosh^3(a+bx)\sinh(a+b}{4b}\\ &= \frac{3x^2}{16}-\frac{3\cosh^2(a+bx)}{16b^2}-\frac{\cosh^4(a+bx)}{16b^2}+\frac{3x\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{x\cosh^3(a+bx)\sinh}{4b}\end{aligned}$$

Mathematica [A] time = 0.175357, size = 53, normalized size = 0.66

$$-\frac{-4bx(8\sinh(2(a+bx))+\sinh(4(a+bx))+6bx)+16\cosh(2(a+bx))+\cosh(4(a+bx))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^4,x]
```

```
[Out] -(16*Cosh[2*(a + b*x)] + Cosh[4*(a + b*x)] - 4*b*x*(6*b*x + 8*Sinh[2*(a + b
*x)] + Sinh[4*(a + b*x)]))/(128*b^2)
```

Maple [A] time = 0.007, size = 120, normalized size = 1.5

$$\frac{1}{b^2}\left(\frac{(bx+a)\sinh(bx+a)(\cosh(bx+a))^3}{4}+\frac{(3\,bx+3\,a)\cosh(bx+a)\sinh(bx+a)}{8}+\frac{3\,(bx+a)^2}{16}-\frac{(\sinh(bx+a))^2}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^4,x)
```

```
[Out] 1/b^2*(1/4*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3+3/8*(b*x+a)*cosh(b*x+a)*sinh(b
*x+a)+3/16*(b*x+a)^2-1/16*sinh(b*x+a)^2*cosh(b*x+a)^2-1/4*cosh(b*x+a)^2-a*(
(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)+3/8*b*x+3/8*a))
```

Maxima [A] time = 1.07999, size = 130, normalized size = 1.62

$$\frac{3}{16}\,x^2+\frac{\left(4\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2}+\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2}-\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 3/16*x^2 + 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 + 1/16*(2*b*x*e^(2
*a) - e^(2*a))*e^(2*b*x)/b^2 - 1/16*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 - 1/25
6*(4*b*x + 1)*e^(-4*b*x - 4*a)/b^2
```

Fricas [A] time = 2.02112, size = 306, normalized size = 3.82

$$\frac{16\,bx\cosh(bx+a)\sinh(bx+a)^3+24\,b^2x^2-\cosh(bx+a)^4-\sinh(bx+a)^4-2\left(3\,\cosh(bx+a)^2+8\right)\sinh(bx+a)}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 24*b^2*x^2 - cosh(b*x + a)^4
- sinh(b*x + a)^4 - 2*(3*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^2 - 16*cosh(b*x
 + a)^2 + 16*(b*x*cosh(b*x + a)^3 + 4*b*x*cosh(b*x + a))*sinh(b*x + a))/b^2
```

Sympy [A] time = 2.37883, size = 144, normalized size = 1.8

$$\begin{cases}\frac{3x^2\sinh^4(a+bx)}{16}-\frac{3x^2\sinh^2(a+bx)\cosh^2(a+bx)}{8}+\frac{3x^2\cosh^4(a+bx)}{16}-\frac{3x\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{5x\sinh(a+bx)\cosh^3(a+bx)}{8b}+\frac{\sinh^4(a+b}{4b^2} \\ \frac{x^2\cosh^4(a)}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x**2*sinh(a + b*x)**4/16 - 3*x**2*sinh(a + b*x)**2*cosh(a + b*
x)**2/8 + 3*x**2*cosh(a + b*x)**4/16 - 3*x*sinh(a + b*x)**3*cosh(a + b*x)/(
8*b) + 5*x*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) + sinh(a + b*x)**4/(4*b**2)
 - 5*sinh(a + b*x)**2*cosh(a + b*x)**2/(16*b**2), Ne(b, 0)), (x**2*cosh(a)*
*4/2, True))
```

Giac [A] time = 1.27422, size = 116, normalized size = 1.45

$$\frac{3}{16}x^2 + \frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2} + \frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{16\,b^2} - \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2} - \frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/16*x^2 + 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 + 1/16*(2*b*x - 1)*e^(2*b*
x + 2*a)/b^2 - 1/16*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 - 1/256*(4*b*x + 1)*e^
(-4*b*x - 4*a)/b^2
```

3.26 $\int (c+dx)^3 \mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=179

$$\frac{6id^2(c+dx)\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{3id(c+d$$

```
[Out] (2*(c + d*x)^3*ArcTan[E^(a + b*x)])/b - ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I
)*E^(a + b*x)])/b^2 + ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(a + b*x)])/b^2 +
 ((6*I)*d^2*(c + d*x)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((6*I)*d^2*(c + d
*x)*PolyLog[3, I*E^(a + b*x)])/b^3 - ((6*I)*d^3*PolyLog[4, (-I)*E^(a + b*x)
])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(a + b*x)])/b^4
```

Rubi [A] time = 0.123387, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4180, 2531, 6609, 2282, 6589}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{3id(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sech[a + b*x],x]
```


```
[Out] (2*(c + d*x)^3*ArcTan[E^(a + b*x)])/b - ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I
)*E^(a + b*x)])/b^2 + ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(a + b*x)])/b^2 +
 ((6*I)*d^2*(c + d*x)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((6*I)*d^2*(c + d
*x)*PolyLog[3, I*E^(a + b*x)])/b^3 - ((6*I)*d^3*PolyLog[4, (-I)*E^(a + b*x)
])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(a + b*x)])/b^4
```


Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\text{sech}(a+bx)\,dx &= \frac{2(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{(3id)\int (c+dx)^2\log\left(1-ie^{a+bx}\right)dx}{b}+\frac{(3id)\int (c+dx)^2\log\left(1+\right.}{b}\\ &= \frac{2(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{3id(c+dx)^2\text{Li}_2\left(ie^{a+bx}\right)}{b^2}+\frac{\left(6id^2\right)\int}{}\\ &= \frac{2(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{3id(c+dx)^2\text{Li}_2\left(ie^{a+bx}\right)}{b^2}+\frac{6id^2(c+}{}\\ &= \frac{2(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{3id(c+dx)^2\text{Li}_2\left(ie^{a+bx}\right)}{b^2}+\frac{6id^2(c+}{}\\ &= \frac{2(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{3id(c+dx)^2\text{Li}_2\left(ie^{a+bx}\right)}{b^2}+\frac{6id^2(c+}{}\end{aligned}$$

Mathematica [A] time = 2.55595, size = 343, normalized size = 1.92

$$\frac{i\left(-3b^2d(c+dx)^2\text{PolyLog}\left(2,-ie^{a+bx}\right)+3b^2d(c+dx)^2\text{PolyLog}\left(2,ie^{a+bx}\right)+6bcd^2\text{PolyLog}\left(3,-ie^{a+bx}\right)-6bcd^2\text{PolyLog}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sech[a + b*x],x]
```

```
[Out] (I*((-2*I)*b^3*c^3*ArcTan[E^(a + b*x)] + 3*b^3*c^2*d*x*Log[1 - I*E^(a + b*x
)] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(a + b*x)] + b^3*d^3*x^3*Log[1 - I*E^(a +
b*x)] - 3*b^3*c^2*d*x*Log[1 + I*E^(a + b*x)] - 3*b^3*c*d^2*x^2*Log[1 + I*E^
(a + b*x)] - b^3*d^3*x^3*Log[1 + I*E^(a + b*x)] - 3*b^2*d*(c + d*x)^2*PolyL
og[2, (-I)*E^(a + b*x)] + 3*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(a + b*x)] + 6
*b*c*d^2*PolyLog[3, (-I)*E^(a + b*x)] + 6*b*d^3*x*PolyLog[3, (-I)*E^(a + b*
x)] - 6*b*c*d^2*PolyLog[3, I*E^(a + b*x)] - 6*b*d^3*x*PolyLog[3, I*E^(a + b
*x)] - 6*d^3*PolyLog[4, (-I)*E^(a + b*x)] + 6*d^3*PolyLog[4, I*E^(a + b*x)]
))/b^4
```

Maple [F] time = 0.216, size = 0, normalized size = 0.

$$\int (dx+c)^3\,\text{sech}\,(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sech(b*x+a),x)
```

```
[Out] int((d*x+c)^3*sech(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,c^{3}\arctan\left(e^{(-bx-a)}\right)}{b}+2\int\frac{\left(d^{3}x^{3}e^{a}+3\,cd^{2}x^{2}e^{a}+3\,c^{2}dxe^{a}\right)e^{(bx)}}{e^{(2\,bx+2\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*c^3*arctan(e^(-b*x - a))/b + 2*integrate((d^3*x^3*e^a + 3*c*d^2*x^2*e^a
+ 3*c^2*d*x*e^a)*e^(b*x)/(e^(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 2.25113, size = 1300, normalized size = 7.26

$$\frac{6i\,d^{3}\mathrm{polylog}\left(4, i\,\cosh\left(bx+a\right)+i\,\sinh\left(bx+a\right)\right)-6i\,d^{3}\mathrm{polylog}\left(4, -i\,\cosh\left(bx+a\right)-i\,\sinh\left(bx+a\right)\right)+\left(3i\,b^{2}d^{3}x^{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (6*I*d^3*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*I*d^3*polylog(4,
 -I*cosh(b*x + a) - I*sinh(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x +
 3*I*b^2*c^2*d)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + (-3*I*b^2*d^3*x^
2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(-I*cosh(b*x + a) - I*sinh(b*x +
a)) + (I*b^3*c^3 - 3*I*a*b^2*c^2*d + 3*I*a^2*b*c*d^2 - I*a^3*d^3)*log(cosh(
b*x + a) + sinh(b*x + a) + I) + (-I*b^3*c^3 + 3*I*a*b^2*c^2*d - 3*I*a^2*b*c
*d^2 + I*a^3*d^3)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (-I*b^3*d^3*x^3
- 3*I*b^3*c*d^2*x^2 - 3*I*b^3*c^2*d*x - 3*I*a*b^2*c^2*d + 3*I*a^2*b*c*d^2 -
 I*a^3*d^3)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (I*b^3*d^3*x^3 + 3
*I*b^3*c*d^2*x^2 + 3*I*b^3*c^2*d*x + 3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + I*
a^3*d^3)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (-6*I*b*d^3*x - 6*I*
b*c*d^2)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + (6*I*b*d^3*x + 6*I
*b*c*d^2)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(c+dx)^{3}\,\mathrm{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sech(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(dx+c)^{3}\,\mathrm{sech}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sech(b*x + a), x)
```

3.27 $\int (c+dx)^2 \mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=119

$$-\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{2id(c+dx)\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{2id^2\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(3,i\right.}{b^3}$$

```
[Out] (2*(c + d*x)^2*ArcTan[E^(a + b*x)])/b - ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*
E^(a + b*x)])/b^2 + ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(a + b*x)])/b^2 + ((2
*I)*d^2*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((2*I)*d^2*PolyLog[3, I*E^(a +
b*x)])/b^3
```

Rubi [A] time = 0.0806661, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4180, 2531, 2282, 6589}

$$-\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{2id(c+dx)\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{2id^2\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(3,i\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sech[a + b*x],x]
```


```
[Out] (2*(c + d*x)^2*ArcTan[E^(a + b*x)])/b - ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*
E^(a + b*x)])/b^2 + ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(a + b*x)])/b^2 + ((2
*I)*d^2*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((2*I)*d^2*PolyLog[3, I*E^(a +
b*x)])/b^3
```


Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\operatorname{sech}(a+bx)\,dx = \frac{2(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{(2id)\int (c+dx)\log\left(1-ie^{a+bx}\right)dx}{b} + \frac{(2id)\int (c+dx)\log\left(1+ie^{\cdot}\right.}{b}$$

$$= \frac{2(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{2id(c+dx)\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^2} + \frac{\left(2id^2\right)\int \mathrm{L}}{}$$

$$= \frac{2(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{2id(c+dx)\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^2} + \frac{\left(2id^2\right)\mathrm{Sul}}{}$$

$$= \frac{2(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{2id(c+dx)\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^2} + \frac{2id^2\mathrm{Li}_3\left(-\right.}{b^3}$$

Mathematica [A] time = 1.45882, size = 199, normalized size = 1.67

$$\frac{i\left(-2bd(c+dx)\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)+2bd(c+dx)\mathrm{PolyLog}\left(2,ie^{a+bx}\right)+2d^2\mathrm{PolyLog}\left(3,-ie^{a+bx}\right)-2d^2\mathrm{PolyLog}\left(3,ie^{a+}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sech[a + b*x],x]
```

```
[Out] (I*((-2*I)*b^2*c^2*ArcTan[E^(a + b*x)] + 2*b^2*c*d*x*Log[1 - I*E^(a + b*x)]
 + b^2*d^2*x^2*Log[1 - I*E^(a + b*x)] - 2*b^2*c*d*x*Log[1 + I*E^(a + b*x)]
- b^2*d^2*x^2*Log[1 + I*E^(a + b*x)] - 2*b*d*(c + d*x)*PolyLog[2, (-I)*E^(a
 + b*x)] + 2*b*d*(c + d*x)*PolyLog[2, I*E^(a + b*x)] + 2*d^2*PolyLog[3, (-I
)*E^(a + b*x)] - 2*d^2*PolyLog[3, I*E^(a + b*x)]))/b^3
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int (dx+c)^2 \operatorname{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sech(b*x+a),x)
```

```
[Out] int((d*x+c)^2*sech(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,c^2\arctan\left(e^{(-bx-a)}\right)}{b} + 2\int \frac{\left(d^2x^2e^a + 2\,cdxe^a\right)e^{(bx)}}{e^{(2\,bx+2\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*c^2*arctan(e^(-b*x - a))/b + 2*integrate((d^2*x^2*e^a + 2*c*d*x*e^a)*e^(
b*x)/(e^(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 2.229, size = 845, normalized size = 7.1

$$-2i\,d^2\text{polylog}(3, i\cosh(bx+a) + i\sinh(bx+a)) + 2i\,d^2\text{polylog}(3, -i\cosh(bx+a) - i\sinh(bx+a)) + \left(2i\,bd^2x + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (-2*I*d^2*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 2*I*d^2*polylog(3
, -I*cosh(b*x + a) - I*sinh(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(I*c
osh(b*x + a) + I*sinh(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-I*cosh(
b*x + a) - I*sinh(b*x + a)) + (I*b^2*c^2 - 2*I*a*b*c*d + I*a^2*d^2)*log(cos
h(b*x + a) + sinh(b*x + a) + I) + (-I*b^2*c^2 + 2*I*a*b*c*d - I*a^2*d^2)*lo
g(cosh(b*x + a) + sinh(b*x + a) - I) + (-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x - 2*
I*a*b*c*d + I*a^2*d^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (I*b^2*
d^2*x^2 + 2*I*b^2*c*d*x + 2*I*a*b*c*d - I*a^2*d^2)*log(-I*cosh(b*x + a) - I
*sinh(b*x + a) + 1))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2\,\text{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sech(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2\,\text{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sech(b*x + a), x)
```

3.28 $\int (c + dx)\mathbf{sech}(a + bx)\, dx$

Optimal. Leaf size=61

$$-\frac{id\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{id\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (2*(c + d*x)*ArcTan[E^(a + b*x)])/b - (I*d*PolyLog[2, (-I)*E^(a + b*x)])/b^
2 + (I*d*PolyLog[2, I*E^(a + b*x)])/b^2
```

Rubi [A] time = 0.037233, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4180, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{id\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sech[a + b*x],x]
```

```
[Out] (2*(c + d*x)*ArcTan[E^(a + b*x)])/b - (I*d*PolyLog[2, (-I)*E^(a + b*x)])/b^
2 + (I*d*PolyLog[2, I*E^(a + b*x)])/b^2
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\text{sech}(a+bx)\,dx &= \frac{2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{(id)\int \log\left(1-ie^{a+bx}\right)dx}{b}+\frac{(id)\int \log\left(1+ie^{a+bx}\right)dx}{b}\\
&= \frac{2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{(id)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^2}+\frac{(id)\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,\right.}{b^2}\\
&= \frac{2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{id\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{id\text{Li}_2\left(ie^{a+bx}\right)}{b^2}
\end{aligned}$$

Mathematica [B] time = 0.0792382, size = 129, normalized size = 2.11

$$\frac{-id\left(\text{PolyLog}\left(2,-ie^{a+bx}\right)-\text{PolyLog}\left(2,ie^{a+bx}\right)\right)+bc\tan^{-1}(\sinh(a+bx))-\frac{1}{2}d(-2ia-2ibx+\pi)\left(\log\left(1-ie^{a+bx}\right)-\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sech[a + b*x],x]
```

```
[Out] (b*c*ArcTan[Sinh[a + b*x]] - (d*((-2*I)*a + Pi - (2*I)*b*x)*(Log[1 - I*E^(a
 + b*x)] - Log[1 + I*E^(a + b*x)]))/2 + (d*((-2*I)*a + Pi)*Log[Cot[((2*I)*a
 + Pi + (2*I)*b*x)/4]])/2 - I*d*(PolyLog[2, (-I)*E^(a + b*x)] - PolyLog[2,
I*E^(a + b*x)]))/b^2
```

Maple [B] time = 0.007, size = 449, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sech(b*x+a),x)
```

```
[Out] I/b^2*d*dilog(-I*cosh(b*x+a)-I*sinh(b*x+a))-I/b*d*ln((1-I)*cosh(1/2*b*x+1/2
*a)+(1+I)*sinh(1/2*b*x+1/2*a))*x+I/b^2*d*ln(-I*cosh(b*x+a)-I*sinh(b*x+a))*l
n((1-I)*cosh(1/2*b*x+1/2*a)+(1+I)*sinh(1/2*b*x+1/2*a))-I/b^2*d*ln((1-I)*cos
h(1/2*b*x+1/2*a)+(1+I)*sinh(1/2*b*x+1/2*a))*a-I/b^2*d*dilog(I*cosh(b*x+a)+I
*sinh(b*x+a))+I/b*d*ln((1+I)*cosh(1/2*b*x+1/2*a)+(1-I)*sinh(1/2*b*x+1/2*a))
*x-I/b^2*d*ln(I*cosh(b*x+a)+I*sinh(b*x+a))*ln((1+I)*cosh(1/2*b*x+1/2*a)+(1-
I)*sinh(1/2*b*x+1/2*a))+I/b^2*d*ln((1+I)*cosh(1/2*b*x+1/2*a)+(1-I)*sinh(1/2
*b*x+1/2*a))*a+1/2*I/b*d*ln(-I*cosh(b*x+a)-I*sinh(b*x+a))*x-1/2*I/b*d*ln(I*
cosh(b*x+a)+I*sinh(b*x+a))*x+1/2*I/b^2*d*ln(-I*cosh(b*x+a)-I*sinh(b*x+a))*a
-1/2*I/b^2*d*ln(I*cosh(b*x+a)+I*sinh(b*x+a))*a-2/b^2*d*a*arctan(exp(b*x+a))
+2/b*c*arctan(exp(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,d\int\frac{xe^{(bx+a)}}{e^{(2\,bx+2\,a)}+1}\,dx-\frac{2\,c\arctan\left(e^{(-bx-a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*d*integrate(x*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x) - 2*c*arctan(e^(-b*x
- a))/b
```

Fricas [B] time = 2.1461, size = 463, normalized size = 7.59

$$\frac{i\,d\text{Li}_2\left(i\,\cosh\left(bx+a\right)+i\,\sinh\left(bx+a\right)\right)-i\,d\text{Li}_2\left(-i\,\cosh\left(bx+a\right)-i\,\sinh\left(bx+a\right)\right)+\left(i\,bc-i\,ad\right)\log\left(\cosh\left(bx+a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (I*d*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - I*d*dilog(-I*cosh(b*x + a)
- I*sinh(b*x + a)) + (I*b*c - I*a*d)*log(cosh(b*x + a) + sinh(b*x + a) + I)
 + (-I*b*c + I*a*d)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (-I*b*d*x - I*
a*d)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (I*b*d*x + I*a*d)*log(-I*
cosh(b*x + a) - I*sinh(b*x + a) + 1))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)\operatorname{sech}(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a),x)
```

```
[Out] Integral((c + d*x)*sech(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\operatorname{sech}(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sech(b*x + a), x)
```

3.29 $\int \frac{\text{sech}(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\text{sech}(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Sech[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.0211822, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{sech}(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Sech[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\text{sech}(a+bx)}{c+dx}\,dx=\int \frac{\text{sech}(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 3.53839, size = 0, normalized size = 0.

$$\int \frac{\text{sech}(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Sech[a + b*x]/(c + d*x), x]
```

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{\text{sech}(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)/(d*x+c),x)
```

```
[Out] int(sech(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx + a)}{dx + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{sech}(bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(a + bx)}{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sech(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx + a)}{dx + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)/(d*x + c), x)
```

3.30 $\int \frac{\text{sech}(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\text{sech}(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Sech[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.0204347, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{sech}(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Sech[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\text{sech}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{sech}(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 7.20085, size = 0, normalized size = 0.

$$\int \frac{\text{sech}(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Sech[a + b*x]/(c + d*x)^2, x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{\text{sech}\,(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(sech(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{sech}(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sech(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)/(d*x + c)^2, x)
```

3.31 $\int (c+dx)^3 \operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=103

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^4}-\frac{3d(c+dx)^2\log\left(e^{2(a+bx)}+1\right)}{b^2}+\frac{(c+dx)^3\tanh(a\cdot}{b}$$

```
[Out] (c + d*x)^3/b - (3*d*(c + d*x)^2*Log[1 + E^(2*(a + b*x))])/b^2 - (3*d^2*(c
+ d*x)*PolyLog[2, -E^(2*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, -E^(2*(a + b*x
))])/(2*b^4) + ((c + d*x)^3*Tanh[a + b*x])/b
```

Rubi [A] time = 0.205963, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4184, 3718, 2190, 2531, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^4}-\frac{3d(c+dx)^2\log\left(e^{2(a+bx)}+1\right)}{b^2}+\frac{(c+dx)^3\tanh(a\cdot}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sech[a + b*x]^2,x]
```

```
[Out] (c + d*x)^3/b - (3*d*(c + d*x)^2*Log[1 + E^(2*(a + b*x))])/b^2 - (3*d^2*(c
+ d*x)*PolyLog[2, -E^(2*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, -E^(2*(a + b*x
))])/(2*b^4) + ((c + d*x)^3*Tanh[a + b*x])/b
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \operatorname{sech}^2(a+bx)\,dx = \frac{(c+dx)^3\tanh(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\tanh(a+bx)\,dx}{b}$$
$$= \frac{(c+dx)^3}{b} + \frac{(c+dx)^3\tanh(a+bx)}{b} - \frac{(6d)\int \frac{e^{2(a+bx)}(c+dx)^2}{1+e^{2(a+bx)}}\,dx}{b}$$
$$= \frac{(c+dx)^3}{b} - \frac{3d(c+dx)^2\log\left(1+e^{2(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\tanh(a+bx)}{b} + \frac{\left(6d^2\right)\int (c+dx)\log}{b^2}$$
$$= \frac{(c+dx)^3}{b} - \frac{3d(c+dx)^2\log\left(1+e^{2(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx)\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh(a}{b}$$
$$= \frac{(c+dx)^3}{b} - \frac{3d(c+dx)^2\log\left(1+e^{2(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx)\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh(a}{b}$$
$$= \frac{(c+dx)^3}{b} - \frac{3d(c+dx)^2\log\left(1+e^{2(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx)\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^3} + \frac{3d^3\text{Li}_3\left(-e^{2(a+bx}\right.}{2b^4}$$

Mathematica [A] time = 2.03888, size = 145, normalized size = 1.41

$$\frac{2\operatorname{sech}(a)\sinh(bx)(c+dx)^3\operatorname{sech}(a+bx) - \frac{e^{2a}d\left(-\frac{3\left(e^{-2a}+1\right)d\left(2b(c+dx)\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)+d\text{PolyLog}\left(3,-e^{-2(a+bx)}\right)\right)}{b^3}+\frac{6\left(e^{-2a}+1\right)(c+dx)^2\log\left(e^{-2(a+bx)}+1\right)}{b}\right.}{e^{2a}+1}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sech[a + b*x]^2,x]
```

```
[Out] (-((d*E^(2*a)*((4*(c + d*x)^3)/(d*E^(2*a)) + (6*(1 + E^(-2*a))*(c + d*x)^2*
Log[1 + E^(-2*(a + b*x))])/b - (3*d*(1 + E^(-2*a))*(2*b*(c + d*x)*PolyLog[2
, -E^(-2*(a + b*x))] + d*PolyLog[3, -E^(-2*(a + b*x))]))/b^3))/(1 + E^(2*a)
)) + 2*(c + d*x)^3*Sech[a]*Sech[a + b*x]*Sinh[b*x])/(2*b)
```

Maple [B] time = 0.067, size = 298, normalized size = 2.9

$$-2\,\frac{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}{b\left(1+e^{2\,bx+2\,a}\right)} - 3\,\frac{c^2d\ln\left(1+e^{2\,bx+2\,a}\right)}{b^2} + 6\,\frac{c^2d\ln\left(e^{bx+a}\right)}{b^2} + 6\,\frac{d^3a^2\ln\left(e^{bx+a}\right)}{b^4} + 2\,\frac{d^3x^3}{b} - 6\,\frac{d^3a^2x}{b^3} - 4\,\frac{a^3}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sech(b*x+a)^2,x)
```

```
[Out] -2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*b*x+2*a))-3*d/b^2*c^2*ln(
1+exp(2*b*x+2*a))+6*d/b^2*c^2*ln(exp(b*x+a))+6*d^3/b^4*a^2*ln(exp(b*x+a))+2
*d^3/b*x^3-6*d^3/b^3*a^2*x-4*d^3/b^4*a^3-3*d^3/b^2*ln(1+exp(2*b*x+2*a))*x^2
-3*d^3/b^3*polylog(2,-exp(2*b*x+2*a))*x+3/2*d^3*polylog(3,-exp(2*b*x+2*a))/
b^4-12*d^2/b^3*c*a*ln(exp(b*x+a))+6*d^2/b*c*x^2+12*d^2/b^2*c*a*x+6*d^2/b^3*
c*a^2-6*d^2/b^2*c*ln(1+exp(2*b*x+2*a))*x-3*d^2/b^3*c*polylog(2,-exp(2*b*x+2
*a))
```

Maxima [B] time = 1.694, size = 321, normalized size = 3.12

$$3\,c^2 d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}+b}-\frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}\right)-\frac{3\left(2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)\right)cd^2}{b^3}+\frac{2\,c^3}{b\left(e^{(-2\,bx-2\,a)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3*c^2*d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) - log((e^(2*b*x + 2*a)
 + 1)*e^(-2*a))/b^2) - 3*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x
+ 2*a)))*c*d^2/b^3 + 2*c^3/(b*(e^(-2*b*x - 2*a) + 1)) - 2*(d^3*x^3 + 3*c*d^
2*x^2)/(b*e^(2*b*x + 2*a) + b) - 3/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) +
2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))*d^3/b^4 + 2*(
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2)/b^4
```

Fricas [C] time = 2.42635, size = 3131, normalized size = 30.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 - 2*(b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*
cosh(b*x + a)^2 - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cosh(b*x + a)*sinh(b*x + a) - 2*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^
3*d^3)*sinh(b*x + a)^2 + 6*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cosh(b*
x + a)^2 + 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b*d^3*x + b
*c*d^2)*sinh(b*x + a)^2)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(b*d^
3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cosh(b*x + a)^2 + 2*(b*d^3*x + b*c*d^2)
*cosh(b*x + a)*sinh(b*x + a) + (b*d^3*x + b*c*d^2)*sinh(b*x + a)^2)*dilog(-
I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3 +
 (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cosh(b*x + a)^2 + 2*(b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)*cosh(b*x + a)*sinh(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) + 3*(b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cosh(b*x
 + a)^2 + 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cosh(b*x + a)*sinh(b*x + a)
 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sinh(b*x + a)^2)*log(cosh(b*x + a) +
 sinh(b*x + a) - I) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^
3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cosh(b*x + a)^2 +
 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cosh(b*x + a)*sinh
(b*x + a) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*sinh(b*x
+ a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2
```

```
*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d
^2 - a^2*d^3)*cosh(b*x + a)^2 + 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^
2 - a^2*d^3)*cosh(b*x + a)*sinh(b*x + a) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2
*a*b*c*d^2 - a^2*d^3)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x +
a) + 1) - 6*(d^3*cosh(b*x + a)^2 + 2*d^3*cosh(b*x + a)*sinh(b*x + a) + d^3*
sinh(b*x + a)^2 + d^3)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(d
^3*cosh(b*x + a)^2 + 2*d^3*cosh(b*x + a)*sinh(b*x + a) + d^3*sinh(b*x + a)^
2 + d^3)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)))/(b^4*cosh(b*x + a)
^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 + b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3 \operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*sech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \operatorname{sech}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sech(b*x + a)^2, x)
```

3.32 $\int (c+dx)^2 \mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=73

$$-\frac{d^2 \text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}-\frac{2d(c+dx)\log\left(e^{2(a+bx)}+1\right)}{b^2}+\frac{(c+dx)^2\tanh(a+bx)}{b}+\frac{(c+dx)^2}{b}$$

```
[Out] (c + d*x)^2/b - (2*d*(c + d*x)*Log[1 + E^(2*(a + b*x))])/b^2 - (d^2*PolyLog
[2, -E^(2*(a + b*x))])/b^3 + ((c + d*x)^2*Tanh[a + b*x])/b
```

Rubi [A] time = 0.132696, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4184, 3718, 2190, 2279, 2391}

$$-\frac{d^2 \text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}-\frac{2d(c+dx)\log\left(e^{2(a+bx)}+1\right)}{b^2}+\frac{(c+dx)^2\tanh(a+bx)}{b}+\frac{(c+dx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sech[a + b*x]^2,x]
```

```
[Out] (c + d*x)^2/b - (2*d*(c + d*x)*Log[1 + E^(2*(a + b*x))])/b^2 - (d^2*PolyLog
[2, -E^(2*(a + b*x))])/b^3 + ((c + d*x)^2*Tanh[a + b*x])/b
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^2\operatorname{sech}^2(a+bx)\,dx = \frac{(c+dx)^2\tanh(a+bx)}{b} - \frac{(2d)\int (c+dx)\tanh(a+bx)\,dx}{b}$$
$$= \frac{(c+dx)^2}{b} + \frac{(c+dx)^2\tanh(a+bx)}{b} - \frac{(4d)\int \frac{e^{2(a+bx)}(c+dx)}{1+e^{2(a+bx)}}\,dx}{b}$$
$$= \frac{(c+dx)^2}{b} - \frac{2d(c+dx)\log\left(1+e^{2(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tanh(a+bx)}{b} + \frac{\left(2d^2\right)\int \log\left(1+e^{2(a\cdot}\right.}{b^2}$$
$$= \frac{(c+dx)^2}{b} - \frac{2d(c+dx)\log\left(1+e^{2(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tanh(a+bx)}{b} + \frac{d^2\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\right.}{b^3}$$
$$= \frac{(c+dx)^2}{b} - \frac{2d(c+dx)\log\left(1+e^{2(a+bx)}\right)}{b^2} - \frac{d^2\operatorname{Li}_2\left(-e^{2(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\tanh(a+bx)}{b}$$

Mathematica [C] time = 6.28069, size = 277, normalized size = 3.79

$$\frac{d^2\operatorname{csch}(a)\operatorname{sech}(a)\left(-b^2x^2e^{-\tanh^{-1}(\coth(a))} + \frac{i\coth(a)\left(i\operatorname{PolyLog}\left(2,e^{2i\left(i\tanh^{-1}(\coth(a))+ibx\right)}\right)-bx\left(-\pi+2i\tanh^{-1}(\coth(a))\right)-2\left(i\tanh^{-1}(\coth(a))+ib\right.}{\cdots}\right.}{b^3\sqrt{\operatorname{csch}^2(a)\left(\sinh^2(a)-\cosh\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*c*d*Sech[a]*(Cosh[a]*Log[Cosh[a]*Cosh[b*x] + Sinh[a]*Sinh[b*x]] - b*x*S
inh[a]))/(b^2*(Cosh[a]^2 - Sinh[a]^2)) + (d^2*Csch[a]*(-((b^2*x^2)/E^ArcTan
h[Coth[a]]) + (I*Coth[a]*(-(b*x*(-Pi + (2*I)*ArcTanh[Coth[a]])) - Pi*Log[1
+ E^(2*b*x)] - 2*(I*b*x + I*ArcTanh[Coth[a]])*Log[1 - E^((2*I)*(I*b*x + I*A
rcTanh[Coth[a]]))] + Pi*Log[Cosh[b*x]] + (2*I)*ArcTanh[Coth[a]]*Log[I*Sinh[
b*x + ArcTanh[Coth[a]]]] + I*PolyLog[2, E^((2*I)*(I*b*x + I*ArcTanh[Coth[a]
]))]))/Sqrt[1 - Coth[a]^2])*Sech[a])/(b^3*Sqrt[Csch[a]^2*(-Cosh[a]^2 + Sinh
[a]^2)]) + (Sech[a]*Sech[a + b*x]*(c^2*Sinh[b*x] + 2*c*d*x*Sinh[b*x] + d^2*
x^2*Sinh[b*x]))/b
```

Maple [B] time = 0.031, size = 159, normalized size = 2.2

$$-2\,\frac{d^2x^2+2\,cdx+c^2}{b\left(1+e^{2\,bx+2\,a}\right)} - 2\,\frac{cd\ln\left(1+e^{2\,bx+2\,a}\right)}{b^2} + 4\,\frac{cd\ln\left(e^{bx+a}\right)}{b^2} + 2\,\frac{d^2x^2}{b} + 4\,\frac{ad^2x}{b^2} + 2\,\frac{a^2d^2}{b^3} - 2\,\frac{d^2\ln\left(1+e^{2\,bx+2\,a}\right)x}{b^2} - \frac{d^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sech(b*x+a)^2,x)
```

```
[Out] -2*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*b*x+2*a))-2*d/b^2*c*ln(1+exp(2*b*x+2*a)
)+4*d/b^2*c*ln(exp(b*x+a))+2*d^2/b*x^2+4*d^2/b^2*a*x+2*d^2/b^3*a^2-2*d^2/b^
2*ln(1+exp(2*b*x+2*a))*x-d^2*polylog(2,-exp(2*b*x+2*a))/b^3-4*d^2/b^3*a*ln(
exp(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\,d^2\left(\frac{x^2}{be^{(2\,bx+2\,a)}+b}-2\int\frac{x}{be^{(2\,bx+2\,a)}+b}\,dx\right)+2\,cd\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}+b}-\frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}\right)+\frac{2\,c^2}{b\left(e^{(-2\,bx-2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*d^2*(x^2/(b*e^(2*b*x + 2*a) + b) - 2*integrate(x/(b*e^(2*b*x + 2*a) + b)
, x)) + 2*c*d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) - log((e^(2*b*x
+ 2*a) + 1)*e^(-2*a))/b^2) + 2*c^2/(b*(e^(-2*b*x - 2*a) + 1))
```

Fricas [C] time = 2.23175, size = 1777, normalized size = 24.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
- a^2*d^2)*cosh(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2
*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
- a^2*d^2)*sinh(b*x + a)^2 + (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sin
h(b*x + a) + d^2*sinh(b*x + a)^2 + d^2)*dilog(I*cosh(b*x + a) + I*sinh(b*x
+ a)) + (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sinh(b*x + a) + d^2*sinh
(b*x + a)^2 + d^2)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (b*c*d - a*d
^2 + (b*c*d - a*d^2)*cosh(b*x + a)^2 + 2*(b*c*d - a*d^2)*cosh(b*x + a)*sinh
(b*x + a) + (b*c*d - a*d^2)*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x +
 a) + I) + (b*c*d - a*d^2 + (b*c*d - a*d^2)*cosh(b*x + a)^2 + 2*(b*c*d - a*
d^2)*cosh(b*x + a)*sinh(b*x + a) + (b*c*d - a*d^2)*sinh(b*x + a)^2)*log(cos
h(b*x + a) + sinh(b*x + a) - I) + (b*d^2*x + a*d^2 + (b*d^2*x + a*d^2)*cosh
(b*x + a)^2 + 2*(b*d^2*x + a*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b*d^2*x +
a*d^2)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (b*d^2
*x + a*d^2 + (b*d^2*x + a*d^2)*cosh(b*x + a)^2 + 2*(b*d^2*x + a*d^2)*cosh(b
*x + a)*sinh(b*x + a) + (b*d^2*x + a*d^2)*sinh(b*x + a)^2)*log(-I*cosh(b*x
+ a) - I*sinh(b*x + a) + 1))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sin
h(b*x + a) + b^3*sinh(b*x + a)^2 + b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2\,\mathrm{sech}^2\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*sech(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \operatorname{sech}(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sech(b*x + a)^2, x)
```

3.33 $\int (c+dx)\text{sech}^2(a+bx)\,dx$

Optimal. Leaf size=29

$$\frac{(c+dx)\tanh(a+bx)}{b}-\frac{d\log(\cosh(a+bx))}{b^2}$$

```
[Out] -((d*Log[Cosh[a + b*x]])/b^2) + ((c + d*x)*Tanh[a + b*x])/b
```

Rubi [A] time = 0.0292901, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4184, 3475}

$$\frac{(c+dx)\tanh(a+bx)}{b}-\frac{d\log(\cosh(a+bx))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sech[a + b*x]^2,x]
```

```
[Out] -((d*Log[Cosh[a + b*x]])/b^2) + ((c + d*x)*Tanh[a + b*x])/b
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\text{sech}^2(a+bx)\,dx &= \frac{(c+dx)\tanh(a+bx)}{b}-\frac{d\int \tanh(a+bx)\,dx}{b}\\ &= -\frac{d\log(\cosh(a+bx))}{b^2}+\frac{(c+dx)\tanh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0898336, size = 51, normalized size = 1.76

$$-\frac{d\log(\cosh(a+bx))}{b^2}+\frac{c\tanh(a+bx)}{b}+\frac{dx\tanh(a)}{b}+\frac{dx\text{sech}(a)\sinh(bx)\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sech[a + b*x]^2,x]
```

```
[Out] -((d*Log[Cosh[a + b*x]])/b^2) + (d*x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b + (
d*x*Tanh[a])/b + (c*Tanh[a + b*x])/b
```

Maple [A] time = 0.023, size = 57, normalized size = 2.

$$2\,{\frac {dx}{b}}+2\,{\frac {da}{{b}^{2}}}-2\,{\frac {dx+c}{b \left(1+{{\rm e}^{2\,bx+2\,a}} \right) }}-{\frac {d\ln \left(1+{{\rm e}^{2\,bx+2\,a}} \right) }{{b}^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sech(b*x+a)^2,x)
```

```
[Out] 2*d/b*x+2*d/b^2*a-2*(d*x+c)/b/(1+exp(2*b*x+2*a))-d/b^2*ln(1+exp(2*b*x+2*a))
```

Maxima [B] time = 1.01154, size = 97, normalized size = 3.34

$$d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}+b}-\frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}\right)+\frac{2\,c}{b\left(e^{(-2\,bx-2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) - log((e^(2*b*x + 2*a) + 1)*
e^(-2*a))/b^2) + 2*c/(b*(e^(-2*b*x - 2*a) + 1))
```

Fricas [B] time = 2.07024, size = 429, normalized size = 14.79

$$\frac{2\,bdx\cosh\left(bx+a\right)^2+4\,bdx\cosh\left(bx+a\right)\sinh\left(bx+a\right)+2\,bdx\sinh\left(bx+a\right)^2-2\,bc-\left(d\cosh\left(bx+a\right)^2+2\,d\cosh\left(bx+\right.\right.}{b^2\cosh\left(bx+a\right)^2+2\,b^2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b^2\sinh\left(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (2*b*d*x*cosh(b*x + a)^2 + 4*b*d*x*cosh(b*x + a)*sinh(b*x + a) + 2*b*d*x*si
nh(b*x + a)^2 - 2*b*c - (d*cosh(b*x + a)^2 + 2*d*cosh(b*x + a)*sinh(b*x + a
) + d*sinh(b*x + a)^2 + d)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x +
a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b
*x + a)^2 + b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c + dx\right) \operatorname{sech}^{2}\left(a + bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*sech(a + b*x)**2, x)
```

Giac [B] time = 1.31527, size = 105, normalized size = 3.62

$$\frac{2\,bdxe^{(2\,bx+2\,a)} - de^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)} + 1\right) - 2\,bc - d\log\left(e^{(2\,bx+2\,a)} + 1\right)}{b^2e^{(2\,bx+2\,a)} + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (2*b*d*x*e^(2*b*x + 2*a) - d*e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) + 1) - 2*b
*c - d*log(e^(2*b*x + 2*a) + 1))/(b^2*e^(2*b*x + 2*a) + b^2)
```

3.34 $\int \frac{\operatorname{sech}^2(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\operatorname{sech}^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Sech[a + b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.0371998, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Sech[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx} \, dx = \int \frac{\operatorname{sech}^2(a+bx)}{c+dx} \, dx$$

Mathematica [A] time = 17.409, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Sech[a + b*x]^2/(c + d*x), x]
```

Maple [A] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(bx+a))^2}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sech(b*x+a)^2/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-4\,d\int \frac{1}{2\left(bd^2x^2+2\,bcdx+bc^2+\left(bd^2x^2e^{(2\,a)}+2\,bcdxe^{(2\,a)}+bc^2e^{(2\,a)}\right)e^{(2\,bx)}\right)}\,dx-\frac{2}{bdx+bc+\left(bdxe^{(2\,a)}+bce^{(2\,a)}\right)e^{(2\,bx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -4*d*integrate(1/2/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2*e^(2*a) + 2*
b*c*d*x*e^(2*a) + b*c^2*e^(2*a))*e^(2*b*x)), x) - 2/(b*d*x + b*c + (b*d*x*e
^(2*a) + b*c*e^(2*a))*e^(2*b*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{sech}\left(bx+a\right)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{sech}^2\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sech(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{sech}\left(bx+a\right)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2/(d*x + c), x)
```

3.35 $\int \frac{\text{sech}^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\text{sech}^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Sech[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [A] time = 0.0354581, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{sech}^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Sech[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\text{sech}^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{sech}^2(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 17.6279, size = 0, normalized size = 0.

$$\int \frac{\text{sech}^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[Sech[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{(\text{sech}\,(bx+a))^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(sech(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-4\,d\int \frac{1}{bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + \left(bd^3x^3e^{(2\,a)} + 3\,bcd^2x^2e^{(2\,a)} + 3\,bc^2dxe^{(2\,a)} + bc^3e^{(2\,a)}\right)e^{(2\,bx)}}\,dx - \frac{\quad}{bd^2x^2 + 2\,\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -4*d*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*
x^3*e^(2*a) + 3*b*c*d^2*x^2*e^(2*a) + 3*b*c^2*d*x*e^(2*a) + b*c^3*e^(2*a))*
e^(2*b*x)), x) - 2/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2*e^(2*a) + 2*
b*c*d*x*e^(2*a) + b*c^2*e^(2*a))*e^(2*b*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{sech}\,(bx+a)^2}{d^2x^2 + 2\,cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{sech}^2\,(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sech(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{sech}\,(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2/(d*x + c)^2, x)
```

3.36 $\int (c+dx)^3 \operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=296

$$\frac{3id^2(c+dx)\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{3id^2(c+dx)\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{3id(c+dx)^2\text{I}}{}$$

```
[Out] (-6*d^2*(c + d*x)*ArcTan[E^(a + b*x)])/b^3 + ((c + d*x)^3*ArcTan[E^(a + b*x
)])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(a + b*x)])/b^4 - (((3*I)/2)*d*(c + d*
x)^2*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - ((3*I)*d^3*PolyLog[2, I*E^(a + b*x
)])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(a + b*x)])/b^2 + ((3*I)*
d^2*(c + d*x)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((3*I)*d^2*(c + d*x)*Poly
Log[3, I*E^(a + b*x)])/b^3 - ((3*I)*d^3*PolyLog[4, (-I)*E^(a + b*x)])/b^4 +
 ((3*I)*d^3*PolyLog[4, I*E^(a + b*x)])/b^4 + (3*d*(c + d*x)^2*Sech[a + b*x]
)/(2*b^2) + ((c + d*x)^3*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rubi [A] time = 0.216765, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4186, 4180, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{3id^2(c+dx)\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{3id^2(c+dx)\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{3id(c+dx)^2\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sech[a + b*x]^3,x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTan[E^(a + b*x)])/b^3 + ((c + d*x)^3*ArcTan[E^(a + b*x
)])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(a + b*x)])/b^4 - (((3*I)/2)*d*(c + d*
x)^2*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - ((3*I)*d^3*PolyLog[2, I*E^(a + b*x
)])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(a + b*x)])/b^2 + ((3*I)*
d^2*(c + d*x)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((3*I)*d^2*(c + d*x)*Poly
Log[3, I*E^(a + b*x)])/b^3 - ((3*I)*d^3*PolyLog[4, (-I)*E^(a + b*x)])/b^4 +
 ((3*I)*d^3*PolyLog[4, I*E^(a + b*x)])/b^4 + (3*d*(c + d*x)^2*Sech[a + b*x]
)/(2*b^2) + ((c + d*x)^3*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \operatorname{sech}^3(a+bx)\,dx = \frac{3d(c+dx)^2\operatorname{sech}(a+bx)}{2b^2} + \frac{(c+dx)^3\operatorname{sech}(a+bx)\tanh(a+bx)}{2b} + \frac{1}{2}\int (c+dx)^3\operatorname{sech}($$

$$= -\frac{6d^2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{3d(c+dx)^2\operatorname{sech}(a+bx)}{2b^2} + \frac{(c}{}$$

$$= -\frac{6d^2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3id(c+dx)^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b^2} + \frac{3i}{}$$

$$= -\frac{6d^2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{3id^3\text{Li}_2\left(-ie^{a+bx}\right)}{b^4} - \frac{3id(c+dx}{}$$

$$= -\frac{6d^2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{3id^3\text{Li}_2\left(-ie^{a+bx}\right)}{b^4} - \frac{3id(c+dx}{}$$

$$= -\frac{6d^2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{3id^3\text{Li}_2\left(-ie^{a+bx}\right)}{b^4} - \frac{3id(c+dx}{}$$

Mathematica [A] time = 27.2441, size = 455, normalized size = 1.54

$$\frac{b^2(c+dx)^2\operatorname{sech}(a+bx)(b(c+dx)\tanh(a+bx)+3d)+i\left(-3d\left(b^2(c+dx)^2-2d^2\right)\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)+3d\left(b^2(c+dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sech[a + b*x]^3,x]
```

```
[Out] (I*((-2*I)*b^3*c^3*ArcTan[E^(a + b*x)] + (12*I)*b*c*d^2*ArcTan[E^(a + b*x)]
 + 3*b^3*c^2*d*x*Log[1 - I*E^(a + b*x)] - 6*b*d^3*x*Log[1 - I*E^(a + b*x)]
+ 3*b^3*c*d^2*x^2*Log[1 - I*E^(a + b*x)] + b^3*d^3*x^3*Log[1 - I*E^(a + b*x
)] - 3*b^3*c^2*d*x*Log[1 + I*E^(a + b*x)] + 6*b*d^3*x*Log[1 + I*E^(a + b*x)
] - 3*b^3*c*d^2*x^2*Log[1 + I*E^(a + b*x)] - b^3*d^3*x^3*Log[1 + I*E^(a + b
*x)] - 3*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, (-I)*E^(a + b*x)] + 3*d*(-
2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, I*E^(a + b*x)] + 6*b*c*d^2*PolyLog[3, (
-I)*E^(a + b*x)] + 6*b*d^3*x*PolyLog[3, (-I)*E^(a + b*x)] - 6*b*c*d^2*PolyL
og[3, I*E^(a + b*x)] - 6*b*d^3*x*PolyLog[3, I*E^(a + b*x)] - 6*d^3*PolyLog[
4, (-I)*E^(a + b*x)] + 6*d^3*PolyLog[4, I*E^(a + b*x)]) + b^2*(c + d*x)^2*S
ech[a + b*x]*(3*d + b*(c + d*x)*Tanh[a + b*x]))/(2*b^4)
```

Maple [F] time = 0.193, size = 0, normalized size = 0.

$$\int (dx+c)^3 (\operatorname{sech}(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sech(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^3*sech(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2d^3\int \frac{x^3e^{(bx+a)}}{b^2e^{(2\,bx+2\,a)}+b^2}\,dx+3\,b^2cd^2\int \frac{x^2e^{(bx+a)}}{b^2e^{(2\,bx+2\,a)}+b^2}\,dx+3\,b^2c^2d\int \frac{xe^{(bx+a)}}{b^2e^{(2\,bx+2\,a)}+b^2}\,dx-c^3\left(\frac{\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{}{b(2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b^2*d^3*integrate(x^3*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) + 3*b^2*c
*d^2*integrate(x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) + 3*b^2*c^2*
d*integrate(x*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - c^3*(arctan(e^(
-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) + e
^(-4*b*x - 4*a) + 1))) - 6*d^3*integrate(x*e^(b*x + a)/(b^2*e^(2*b*x + 2*a)
 + b^2), x) - 6*c*d^2*arctan(e^(b*x + a))/b^3 + ((b*d^3*x^3*e^(3*a) + 3*c^2
*d*e^(3*a) + 3*(b*c*d^2 + d^3)*x^2*e^(3*a) + 3*(b*c^2*d + 2*c*d^2)*x*e^(3*a
))*e^(3*b*x) - (b*d^3*x^3*e^a - 3*c^2*d*e^a + 3*(b*c*d^2 - d^3)*x^2*e^a + 3
*(b*c^2*d - 2*c*d^2)*x*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x
+ 2*a) + b^2)
```

Fricas [C] time = 3.43542, size = 11732, normalized size = 39.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 +
 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*cosh(b*x + a)^3 + 6*(b^3*d^3*x^3 + b^3*c^3
+ 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x
)*cosh(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d +
3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*sinh(b*x + a)^
3 - 2*(b^3*d^3*x^3 + b^3*c^3 - 3*b^2*c^2*d + 3*(b^3*c*d^2 - b^2*d^3)*x^2 +
3*(b^3*c^2*d - 2*b^2*c*d^2)*x)*cosh(b*x + a) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c
*d^2*x + 3*I*b^2*c^2*d + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d
 - 6*I*d^3)*cosh(b*x + a)^4 + (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x + 12*I*b
^2*c^2*d - 24*I*d^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (3*I*b^2*d^3*x^2 + 6*I
*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*sinh(b*x + a)^4 - 6*I*d^3 + (6*I*b^
2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 12*I*d^3)*cosh(b*x + a)^2 +
(6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 12*I*d^3 + (18*I*b^2*
d^3*x^2 + 36*I*b^2*c*d^2*x + 18*I*b^2*c^2*d - 36*I*d^3)*cosh(b*x + a)^2)*si
nh(b*x + a)^2 + ((12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d - 24
*I*d^3)*cosh(b*x + a)^3 + (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x + 12*I*b^2*c
^2*d - 24*I*d^3)*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*si
nh(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + (-3*I*
b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*cosh(b*x + a)^4 +
(-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d + 24*I*d^3)*cosh(b*x
 + a)*sinh(b*x + a)^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d
 + 6*I*d^3)*sinh(b*x + a)^4 + 6*I*d^3 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*
x - 6*I*b^2*c^2*d + 12*I*d^3)*cosh(b*x + a)^2 + (-6*I*b^2*d^3*x^2 - 12*I*b^
2*c*d^2*x - 6*I*b^2*c^2*d + 12*I*d^3 + (-18*I*b^2*d^3*x^2 - 36*I*b^2*c*d^2*
x - 18*I*b^2*c^2*d + 36*I*d^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + ((-12*I*b
^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d + 24*I*d^3)*cosh(b*x + a)^3
+ (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d + 24*I*d^3)*cosh(b
*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (I*b^3*
c^3 - 3*I*a*b^2*c^2*d + 3*I*(a^2 - 2)*b*c*d^2 + (I*b^3*c^3 - 3*I*a*b^2*c^2*
d + 3*I*(a^2 - 2)*b*c*d^2 - I*(a^3 - 6*a)*d^3)*cosh(b*x + a)^4 + (4*I*b^3*c
^3 - 12*I*a*b^2*c^2*d + 12*I*(a^2 - 2)*b*c*d^2 - 4*I*(a^3 - 6*a)*d^3)*cosh(
b*x + a)*sinh(b*x + a)^3 + (I*b^3*c^3 - 3*I*a*b^2*c^2*d + 3*I*(a^2 - 2)*b*c
*d^2 - I*(a^3 - 6*a)*d^3)*sinh(b*x + a)^4 - I*(a^3 - 6*a)*d^3 + (2*I*b^3*c^
3 - 6*I*a*b^2*c^2*d + 6*I*(a^2 - 2)*b*c*d^2 - 2*I*(a^3 - 6*a)*d^3)*cosh(b*x
 + a)^2 + (2*I*b^3*c^3 - 6*I*a*b^2*c^2*d + 6*I*(a^2 - 2)*b*c*d^2 - 2*I*(a^3
 - 6*a)*d^3 + (6*I*b^3*c^3 - 18*I*a*b^2*c^2*d + 18*I*(a^2 - 2)*b*c*d^2 - 6*
I*(a^3 - 6*a)*d^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + ((4*I*b^3*c^3 - 12*I*
a*b^2*c^2*d + 12*I*(a^2 - 2)*b*c*d^2 - 4*I*(a^3 - 6*a)*d^3)*cosh(b*x + a)^3
 + (4*I*b^3*c^3 - 12*I*a*b^2*c^2*d + 12*I*(a^2 - 2)*b*c*d^2 - 4*I*(a^3 - 6*
a)*d^3)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I
) + (-I*b^3*c^3 + 3*I*a*b^2*c^2*d - 3*I*(a^2 - 2)*b*c*d^2 + (-I*b^3*c^3 + 3
*I*a*b^2*c^2*d - 3*I*(a^2 - 2)*b*c*d^2 + I*(a^3 - 6*a)*d^3)*cosh(b*x + a)^4
 + (-4*I*b^3*c^3 + 12*I*a*b^2*c^2*d - 12*I*(a^2 - 2)*b*c*d^2 + 4*I*(a^3 - 6
*a)*d^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^3*c^3 + 3*I*a*b^2*c^2*d - 3*
I*(a^2 - 2)*b*c*d^2 + I*(a^3 - 6*a)*d^3)*sinh(b*x + a)^4 + I*(a^3 - 6*a)*d^
3 + (-2*I*b^3*c^3 + 6*I*a*b^2*c^2*d - 6*I*(a^2 - 2)*b*c*d^2 + 2*I*(a^3 - 6*
a)*d^3)*cosh(b*x + a)^2 + (-2*I*b^3*c^3 + 6*I*a*b^2*c^2*d - 6*I*(a^2 - 2)*b
*c*d^2 + 2*I*(a^3 - 6*a)*d^3 + (-6*I*b^3*c^3 + 18*I*a*b^2*c^2*d - 18*I*(a^2
 - 2)*b*c*d^2 + 6*I*(a^3 - 6*a)*d^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + ((-
4*I*b^3*c^3 + 12*I*a*b^2*c^2*d - 12*I*(a^2 - 2)*b*c*d^2 + 4*I*(a^3 - 6*a)*d
^3)*cosh(b*x + a)^3 + (-4*I*b^3*c^3 + 12*I*a*b^2*c^2*d - 12*I*(a^2 - 2)*b*c
```

```
*d^2 + 4*I*(a^3 - 6*a)*d^3)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a)
 + sinh(b*x + a) - I) + (-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 - 3*I*a*b^2*c^2
*d + 3*I*a^2*b*c*d^2 + (-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 - 3*I*a*b^2*c^2*
d + 3*I*a^2*b*c*d^2 - I*(a^3 - 6*a)*d^3 - 3*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh
(b*x + a)^4 + (-4*I*b^3*d^3*x^3 - 12*I*b^3*c*d^2*x^2 - 12*I*a*b^2*c^2*d + 1
2*I*a^2*b*c*d^2 - 4*I*(a^3 - 6*a)*d^3 - 12*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh(
b*x + a)*sinh(b*x + a)^3 + (-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 - 3*I*a*b^2*
c^2*d + 3*I*a^2*b*c*d^2 - I*(a^3 - 6*a)*d^3 - 3*I*(b^3*c^2*d - 2*b*d^3)*x)*
sinh(b*x + a)^4 - I*(a^3 - 6*a)*d^3 + (-2*I*b^3*d^3*x^3 - 6*I*b^3*c*d^2*x^2
 - 6*I*a*b^2*c^2*d + 6*I*a^2*b*c*d^2 - 2*I*(a^3 - 6*a)*d^3 - 6*I*(b^3*c^2*d
 - 2*b*d^3)*x)*cosh(b*x + a)^2 + (-2*I*b^3*d^3*x^3 - 6*I*b^3*c*d^2*x^2 - 6*
I*a*b^2*c^2*d + 6*I*a^2*b*c*d^2 - 2*I*(a^3 - 6*a)*d^3 + (-6*I*b^3*d^3*x^3 -
 18*I*b^3*c*d^2*x^2 - 18*I*a*b^2*c^2*d + 18*I*a^2*b*c*d^2 - 6*I*(a^3 - 6*a)
*d^3 - 18*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^2 - 6*I*(b^3*c^2*d - 2*b
*d^3)*x)*sinh(b*x + a)^2 - 3*I*(b^3*c^2*d - 2*b*d^3)*x + ((-4*I*b^3*d^3*x^3
 - 12*I*b^3*c*d^2*x^2 - 12*I*a*b^2*c^2*d + 12*I*a^2*b*c*d^2 - 4*I*(a^3 - 6*
a)*d^3 - 12*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^3 + (-4*I*b^3*d^3*x^3
- 12*I*b^3*c*d^2*x^2 - 12*I*a*b^2*c^2*d + 12*I*a^2*b*c*d^2 - 4*I*(a^3 - 6*a
)*d^3 - 12*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a))*sinh(b*x + a))*log(I*c
osh(b*x + a) + I*sinh(b*x + a) + 1) + (I*b^3*d^3*x^3 + 3*I*b^3*c*d^2*x^2 +
3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + (I*b^3*d^3*x^3 + 3*I*b^3*c*d^2*x^2 + 3*
I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + I*(a^3 - 6*a)*d^3 + 3*I*(b^3*c^2*d - 2*b*
d^3)*x)*cosh(b*x + a)^4 + (4*I*b^3*d^3*x^3 + 12*I*b^3*c*d^2*x^2 + 12*I*a*b^
2*c^2*d - 12*I*a^2*b*c*d^2 + 4*I*(a^3 - 6*a)*d^3 + 12*I*(b^3*c^2*d - 2*b*d^
3)*x)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b^3*d^3*x^3 + 3*I*b^3*c*d^2*x^2 +
3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + I*(a^3 - 6*a)*d^3 + 3*I*(b^3*c^2*d - 2*
b*d^3)*x)*sinh(b*x + a)^4 + I*(a^3 - 6*a)*d^3 + (2*I*b^3*d^3*x^3 + 6*I*b^3*
c*d^2*x^2 + 6*I*a*b^2*c^2*d - 6*I*a^2*b*c*d^2 + 2*I*(a^3 - 6*a)*d^3 + 6*I*(
b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^2 + (2*I*b^3*d^3*x^3 + 6*I*b^3*c*d^2*
x^2 + 6*I*a*b^2*c^2*d - 6*I*a^2*b*c*d^2 + 2*I*(a^3 - 6*a)*d^3 + (6*I*b^3*d^
3*x^3 + 18*I*b^3*c*d^2*x^2 + 18*I*a*b^2*c^2*d - 18*I*a^2*b*c*d^2 + 6*I*(a^3
 - 6*a)*d^3 + 18*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^2 + 6*I*(b^3*c^2*
d - 2*b*d^3)*x)*sinh(b*x + a)^2 + 3*I*(b^3*c^2*d - 2*b*d^3)*x + ((4*I*b^3*d
^3*x^3 + 12*I*b^3*c*d^2*x^2 + 12*I*a*b^2*c^2*d - 12*I*a^2*b*c*d^2 + 4*I*(a^
3 - 6*a)*d^3 + 12*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^3 + (4*I*b^3*d^3
*x^3 + 12*I*b^3*c*d^2*x^2 + 12*I*a*b^2*c^2*d - 12*I*a^2*b*c*d^2 + 4*I*(a^3
- 6*a)*d^3 + 12*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a))*sinh(b*x + a))*lo
g(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (6*I*d^3*cosh(b*x + a)^4 + 24*I
*d^3*cosh(b*x + a)*sinh(b*x + a)^3 + 6*I*d^3*sinh(b*x + a)^4 + 12*I*d^3*cos
h(b*x + a)^2 + 6*I*d^3 + (36*I*d^3*cosh(b*x + a)^2 + 12*I*d^3)*sinh(b*x + a
)^2 + (24*I*d^3*cosh(b*x + a)^3 + 24*I*d^3*cosh(b*x + a))*sinh(b*x + a))*po
lylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) + (-6*I*d^3*cosh(b*x + a)^4 - 2
4*I*d^3*cosh(b*x + a)*sinh(b*x + a)^3 - 6*I*d^3*sinh(b*x + a)^4 - 12*I*d^3*
cosh(b*x + a)^2 - 6*I*d^3 + (-36*I*d^3*cosh(b*x + a)^2 - 12*I*d^3)*sinh(b*x
 + a)^2 + (-24*I*d^3*cosh(b*x + a)^3 - 24*I*d^3*cosh(b*x + a))*sinh(b*x + a
))*polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + (-6*I*b*d^3*x + (-6*I*b
*d^3*x - 6*I*b*c*d^2)*cosh(b*x + a)^4 + (-24*I*b*d^3*x - 24*I*b*c*d^2)*cosh
(b*x + a)*sinh(b*x + a)^3 + (-6*I*b*d^3*x - 6*I*b*c*d^2)*sinh(b*x + a)^4 -
6*I*b*c*d^2 + (-12*I*b*d^3*x - 12*I*b*c*d^2)*cosh(b*x + a)^2 + (-12*I*b*d^3
*x - 12*I*b*c*d^2 + (-36*I*b*d^3*x - 36*I*b*c*d^2)*cosh(b*x + a)^2)*sinh(b*
x + a)^2 + ((-24*I*b*d^3*x - 24*I*b*c*d^2)*cosh(b*x + a)^3 + (-24*I*b*d^3*x
 - 24*I*b*c*d^2)*cosh(b*x + a))*sinh(b*x + a))*polylog(3, I*cosh(b*x + a) +
 I*sinh(b*x + a)) + (6*I*b*d^3*x + (6*I*b*d^3*x + 6*I*b*c*d^2)*cosh(b*x + a
)^4 + (24*I*b*d^3*x + 24*I*b*c*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (6*I*b*
d^3*x + 6*I*b*c*d^2)*sinh(b*x + a)^4 + 6*I*b*c*d^2 + (12*I*b*d^3*x + 12*I*b
*c*d^2)*cosh(b*x + a)^2 + (12*I*b*d^3*x + 12*I*b*c*d^2 + (36*I*b*d^3*x + 36
*I*b*c*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + ((24*I*b*d^3*x + 24*I*b*c*d^
2)*cosh(b*x + a)^3 + (24*I*b*d^3*x + 24*I*b*c*d^2)*cosh(b*x + a))*sinh(b*x
+ a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b^3*d^3*x^3 + b^3
```

```
*c^3 - 3*b^2*c^2*d + 3*(b^3*c*d^2 - b^2*d^3)*x^2 - 3*(b^3*d^3*x^3 + b^3*c^3
 + 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*
x)*cosh(b*x + a)^2 + 3*(b^3*c^2*d - 2*b^2*c*d^2)*x)*sinh(b*x + a))/(b^4*cos
h(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 +
2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 + b^4)*sinh(b*x + a)
^2 + 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \operatorname{sech}^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*sech(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \operatorname{sech}(bx + a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sech(b*x + a)^3, x)
```

3.37 $\int (c+dx)^2 \operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=175

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{id(c+dx)\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{id^2\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{id^2\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}+$$

```
[Out] ((c + d*x)^2*ArcTan[E^(a + b*x)])/b - (d^2*ArcTan[Sinh[a + b*x]])/b^3 - (I*
d*(c + d*x)*PolyLog[2, (-I)*E^(a + b*x)])/b^2 + (I*d*(c + d*x)*PolyLog[2, I
*E^(a + b*x)])/b^2 + (I*d^2*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - (I*d^2*Poly
Log[3, I*E^(a + b*x)])/b^3 + (d*(c + d*x)*Sech[a + b*x])/b^2 + ((c + d*x)^2
*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rubi [A] time = 0.132825, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4186, 3770, 4180, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{id(c+dx)\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{id^2\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{id^2\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sech[a + b*x]^3,x]
```


```
[Out] ((c + d*x)^2*ArcTan[E^(a + b*x)])/b - (d^2*ArcTan[Sinh[a + b*x]])/b^3 - (I*
d*(c + d*x)*PolyLog[2, (-I)*E^(a + b*x)])/b^2 + (I*d*(c + d*x)*PolyLog[2, I
*E^(a + b*x)])/b^2 + (I*d^2*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - (I*d^2*Poly
Log[3, I*E^(a + b*x)])/b^3 + (d*(c + d*x)*Sech[a + b*x])/b^2 + ((c + d*x)^2
*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```


Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\text{sech}^3(a+bx)\,dx = \frac{d(c+dx)\text{sech}(a+bx)}{b^2}+\frac{(c+dx)^2\text{sech}(a+bx)\tanh(a+bx)}{2b}+\frac{1}{2}\int (c+dx)^2\text{sech}(a-$$
$$=\frac{(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tan^{-1}(\sinh(a+bx))}{b^3}+\frac{d(c+dx)\text{sech}(a+bx)}{b^2}+\frac{(c+dx)^2}{}$$
$$=\frac{(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tan^{-1}(\sinh(a+bx))}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{id(c+d}{}$$
$$=\frac{(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tan^{-1}(\sinh(a+bx))}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{id(c+d}{}$$
$$=\frac{(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tan^{-1}(\sinh(a+bx))}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{id(c+d}{}$$

Mathematica [A] time = 5.31035, size = 270, normalized size = 1.54

$$\frac{i\left(-2bd(c+dx)\text{PolyLog}\left(2,-ie^{a+bx}\right)+2bd(c+dx)\text{PolyLog}\left(2,ie^{a+bx}\right)+2d^2\text{PolyLog}\left(3,-ie^{a+bx}\right)-2d^2\text{PolyLog}\left(3,i\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sech[a + b*x]^3,x]
```

```
[Out] (I*((-2*I)*b^2*c^2*ArcTan[E^(a + b*x)] + (4*I)*d^2*ArcTan[E^(a + b*x)] + 2*
b^2*c*d*x*Log[1 - I*E^(a + b*x)] + b^2*d^2*x^2*Log[1 - I*E^(a + b*x)] - 2*b
^2*c*d*x*Log[1 + I*E^(a + b*x)] - b^2*d^2*x^2*Log[1 + I*E^(a + b*x)] - 2*b*
d*(c + d*x)*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*d*(c + d*x)*PolyLog[2, I*E^(
a + b*x)] + 2*d^2*PolyLog[3, (-I)*E^(a + b*x)] - 2*d^2*PolyLog[3, I*E^(a +
b*x)]) + b^2*(c + d*x)^2*Sech[a]*Sech[a + b*x]^2*Sinh[b*x] + b*(c + d*x)*Se
ch[a + b*x]*(2*d + b*(c + d*x)*Tanh[a]))/(2*b^3)
```

Maple [F] time = 0.164, size = 0, normalized size = 0.

$$\int (dx+c)^2 (\text{sech}\,(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sech(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^2*sech(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2d^2\int\frac{x^2e^{(bx+a)}}{b^2e^{(2\,bx+2\,a)}+b^2}\,dx+2\,b^2cd\int\frac{xe^{(bx+a)}}{b^2e^{(2\,bx+2\,a)}+b^2}\,dx-c^2\left(\frac{\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1\right)}\right)+\frac{(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b^2*d^2*integrate(x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) + 2*b^2*c
*d*integrate(x*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - c^2*(arctan(e^
(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) +
e^(-4*b*x - 4*a) + 1))) + ((b*d^2*x^2*e^(3*a) + 2*c*d*e^(3*a) + 2*(b*c*d +
d^2)*x*e^(3*a))*e^(3*b*x) - (b*d^2*x^2*e^a - 2*c*d*e^a + 2*(b*c*d - d^2)*x*
e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) - 2*d^2*a
rctan(e^(b*x + a))/b^3
```

Fricas [C] time = 2.83246, size = 6791, normalized size = 38.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(b*x +
 a)^3 + 6*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(b*
x + a)*sinh(b*x + a)^2 + 2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d +
b*d^2)*x)*sinh(b*x + a)^3 - 2*(b^2*d^2*x^2 + b^2*c^2 - 2*b*c*d + 2*(b^2*c*d
 - b*d^2)*x)*cosh(b*x + a) + ((2*I*b*d^2*x + 2*I*b*c*d)*cosh(b*x + a)^4 + (
8*I*b*d^2*x + 8*I*b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (2*I*b*d^2*x + 2*I
*b*c*d)*sinh(b*x + a)^4 + 2*I*b*d^2*x + 2*I*b*c*d + (4*I*b*d^2*x + 4*I*b*c*
d)*cosh(b*x + a)^2 + (4*I*b*d^2*x + 4*I*b*c*d + (12*I*b*d^2*x + 12*I*b*c*d)
*cosh(b*x + a)^2)*sinh(b*x + a)^2 + ((8*I*b*d^2*x + 8*I*b*c*d)*cosh(b*x + a
)^3 + (8*I*b*d^2*x + 8*I*b*c*d)*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(
b*x + a) + I*sinh(b*x + a)) + ((-2*I*b*d^2*x - 2*I*b*c*d)*cosh(b*x + a)^4 +
 (-8*I*b*d^2*x - 8*I*b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (-2*I*b*d^2*x -
 2*I*b*c*d)*sinh(b*x + a)^4 - 2*I*b*d^2*x - 2*I*b*c*d + (-4*I*b*d^2*x - 4*I
*b*c*d)*cosh(b*x + a)^2 + (-4*I*b*d^2*x - 4*I*b*c*d + (-12*I*b*d^2*x - 12*I
*b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + ((-8*I*b*d^2*x - 8*I*b*c*d)*cosh
(b*x + a)^3 + (-8*I*b*d^2*x - 8*I*b*c*d)*cosh(b*x + a))*sinh(b*x + a))*dilo
g(-I*cosh(b*x + a) - I*sinh(b*x + a)) + ((I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2
- 2)*d^2)*cosh(b*x + a)^4 + (4*I*b^2*c^2 - 8*I*a*b*c*d + 4*I*(a^2 - 2)*d^2)
*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2
)*sinh(b*x + a)^4 + I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2 + (2*I*b^2*c^
2 - 4*I*a*b*c*d + 2*I*(a^2 - 2)*d^2)*cosh(b*x + a)^2 + (2*I*b^2*c^2 - 4*I*a
*b*c*d + 2*I*(a^2 - 2)*d^2 + (6*I*b^2*c^2 - 12*I*a*b*c*d + 6*I*(a^2 - 2)*d^
2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + ((4*I*b^2*c^2 - 8*I*a*b*c*d + 4*I*(a^
```

```
2 - 2)*d^2)*cosh(b*x + a)^3 + (4*I*b^2*c^2 - 8*I*a*b*c*d + 4*I*(a^2 - 2)*d^
2)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + (
(-I*b^2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)*d^2)*cosh(b*x + a)^4 + (-4*I*b^2*c^
2 + 8*I*a*b*c*d - 4*I*(a^2 - 2)*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^
2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)*d^2)*sinh(b*x + a)^4 - I*b^2*c^2 + 2*I*a*
b*c*d - I*(a^2 - 2)*d^2 + (-2*I*b^2*c^2 + 4*I*a*b*c*d - 2*I*(a^2 - 2)*d^2)*
cosh(b*x + a)^2 + (-2*I*b^2*c^2 + 4*I*a*b*c*d - 2*I*(a^2 - 2)*d^2 + (-6*I*b
^2*c^2 + 12*I*a*b*c*d - 6*I*(a^2 - 2)*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2
 + ((-4*I*b^2*c^2 + 8*I*a*b*c*d - 4*I*(a^2 - 2)*d^2)*cosh(b*x + a)^3 + (-4*
I*b^2*c^2 + 8*I*a*b*c*d - 4*I*(a^2 - 2)*d^2)*cosh(b*x + a))*sinh(b*x + a))*
log(cosh(b*x + a) + sinh(b*x + a) - I) + (-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x +
(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x - 2*I*a*b*c*d + I*a^2*d^2)*cosh(b*x + a)^4
+ (-4*I*b^2*d^2*x^2 - 8*I*b^2*c*d*x - 8*I*a*b*c*d + 4*I*a^2*d^2)*cosh(b*x +
 a)*sinh(b*x + a)^3 + (-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x - 2*I*a*b*c*d + I*a^2
*d^2)*sinh(b*x + a)^4 - 2*I*a*b*c*d + I*a^2*d^2 + (-2*I*b^2*d^2*x^2 - 4*I*b
^2*c*d*x - 4*I*a*b*c*d + 2*I*a^2*d^2)*cosh(b*x + a)^2 + (-2*I*b^2*d^2*x^2 -
 4*I*b^2*c*d*x - 4*I*a*b*c*d + 2*I*a^2*d^2 + (-6*I*b^2*d^2*x^2 - 12*I*b^2*c
*d*x - 12*I*a*b*c*d + 6*I*a^2*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + ((-4*
I*b^2*d^2*x^2 - 8*I*b^2*c*d*x - 8*I*a*b*c*d + 4*I*a^2*d^2)*cosh(b*x + a)^3
+ (-4*I*b^2*d^2*x^2 - 8*I*b^2*c*d*x - 8*I*a*b*c*d + 4*I*a^2*d^2)*cosh(b*x +
 a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (I*b^2*d^2
*x^2 + 2*I*b^2*c*d*x + (I*b^2*d^2*x^2 + 2*I*b^2*c*d*x + 2*I*a*b*c*d - I*a^2
*d^2)*cosh(b*x + a)^4 + (4*I*b^2*d^2*x^2 + 8*I*b^2*c*d*x + 8*I*a*b*c*d - 4*
I*a^2*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b^2*d^2*x^2 + 2*I*b^2*c*d*x +
 2*I*a*b*c*d - I*a^2*d^2)*sinh(b*x + a)^4 + 2*I*a*b*c*d - I*a^2*d^2 + (2*I*
b^2*d^2*x^2 + 4*I*b^2*c*d*x + 4*I*a*b*c*d - 2*I*a^2*d^2)*cosh(b*x + a)^2 +
(2*I*b^2*d^2*x^2 + 4*I*b^2*c*d*x + 4*I*a*b*c*d - 2*I*a^2*d^2 + (6*I*b^2*d^2
*x^2 + 12*I*b^2*c*d*x + 12*I*a*b*c*d - 6*I*a^2*d^2)*cosh(b*x + a)^2)*sinh(b
*x + a)^2 + ((4*I*b^2*d^2*x^2 + 8*I*b^2*c*d*x + 8*I*a*b*c*d - 4*I*a^2*d^2)*
cosh(b*x + a)^3 + (4*I*b^2*d^2*x^2 + 8*I*b^2*c*d*x + 8*I*a*b*c*d - 4*I*a^2*
d^2)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) +
 1) + (-2*I*d^2*cosh(b*x + a)^4 - 8*I*d^2*cosh(b*x + a)*sinh(b*x + a)^3 - 2
*I*d^2*sinh(b*x + a)^4 - 4*I*d^2*cosh(b*x + a)^2 + (-12*I*d^2*cosh(b*x + a)
^2 - 4*I*d^2)*sinh(b*x + a)^2 - 2*I*d^2 + (-8*I*d^2*cosh(b*x + a)^3 - 8*I*d
^2*cosh(b*x + a))*sinh(b*x + a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x +
a)) + (2*I*d^2*cosh(b*x + a)^4 + 8*I*d^2*cosh(b*x + a)*sinh(b*x + a)^3 + 2*
I*d^2*sinh(b*x + a)^4 + 4*I*d^2*cosh(b*x + a)^2 + (12*I*d^2*cosh(b*x + a)^2
 + 4*I*d^2)*sinh(b*x + a)^2 + 2*I*d^2 + (8*I*d^2*cosh(b*x + a)^3 + 8*I*d^2*
cosh(b*x + a))*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)
) - 2*(b^2*d^2*x^2 + b^2*c^2 - 2*b*c*d - 3*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d
 + 2*(b^2*c*d + b*d^2)*x)*cosh(b*x + a)^2 + 2*(b^2*c*d - b*d^2)*x)*sinh(b*x
 + a))/(b^3*cosh(b*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sin
h(b*x + a)^4 + 2*b^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3
)*sinh(b*x + a)^2 + 4*(b^3*cosh(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x +
a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \operatorname{sech}^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sech(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*sech(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sech(b*x + a)^3, x)
```

3.38 $\int (c+dx)\text{sech}^3(a+bx)\,dx$

Optimal. Leaf size=102

$$-\frac{id\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{id\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}+\frac{d\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b}+\frac{(c+dx)\tanh(a+bx)\text{s}}{2b}$$

```
[Out] ((c + d*x)*ArcTan[E^(a + b*x)])/b - ((I/2)*d*PolyLog[2, (-I)*E^(a + b*x)])/
b^2 + ((I/2)*d*PolyLog[2, I*E^(a + b*x)])/b^2 + (d*Sech[a + b*x])/(2*b^2) +
 ((c + d*x)*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rubi [A] time = 0.0628146, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4185, 4180, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{id\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}+\frac{d\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b}+\frac{(c+dx)\tanh(a+bx)\text{s}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sech[a + b*x]^3,x]
```


```
[Out] ((c + d*x)*ArcTan[E^(a + b*x)])/b - ((I/2)*d*PolyLog[2, (-I)*E^(a + b*x)])/
b^2 + ((I/2)*d*PolyLog[2, I*E^(a + b*x)])/b^2 + (d*Sech[a + b*x])/(2*b^2) +
 ((c + d*x)*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```


Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\text{sech}^3(a+bx)\,dx = \frac{d\text{sech}(a+bx)}{2b^2} + \frac{(c+dx)\text{sech}(a+bx)\tanh(a+bx)}{2b} + \frac{1}{2}\int (c+dx)\text{sech}(a+bx)\,dx$$
$$= \frac{(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{d\text{sech}(a+bx)}{2b^2} + \frac{(c+dx)\text{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{(id)\int \log}{}$$
$$= \frac{(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{d\text{sech}(a+bx)}{2b^2} + \frac{(c+dx)\text{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{(id)\,\text{Subst}}{}$$
$$= \frac{(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{id\text{Li}_2\left(-ie^{a+bx}\right)}{2b^2} + \frac{id\text{Li}_2\left(ie^{a+bx}\right)}{2b^2} + \frac{d\text{sech}(a+bx)}{2b^2} + \frac{(c+dx)\text{sech}}{}$$

Mathematica [A] time = 2.85816, size = 180, normalized size = 1.76

$$\frac{-id\left(\text{PolyLog}\left(2,-ie^{a+bx}\right)-\text{PolyLog}\left(2,ie^{a+bx}\right)\right)+bc\tan^{-1}(\sinh(a+bx))+bc\tanh(a+bx)\text{sech}(a+bx)-\frac{1}{2}d(-2ia-2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sech[a + b*x]^3,x]
```

```
[Out] (b*c*ArcTan[Sinh[a + b*x]] - (d*((-2*I)*a + Pi - (2*I)*b*x)*(Log[1 - I*E^(a
 + b*x)] - Log[1 + I*E^(a + b*x)]))/2 + (d*((-2*I)*a + Pi)*Log[Cot[((2*I)*a
 + Pi + (2*I)*b*x)/4]])/2 - I*d*(PolyLog[2, (-I)*E^(a + b*x)] - PolyLog[2,
I*E^(a + b*x)]) + b*d*x*Sech[a]*Sech[a + b*x]^2*Sinh[b*x] + d*Sech[a + b*x]
*(1 + b*x*Tanh[a]) + b*c*Sech[a + b*x]*Tanh[a + b*x])/(2*b^2)
```

Maple [B] time = 0.053, size = 216, normalized size = 2.1

$$\frac{e^{bx+a}\left(bdxe^{2bx+2a}+bce^{2bx+2a}-bdx+de^{2bx+2a}-cb+d\right)}{b^2\left(1+e^{2bx+2a}\right)^2} + \frac{c\arctan\left(e^{bx+a}\right)}{b} - \frac{\frac{i}{2}d\ln\left(1+ie^{bx+a}\right)x}{b} - \frac{\frac{i}{2}d\ln\left(1+ie^{bx+a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sech(b*x+a)^3,x)
```

```
[Out] exp(b*x+a)*(b*d*x*exp(2*b*x+2*a)+b*c*exp(2*b*x+2*a)-b*d*x+d*exp(2*b*x+2*a)-
c*b+d)/b^2/(1+exp(2*b*x+2*a))^2+1/b*c*arctan(exp(b*x+a))-1/2*I/b*d*ln(1+I*e
xp(b*x+a))*x-1/2*I/b^2*d*ln(1+I*exp(b*x+a))*a+1/2*I/b*d*ln(1-I*exp(b*x+a))*
x+1/2*I/b^2*d*ln(1-I*exp(b*x+a))*a-1/2*I/b^2*d*dilog(1+I*exp(b*x+a))+1/2*I/
b^2*d*dilog(1-I*exp(b*x+a))-1/b^2*d*a*arctan(exp(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d\left(\frac{\left(bxe^{(3a)}+e^{(3a)}\right)e^{(3bx)}-\left(bxe^{a}-e^{a}\right)e^{(bx)}}{b^2e^{(4bx+4a)}+2b^2e^{(2bx+2a)}+b^2}+8\int\frac{xe^{(bx+a)}}{8\left(e^{(2bx+2a)}+1\right)}\,dx\right)-c\left(\frac{\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{e^{(-bx-a)}-e^{(-3bx-3a)}}{b\left(2e^{(-2bx-2a)}+e^{(-4bx-4a)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] d*(((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) - (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4*
b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 8*integrate(1/8*x*e^(b*x + a)/(
e^(2*b*x + 2*a) + 1), x)) - c*(arctan(e^(-b*x - a))/b - (e^(-b*x - a) - e^(
-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) + 1)))
```

Fricas [B] time = 2.52088, size = 3492, normalized size = 34.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*d*x + b*c + d)*cosh(b*x + a)^3 + 6*(b*d*x + b*c + d)*cosh(b*x + a
)*sinh(b*x + a)^2 + 2*(b*d*x + b*c + d)*sinh(b*x + a)^3 - 2*(b*d*x + b*c -
d)*cosh(b*x + a) + (I*d*cosh(b*x + a)^4 + 4*I*d*cosh(b*x + a)*sinh(b*x + a)
^3 + I*d*sinh(b*x + a)^4 + 2*I*d*cosh(b*x + a)^2 + (6*I*d*cosh(b*x + a)^2 +
 2*I*d)*sinh(b*x + a)^2 + (4*I*d*cosh(b*x + a)^3 + 4*I*d*cosh(b*x + a))*sin
h(b*x + a) + I*d)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + (-I*d*cosh(b*x
 + a)^4 - 4*I*d*cosh(b*x + a)*sinh(b*x + a)^3 - I*d*sinh(b*x + a)^4 - 2*I*d
*cosh(b*x + a)^2 + (-6*I*d*cosh(b*x + a)^2 - 2*I*d)*sinh(b*x + a)^2 + (-4*I
*d*cosh(b*x + a)^3 - 4*I*d*cosh(b*x + a))*sinh(b*x + a) - I*d)*dilog(-I*cos
h(b*x + a) - I*sinh(b*x + a)) + ((I*b*c - I*a*d)*cosh(b*x + a)^4 + (4*I*b*c
 - 4*I*a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b*c - I*a*d)*sinh(b*x + a)^4
 + (2*I*b*c - 2*I*a*d)*cosh(b*x + a)^2 + ((6*I*b*c - 6*I*a*d)*cosh(b*x + a)
^2 + 2*I*b*c - 2*I*a*d)*sinh(b*x + a)^2 + I*b*c - I*a*d + ((4*I*b*c - 4*I*a
*d)*cosh(b*x + a)^3 + (4*I*b*c - 4*I*a*d)*cosh(b*x + a))*sinh(b*x + a))*log
(cosh(b*x + a) + sinh(b*x + a) + I) + ((-I*b*c + I*a*d)*cosh(b*x + a)^4 + (
-4*I*b*c + 4*I*a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*c + I*a*d)*sinh(b
*x + a)^4 + (-2*I*b*c + 2*I*a*d)*cosh(b*x + a)^2 + ((-6*I*b*c + 6*I*a*d)*co
sh(b*x + a)^2 - 2*I*b*c + 2*I*a*d)*sinh(b*x + a)^2 - I*b*c + I*a*d + ((-4*I
*b*c + 4*I*a*d)*cosh(b*x + a)^3 + (-4*I*b*c + 4*I*a*d)*cosh(b*x + a))*sinh(
b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + ((-I*b*d*x - I*a*d)*cosh
(b*x + a)^4 + (-4*I*b*d*x - 4*I*a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*
d*x - I*a*d)*sinh(b*x + a)^4 - I*b*d*x + (-2*I*b*d*x - 2*I*a*d)*cosh(b*x +
a)^2 + (-2*I*b*d*x + (-6*I*b*d*x - 6*I*a*d)*cosh(b*x + a)^2 - 2*I*a*d)*sinh
(b*x + a)^2 - I*a*d + ((-4*I*b*d*x - 4*I*a*d)*cosh(b*x + a)^3 + (-4*I*b*d*x
 - 4*I*a*d)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x
+ a) + 1) + ((I*b*d*x + I*a*d)*cosh(b*x + a)^4 + (4*I*b*d*x + 4*I*a*d)*cosh
(b*x + a)*sinh(b*x + a)^3 + (I*b*d*x + I*a*d)*sinh(b*x + a)^4 + I*b*d*x + (
2*I*b*d*x + 2*I*a*d)*cosh(b*x + a)^2 + (2*I*b*d*x + (6*I*b*d*x + 6*I*a*d)*c
osh(b*x + a)^2 + 2*I*a*d)*sinh(b*x + a)^2 + I*a*d + ((4*I*b*d*x + 4*I*a*d)*
cosh(b*x + a)^3 + (4*I*b*d*x + 4*I*a*d)*cosh(b*x + a))*sinh(b*x + a))*log(-
I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 2*(b*d*x - 3*(b*d*x + b*c + d)*cos
h(b*x + a)^2 + b*c - d)*sinh(b*x + a))/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh(b*
x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 + 2*b^2*cosh(b*x + a)^2 + 2*(3
*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a)^3
+ b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \operatorname{sech}^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*sech(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\operatorname{sech}(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sech(b*x + a)^3, x)
```

3.39 $\int \frac{\text{sech}^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\text{sech}^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Sech[a + b*x]^3/(c + d*x), x]
```

Rubi [A] time = 0.0366865, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{sech}^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[a + b*x]^3/(c + d*x),x]
```

```
[Out] Defer[Int][Sech[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\text{sech}^3(a+bx)}{c+dx}\,dx=\int \frac{\text{sech}^3(a+bx)}{c+dx}\,dx$$

Mathematica [F] time = 180.016, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sech[a + b*x]^3/(c + d*x),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.3, size = 0, normalized size = 0.

$$\int \frac{(\text{sech}\,(bx+a))^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(sech(b*x+a)^3/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bdxe^{(3a)} + (bc-d)e^{(3a)}\right)e^{(3bx)} - (bdxe^{a} + (bc+d)e^{a})e^{(bx)}}{b^2d^2x^2 + 2b^2cdx + b^2c^2 + \left(b^2d^2x^2e^{(4a)} + 2b^2cdxe^{(4a)} + b^2c^2e^{(4a)}\right)e^{(4bx)} + 2\left(b^2d^2x^2e^{(2a)} + 2b^2cdxe^{(2a)} + b^2c^2e^{(2a)}\right)e^{(2bx)}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] ((b*d*x*e^(3*a) + (b*c - d)*e^(3*a))*e^(3*b*x) - (b*d*x*e^a + (b*c + d)*e^a
)*e^(b*x))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2*e^(4*a) + 2*
b^2*c*d*x*e^(4*a) + b^2*c^2*e^(4*a))*e^(4*b*x) + 2*(b^2*d^2*x^2*e^(2*a) + 2
*b^2*c*d*x*e^(2*a) + b^2*c^2*e^(2*a))*e^(2*b*x)) + 8*integrate(1/8*(b^2*d^2
*x^2*e^a + 2*b^2*c*d*x*e^a + (b^2*c^2 - 2*d^2)*e^a)*e^(b*x)/(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3*e^(2*a) + 3*b^2*c*
d^2*x^2*e^(2*a) + 3*b^2*c^2*d*x*e^(2*a) + b^2*c^3*e^(2*a))*e^(2*b*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{sech}(bx+a)^3}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sech(a + b*x)**3/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)^3}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3/(d*x + c), x)
```

3.40 $\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Sech[a + b*x]^3/(c + d*x)^2, x]
```

Rubi [A] time = 0.034078, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Sech[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [F] time = 180.015, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sech[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.422, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(bx+a))^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(sech(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bdxe^{(3a)} + (bc - 2d)e^{(3a)}\right)e^{(3bx)} - (bdxe^{a} + (bc + 2d)e^{a})e^{(bx)}}{b^2d^3x^3 + 3b^2cd^2x^2 + 3b^2c^2dx + b^2c^3 + \left(b^2d^3x^3e^{(4a)} + 3b^2cd^2x^2e^{(4a)} + 3b^2c^2dxe^{(4a)} + b^2c^3e^{(4a)}\right)e^{(4bx)} + 2\left(b^2d^3x^3e^{(2a)} + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] ((b*d*x*e^(3*a) + (b*c - 2*d)*e^(3*a))*e^(3*b*x) - (b*d*x*e^a + (b*c + 2*d)
*e^a)*e^(b*x))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (
b^2*d^3*x^3*e^(4*a) + 3*b^2*c*d^2*x^2*e^(4*a) + 3*b^2*c^2*d*x*e^(4*a) + b^2
*c^3*e^(4*a))*e^(4*b*x) + 2*(b^2*d^3*x^3*e^(2*a) + 3*b^2*c*d^2*x^2*e^(2*a)
+ 3*b^2*c^2*d*x*e^(2*a) + b^2*c^3*e^(2*a))*e^(2*b*x)) + 8*integrate(1/8*(b^
2*d^2*x^2*e^a + 2*b^2*c*d*x*e^a + (b^2*c^2 - 6*d^2)*e^a)*e^(b*x)/(b^2*d^4*x
^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d
^4*x^4*e^(2*a) + 4*b^2*c*d^3*x^3*e^(2*a) + 6*b^2*c^2*d^2*x^2*e^(2*a) + 4*b^
2*c^3*d*x*e^(2*a) + b^2*c^4*e^(2*a))*e^(2*b*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}(bx + a)^3}{d^2x^2 + 2\,cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx + a)^3}{(dx + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3/(d*x + c)^2, x)
```

3.41 $\int (c+dx)^{5/2}\cosh(a+bx)\,dx$

Optimal. Leaf size=171

$$\frac{15\sqrt{\pi}d^{5/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sinh(a+bx)}{4b^3}-\frac{5d(c+dx)^{3/2}\cosh(a+b}{2b^2}$$

```
[Out] (-5*d*(c + d*x)^(3/2)*Cosh[a + b*x])/(2*b^2) + (15*d^(5/2)*E^(-a + (b*c)/d)
*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (15*d^(5/2)*
E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)
) + (15*d^2*Sqrt[c + d*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^(5/2)*Sinh[a
+ b*x])/b
```

Rubi [A] time = 0.329771, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3296, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\pi}d^{5/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sinh(a+bx)}{4b^3}-\frac{5d(c+dx)^{3/2}\cosh(a+b}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cosh[a + b*x],x]
```


```
[Out] (-5*d*(c + d*x)^(3/2)*Cosh[a + b*x])/(2*b^2) + (15*d^(5/2)*E^(-a + (b*c)/d)
*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (15*d^(5/2)*
E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)
) + (15*d^2*Sqrt[c + d*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^(5/2)*Sinh[a
+ b*x])/b
```


Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (c+dx)^{5/2}\cosh(a+bx)\,dx = \frac{(c+dx)^{5/2}\sinh(a+bx)}{b} - \frac{(5d)\int (c+dx)^{3/2}\sinh(a+bx)\,dx}{2b}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2} + \frac{(c+dx)^{5/2}\sinh(a+bx)}{b} + \frac{\left(15d^2\right)\int \sqrt{c+dx}\cosh(a+bx)}{4b^2}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2} + \frac{15d^2\sqrt{c+dx}\sinh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sinh(a+bx)}{b} - $$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2} + \frac{15d^2\sqrt{c+dx}\sinh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sinh(a+bx)}{b} - $$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2} + \frac{15d^2\sqrt{c+dx}\sinh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sinh(a+bx)}{b} + $$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2} + \frac{15d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{15d^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+}}{\sqrt{d}}\right)}{16b^{7/2}}$$

Mathematica [A] time = 0.0523993, size = 107, normalized size = 0.63

$$-\frac{d^3e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{7}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{7}{2},\frac{b(c+dx)}{d}\right)\right)}{2b^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cosh[a + b*x],x]
```

```
[Out] -(d^3*E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[7/2, -((b*(c
 + d*x))/d)] + E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[7/2, (b*(c + d*x))
/d]))/(2*b^4*Sqrt[c + d*x])
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{2}}\cosh(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cosh(b*x+a),x)
```

```
[Out] int((d*x+c)^(5/2)*cosh(b*x+a),x)
```

Maxima [B] time = 1.08342, size = 416, normalized size = 2.43

$$\frac{32(dx+c)^{\frac{7}{2}}\cosh(bx+a) - \frac{\left(\frac{105\sqrt{\pi}d^4\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^4\sqrt{-\frac{b}{d}}} - \frac{105\sqrt{\pi}d^4\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^4\sqrt{\frac{b}{d}}} + \frac{2\left(8(dx+c)^{\frac{7}{2}}b^3de^{\left(\frac{bc}{d}\right)}+28(dx+c)^{\frac{5}{2}}b^2d^2e^{\left(\frac{bc}{d}\right)}+70(dx+c)^{\frac{3}{2}}bd^3e^{(}\right.}{b^4}}{d}}{112d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/112*(32*(d*x + c)^(7/2)*cosh(b*x + a) - (105*sqrt(pi)*d^4*erf(sqrt(d*x +
c)*sqrt(-b/d))*e^(a - b*c/d)/(b^4*sqrt(-b/d)) - 105*sqrt(pi)*d^4*erf(sqrt(d
*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^4*sqrt(b/d)) + 2*(8*(d*x + c)^(7/2)*b^
3*d*e^(b*c/d) + 28*(d*x + c)^(5/2)*b^2*d^2*e^(b*c/d) + 70*(d*x + c)^(3/2)*b
*d^3*e^(b*c/d) + 105*sqrt(d*x + c)*d^4*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^
4 + 2*(8*(d*x + c)^(7/2)*b^3*d*e^a - 28*(d*x + c)^(5/2)*b^2*d^2*e^a + 70*(d
*x + c)^(3/2)*b*d^3*e^a - 105*sqrt(d*x + c)*d^4*e^a)*e^((d*x + c)*b/d - b*c
/d)/b^4)*b/d)/d
```

Fricas [B] time = 2.12818, size = 1177, normalized size = 6.88

$$\frac{15\,\sqrt{\pi}\left(d^{3}\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-d^{3}\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d^{3}\cosh\left(-\frac{bc-ad}{d}\right)-d^{3}\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/16*(15*sqrt(pi)*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x +
a)*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) - d^3*sinh(-(b*c - a*d)
/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 15*sqrt(pi)*(d
^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-(b*c - a*d)
/d) + (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*
sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 1
0*b^2*c*d + 15*b*d^2 - (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 +
 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^2 - 2*(4*b^3*d^2*x^2 + 4*b^3*c^
2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)*sinh
(b*x + a) - (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c
*d - 5*b^2*d^2)*x)*sinh(b*x + a)^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*x)*sqrt(d*x
+ c))/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cosh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47911, size = 313, normalized size = 1.83

$$-\frac{\frac{15\,\sqrt{\pi}d^{4}\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b^{3}}-\frac{15\,\sqrt{\pi}d^{4}\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b^{3}}-\frac{2\left(4\,(dx+c)^{\frac{5}{2}}b^{2}d-10\,(dx+c)^{\frac{3}{2}}bd^{2}+15\,\sqrt{dx+c}d^{3}\right)e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^{3}}+\frac{2\left(4\,(dx\ldots\right.}{\ldots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(15*sqrt(pi)*d^4*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(s
qrt(b*d)*b^3) - 15*sqrt(pi)*d^4*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c -
 a*d)/d)/(sqrt(-b*d)*b^3) - 2*(4*(d*x + c)^(5/2)*b^2*d - 10*(d*x + c)^(3/2)
*b*d^2 + 15*sqrt(d*x + c)*d^3)*e^(((d*x + c)*b - b*c + a*d)/d)/b^3 + 2*(4*(
d*x + c)^(5/2)*b^2*d + 10*(d*x + c)^(3/2)*b*d^2 + 15*sqrt(d*x + c)*d^3)*e^(
-((d*x + c)*b - b*c + a*d)/d)/b^3)/d
```

3.42 $\int (c+dx)^{3/2}\cosh(a+bx)\,dx$

Optimal. Leaf size=146

$$\frac{3\sqrt{\pi}d^{3/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3\sqrt{\pi}d^{3/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3d\sqrt{c+dx}\cosh(a+bx)}{2b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}$$

```
[Out] (-3*d*Sqrt[c + d*x]*Cosh[a + b*x])/(2*b^2) + (3*d^(3/2)*E^(-a + (b*c)/d)*Sq
rt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (3*d^(3/2)*E^(a
- (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + ((
c + d*x)^(3/2)*Sinh[a + b*x])/b
```

Rubi [A] time = 0.242839, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3296, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}d^{3/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3\sqrt{\pi}d^{3/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3d\sqrt{c+dx}\cosh(a+bx)}{2b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cosh[a + b*x],x]
```

```
[Out] (-3*d*Sqrt[c + d*x]*Cosh[a + b*x])/(2*b^2) + (3*d^(3/2)*E^(-a + (b*c)/d)*Sq
rt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (3*d^(3/2)*E^(a
- (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + ((
c + d*x)^(3/2)*Sinh[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (c+dx)^{3/2}\cosh(a+bx)\,dx = \frac{(c+dx)^{3/2}\sinh(a+bx)}{b} - \frac{(3d)\int \sqrt{c+dx}\sinh(a+bx)\,dx}{2b}$$

$$= -\frac{3d\sqrt{c+dx}\cosh(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sinh(a+bx)}{b} + \frac{\left(3d^2\right)\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{4b^2}$$

$$= -\frac{3d\sqrt{c+dx}\cosh(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sinh(a+bx)}{b} + \frac{\left(3d^2\right)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{8b^2} + \frac{\left(3d^2\right)\int \frac{e^{i(ia}}{\sqrt{c}}}{8b^2}$$

$$= -\frac{3d\sqrt{c+dx}\cosh(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sinh(a+bx)}{b} + \frac{(3d)\,\mathrm{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x,\right.}{4b^2}$$

$$= -\frac{3d\sqrt{c+dx}\cosh(a+bx)}{2b^2} + \frac{3d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{3d^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}$$

Mathematica [A] time = 0.101172, size = 107, normalized size = 0.73

$$\frac{d\sqrt{c+dx}e^{-a-\frac{bc}{d}}\left(-\frac{e^{2a}\mathrm{Gamma}\left(\frac{5}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}}-\frac{e^{\frac{2bc}{d}}\mathrm{Gamma}\left(\frac{5}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cosh[a + b*x],x]
```

```
[Out] (d*E^(-a - (b*c)/d)*Sqrt[c + d*x]*(-((E^(2*a)*Gamma[5/2, -((b*(c + d*x))/d)
])/Sqrt[-((b*(c + d*x))/d)]) - (E^((2*b*c)/d)*Gamma[5/2, (b*(c + d*x))/d])/
Sqrt[(b*(c + d*x))/d]))/(2*b^2)
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{2}}\cosh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cosh(b*x+a),x)
```

```
[Out] int((d*x+c)^(3/2)*cosh(b*x+a),x)
```

Maxima [B] time = 1.08615, size = 362, normalized size = 2.48

$$\frac{16(dx+c)^{\frac{5}{2}}\cosh(bx+a) + \frac{\left(\frac{15\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} + \frac{15\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} - \frac{2\left(4(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{bc}{d}\right)}+10(dx+c)^{\frac{3}{2}}bd^2e^{\left(\frac{bc}{d}\right)}+15\sqrt{dx+c}d^3e^{\left(\frac{bc}{d}\right)}\right)e^{(}}{b^3}\right.}{d}}{40d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/40*(16*(d*x + c)^(5/2)*cosh(b*x + a) + (15*sqrt(pi)*d^3*erf(sqrt(d*x + c)
*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) + 15*sqrt(pi)*d^3*erf(sqrt(d*x
+ c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) - 2*(4*(d*x + c)^(5/2)*b^2*d
*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt(d*x + c)*d^3*e^(b
*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 2*(4*(d*x + c)^(5/2)*b^2*d*e^a - 10*(d*
x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d - b*c/d
)/b^3)*b/d)/d
```

Fricas [B] time = 2.16558, size = 892, normalized size = 6.11

$$\frac{3\sqrt{\pi}\left(d^2\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-d^2\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d^2\cosh\left(-\frac{bc-ad}{d}\right)-d^2\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(pi)*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)
*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh(-(b*c - a*d)/d
))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 3*sqrt(pi)*(d^2*
cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d)
 + (d^2*cosh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqr
t(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(2*b^2*d*x + 2*b^2*c - (2*b^2*d*x
 + 2*b^2*c - 3*b*d)*cosh(b*x + a)^2 - 2*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(
b*x + a)*sinh(b*x + a) - (2*b^2*d*x + 2*b^2*c - 3*b*d)*sinh(b*x + a)^2 + 3*
b*d)*sqrt(d*x + c))/(b^3*cosh(b*x + a) + b^3*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^{\frac{3}{2}}\cosh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cosh(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*cosh(a + b*x), x)
```

Giac [A] time = 1.46597, size = 273, normalized size = 1.87

$$-\frac{\frac{3\sqrt{\pi}d^3\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b^2}+\frac{3\sqrt{\pi}d^3\,\mathrm{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b^2}-\frac{2\left(2\left(dx+c\right)^{\frac{3}{2}}bd-3\sqrt{dx+c}d^2\right)e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2}+\frac{2\left(2\left(dx+c\right)^{\frac{3}{2}}bd+3\sqrt{dx+c}d^2\right)}{b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(3*sqrt(pi)*d^3*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqr
t(b*d)*b^2) + 3*sqrt(pi)*d^3*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*
d)/d)/(sqrt(-b*d)*b^2) - 2*(2*(d*x + c)^(3/2)*b*d - 3*sqrt(d*x + c)*d^2)*e^
(((d*x + c)*b - b*c + a*d)/d)/b^2 + 2*(2*(d*x + c)^(3/2)*b*d + 3*sqrt(d*x +
 c)*d^2)*e^(-((d*x + c)*b - b*c + a*d)/d)/b^2)/d
```

3.43 $\int \sqrt{c+dx}\cosh(a+bx)\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{c+dx}\sinh(a+bx)}{b}$$

```
[Out] (Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4
*b^(3/2)) - (Sqrt[d]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sinh[a + b*x])/b
```

Rubi [A] time = 0.177666, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3296, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{c+dx}\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cosh[a + b*x],x]
```

```
[Out] (Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4
*b^(3/2)) - (Sqrt[d]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sinh[a + b*x])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{c+dx}\cosh(a+bx)\,dx &= \frac{\sqrt{c+dx}\sinh(a+bx)}{b} - \frac{d\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{2b} \\
&= \frac{\sqrt{c+dx}\sinh(a+bx)}{b} - \frac{d\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4b} + \frac{d\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4b} \\
&= \frac{\sqrt{c+dx}\sinh(a+bx)}{b} + \frac{\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{2b} - \frac{\text{Subst}\left(\int e^{-i\left(ia-\frac{ibc}{d}\right)+\frac{bx^2}{d}}\,dx\right.}{2b} \\
&= \frac{\sqrt{d}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{d}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{c+dx}\sinh(a+bx)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0920261, size = 105, normalized size = 0.85

$$
\frac{\sqrt{c+dx}e^{-a-\frac{bc}{d}}\left(\frac{e^{2a}\text{Gamma}\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}} - \frac{e^{\frac{2bc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cosh[a + b*x],x]
```

```
[Out] (E^(-a - (b*c)/d)*Sqrt[c + d*x]*((E^(2*a)*Gamma[3/2, -((b*(c + d*x))/d)])/S
qrt[-((b*(c + d*x))/d)] - (E^((2*b*c)/d)*Gamma[3/2, (b*(c + d*x))/d])/Sqrt[
(b*(c + d*x))/d]))/(2*b)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$
\int \cosh(bx+a)\sqrt{dx+c}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] int(cosh(b*x+a)*(d*x+c)^(1/2),x)
```

Maxima [B] time = 1.11679, size = 311, normalized size = 2.53

$$
\frac{8(dx+c)^{\frac{3}{2}}\cosh(bx+a) - \frac{\left(\frac{3\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} - \frac{3\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} + \frac{2\left(2(dx+c)^{\frac{3}{2}}bde^{\left(\frac{bc}{d}\right)}+3\sqrt{dx+c}d^2e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^2} + \frac{2\left(2(dx+c)^{\frac{3}{2}}\right.}{}}{d}}{12d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(8*(d*x + c)^(3/2)*cosh(b*x + a) - (3*sqrt(pi)*d^2*erf(sqrt(d*x + c)*s
qrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) - 3*sqrt(pi)*d^2*erf(sqrt(d*x + c
)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) + 2*(2*(d*x + c)^(3/2)*b*d*e^(b
*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^2 + 2*(2*(d
*x + c)^(3/2)*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*c/d)/
b^2)*b/d)/d
```

Fricas [B] time = 2.08264, size = 717, normalized size = 5.83

$$\frac{\sqrt{\pi}\left(d\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-d\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d\cosh\left(-\frac{bc-ad}{d}\right)-d\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh\left(bx+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh(
-(b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(b*
x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + sqrt(pi)*(d*cosh(b*x + a)*
cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + (d*cosh(-(b*c
 - a*d)/d) + d*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x
 + c)*sqrt(-b/d)) + 2*(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a)
+ b*sinh(b*x + a)^2 - b)*sqrt(d*x + c))/(b^2*cosh(b*x + a) + b^2*sinh(b*x +
 a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\cosh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*cosh(a + b*x), x)
```

Giac [A] time = 1.44222, size = 228, normalized size = 1.85

$$-\frac{\frac{\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b}-\frac{\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b}-\frac{2\sqrt{dx+c}de^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b}+\frac{2\sqrt{dx+c}de^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqrt(
b*d)*b) - sqrt(pi)*d^2*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*d)/d)/
(sqrt(-b*d)*b) - 2*sqrt(d*x + c)*d*e^(((d*x + c)*b - b*c + a*d)/d)/b + 2*sq
rt(d*x + c)*d*e^(-((d*x + c)*b - b*c + a*d)/d)/b)/d
```

3.44 $\int \frac{\cosh(a+bx)}{\sqrt{c+dx}} \, dx$

Optimal. Leaf size=104

$$\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

```
[Out] (E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]
*Sqrt[d]) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]
)/(2*Sqrt[b]*Sqrt[d])
```

Rubi [A] time = 0.129774, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]
*Sqrt[d]) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]
)/(2*Sqrt[b]*Sqrt[d])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx = \frac{1}{2}\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx + \frac{1}{2}\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx$$
$$= \frac{\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d} + \frac{\text{Subst}\left(\int e^{-i\left(ia-\frac{ibc}{d}\right)+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

Mathematica [A] time = 0.0428036, size = 105, normalized size = 1.01

$$\frac{e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)-e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*
x))/d)] - E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]))
/(2*b*Sqrt[c + d*x])
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \cosh(bx+a)\frac{1}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] int(cosh(b*x+a)/(d*x+c)^(1/2),x)
```

Maxima [B] time = 1.0842, size = 243, normalized size = 2.34

$$\frac{4\sqrt{dx+c}\cosh(bx+a)+\frac{\left(\frac{\sqrt{\pi}d\,\text{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}+\frac{\sqrt{\pi}d\,\text{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}-\frac{2\sqrt{dx+c}de^{\left(a+\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b}-\frac{2\sqrt{dx+c}de^{\left(-a-\frac{(dx+c)b}{d}+\frac{bc}{d}\right)}}{b}\right)b}{d}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(d*x + c)*cosh(b*x + a) + (sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-b/
d))*e^(a - b*c/d)/(b*sqrt(-b/d)) + sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(b/d))*
e^(-a + b*c/d)/(b*sqrt(b/d)) - 2*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d - b*c
/d)/b - 2*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b)*b/d)/d
```

Fricas [A] time = 2.0964, size = 271, normalized size = 2.61

$$\frac{\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)-\sinh\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)+\sinh\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(b/d)) - sqrt(pi)*sqrt(-b/d)*(cosh(-(b*c - a*d)/d) + sinh(
-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cosh(a + b*x)/sqrt(c + d*x), x)
```

Giac [A] time = 1.39583, size = 123, normalized size = 1.18

$$-\frac{\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}}+\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/sqrt(b*d
) + sqrt(pi)*d*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*d)/d)/sqrt(-b*
d))/d
```

3.45 $\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}} dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{\pi}\sqrt{b}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\pi}\sqrt{b}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}$$

```
[Out] (-2*Cosh[a + b*x])/(d*Sqrt[c + d*x]) - (Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[Pi]*E
rf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[b]*E^(a - (b*c)/d)*Sqr
t[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2)
```

Rubi [A] time = 0.1827, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3297, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\pi}\sqrt{b}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(d*Sqrt[c + d*x]) - (Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[Pi]*E
rf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[b]*E^(a - (b*c)/d)*Sqr
t[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}+\frac{(2b)\int\frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}+\frac{b\int\frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{d}-\frac{b\int\frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{(2b)\,\mathrm{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d^2}+\frac{(2b)\,\mathrm{Subst}\left(\int e^{-i\left(ia-\frac{ibc}{d}\right)+\frac{bx^2}{d}}\,dx,x,\right.}{d^2}\\
&= -\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{\sqrt{b}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{b}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.350113, size = 118, normalized size = 0.99

$$\frac{e^{-a}\left(e^{2a-\frac{bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},b\left(\frac{c}{d}+x\right)\right)-e^{-bx}\left(e^{2(a+bx)}+1\right)\right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-((1 + E^(2*(a + b*x)))/E^(b*x)) + E^((b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma
[1/2, b*(c/d + x)] + E^(2*a - (b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2,
-((b*(c + d*x))/d)])/(d*E^a*Sqrt[c + d*x])
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \cosh(bx+a)\,(dx+c)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c)^(3/2),x)
```

```
[Out] int(cosh(b*x+a)/(d*x+c)^(3/2),x)
```

Maxima [A] time = 1.0513, size = 140, normalized size = 1.18

$$\frac{\frac{\left(\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}\right)b}{d}-\frac{2\cosh(bx+a)}{\sqrt{dx+c}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] ((sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/sqrt(-b/d) - sqrt(pi
)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/sqrt(b/d))*b/d - 2*cosh(b*x +
 a)/sqrt(d*x + c))/d
```

Fricas [B] time = 2.10143, size = 815, normalized size = 6.85

$$-\frac{\sqrt{\pi}\left((dx+c)\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-(dx+c)\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left((dx+c)\cosh\left(-\frac{bc-ad}{d}\right)-(dx+c)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b
*x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) - (d*x + c)*
sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d))
 + sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + (d*x + c)*cosh(
b*x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) + (d*x + c)
*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/
d)) + sqrt(d*x + c)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh
(b*x + a)^2 + 1))/((d^2*x + c*d)*cosh(b*x + a) + (d^2*x + c*d)*sinh(b*x + a
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(a+bx\right)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(cosh(a + b*x)/(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(bx+a\right)}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)/(d*x + c)^(3/2), x)
```

3.46 $\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=149

$$\frac{2\sqrt{\pi}b^{3/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{\pi}b^{3/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4b\sinh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}$$

```
[Out] (-2*Cosh[a + b*x])/(3*d*(c + d*x)^(3/2)) + (2*b^(3/2)*E^(-a + (b*c)/d)*Sqrt
[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (2*b^(3/2)*E^(a -
(b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) - (4*b
*Sinh[a + b*x])/(3*d^2*Sqrt[c + d*x])
```

Rubi [A] time = 0.248108, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3297, 3307, 2180, 2204, 2205}

$$\frac{2\sqrt{\pi}b^{3/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{\pi}b^{3/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4b\sinh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(3*d*(c + d*x)^(3/2)) + (2*b^(3/2)*E^(-a + (b*c)/d)*Sqrt
[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (2*b^(3/2)*E^(a -
(b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) - (4*b
*Sinh[a + b*x])/(3*d^2*Sqrt[c + d*x])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}} + \frac{(2b)\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx}{3d}$$

$$= -\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b\sinh(a+bx)}{3d^2\sqrt{c+dx}} + \frac{\left(4b^2\right)\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b\sinh(a+bx)}{3d^2\sqrt{c+dx}} + \frac{\left(2b^2\right)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{\left(2b^2\right)\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b\sinh(a+bx)}{3d^2\sqrt{c+dx}} + \frac{\left(4b^2\right)\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{3d^3} + \frac{\left(4b^2\right)\text{Subs}}{}$$

$$= -\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}} + \frac{2b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4b\sinh(a+b}{3d^2\sqrt{c+dx}}$$

Mathematica [A] time = 0.708778, size = 150, normalized size = 1.01

$$\frac{e^{-a}\left(-2de^{2a-\frac{bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)-e^{-bx}\left(2de^{b\left(\frac{c}{d}+x\right)}\left(\frac{b(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},b\left(\frac{c}{d}+x\right)\right)+2b\left(e^{2(a+bx)}-\right.\right.}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (-((d*(1 + E^(2*(a + b*x))) + 2*b*(-1 + E^(2*(a + b*x)))*(c + d*x) + 2*d*E^
(b*(c/d + x))*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, b*(c/d + x)])/E^(b*x)) - 2
*d*E^(2*a - (b*c)/d)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/
d)])/(3*d^2*E^a*(c + d*x)^(3/2))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \cosh(bx+a)\,(dx+c)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c)^(5/2),x)
```

```
[Out] int(cosh(b*x+a)/(d*x+c)^(5/2),x)
```

Maxima [A] time = 1.20593, size = 155, normalized size = 1.04

$$\frac{\frac{\left(\frac{\sqrt{\frac{(dx+c)b}{d}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}-\frac{\sqrt{-\frac{(dx+c)b}{d}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}\right)b}{d}-\frac{2\cosh(bx+a)}{(dx+c)^{\frac{3}{2}}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*((sqrt((d*x + c)*b/d)*e^(-a + b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d*
x + c) - sqrt(-(d*x + c)*b/d)*e^(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sqr
t(d*x + c))*b/d - 2*cosh(b*x + a)/(d*x + c)^(3/2))/d
```

Fricas [B] time = 2.15731, size = 1224, normalized size = 8.21

$$2\sqrt{\pi}\left(\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(\left(bd^2x^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-(b*c -
 a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-(b*c - a*d)/
d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2
*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d
*x + c)*sqrt(b/d)) - 2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
 a)*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*si
nh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d)
+ (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt
(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (2*b*d*x - (2*b*d*x + 2*b*c + d)*cos
h(b*x + a)^2 - 2*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)*sinh(b*x + a) - (2*b*d
*x + 2*b*c + d)*sinh(b*x + a)^2 + 2*b*c - d)*sqrt(d*x + c))/((d^4*x^2 + 2*c
*d^3*x + c^2*d^2)*cosh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d^2)*sinh(b*x
+ a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(a+bx\right)}{\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(cosh(a + b*x)/(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)/(d*x + c)^(5/2), x)
```

3.47 $\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{4\sqrt{\pi}b^{5/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{4\sqrt{\pi}b^{5/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}}-\frac{4b\sinh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}$$

```
[Out] (-2*Cosh[a + b*x])/(5*d*(c + d*x)^(5/2)) - (8*b^2*Cosh[a + b*x])/(15*d^3*Sq
rt[c + d*x]) - (4*b^(5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d
*x])/Sqrt[d]])/(15*d^(7/2)) + (4*b^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqr
t[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) - (4*b*Sinh[a + b*x])/(15*d^2*(c
 + d*x)^(3/2))
```

Rubi [A] time = 0.312007, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3297, 3308, 2180, 2204, 2205}

$$-\frac{4\sqrt{\pi}b^{5/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{4\sqrt{\pi}b^{5/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}}-\frac{4b\sinh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x)^(7/2),x]
```


```
[Out] (-2*Cosh[a + b*x])/(5*d*(c + d*x)^(5/2)) - (8*b^2*Cosh[a + b*x])/(15*d^3*Sq
rt[c + d*x]) - (4*b^(5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d
*x])/Sqrt[d]])/(15*d^(7/2)) + (4*b^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqr
t[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) - (4*b*Sinh[a + b*x])/(15*d^2*(c
 + d*x)^(3/2))
```


Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} + \frac{(2b)\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx}{5d}$$

$$= -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{4b\sinh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(4b^2\right)\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}} - \frac{4b\sinh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(8b^3\right)\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{15d^3}$$

$$= -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}} - \frac{4b\sinh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(4b^3\right)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{15d^3} - \frac{\left(4b^3\right)\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{15d^3}$$

$$= -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}} - \frac{4b\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{\left(8b^3\right)\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c}\right.}{15d^4}$$

$$= -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}} - \frac{4b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{4b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+}}{\sqrt{d}}\right.}{15d^{7/2}}$$

Mathematica [A] time = 0.395199, size = 191, normalized size = 1.1

$$\frac{e^{-a}\left(2e^{2a}\left(-2be^{-\frac{bc}{d}}(c+dx)\left(2d\left(-\frac{b(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)+e^{b\left(\frac{c}{d}+x\right)}(2b(c+dx)+d)\right)-3d^2e^{bx}\right)+e^{-bx}\left(8d^2e^{b\left(\frac{c}{d}\right.}\right.}{30d^3(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (2*E^(2*a)*(-3*d^2*E^(b*x) - (2*b*(c + d*x)*(E^(b*(c/d + x))*(d + 2*b*(c +
d*x)) + 2*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)]))/E^(
(b*c)/d)) + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*d^2*E^(b*(c/d
 + x))*((b*(c + d*x))/d)^(5/2)*Gamma[1/2, (b*(c + d*x))/d])/E^(b*x))/(30*d^
3*E^a*(c + d*x)^(5/2))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \cosh(bx+a)\,(dx+c)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c)^(7/2),x)
```

```
[Out] int(cosh(b*x+a)/(d*x+c)^(7/2),x)
```

Maxima [A] time = 1.18735, size = 155, normalized size = 0.89

$$\frac{\frac{\left(\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}-\frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}\right)b}{d}-\frac{2\cosh(bx+a)}{(dx+c)^{\frac{5}{2}}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*((((d*x + c)*b/d)^(3/2)*e^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*x
+ c)^(3/2) - (-(d*x + c)*b/d)^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)*b/
d)/(d*x + c)^(3/2))*b/d - 2*cosh(b*x + a)/(d*x + c)^(5/2))/d
```

Fricas [B] time = 2.29256, size = 1821, normalized size = 10.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(4*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) - (b^2*d^3*
x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh
(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 4*sqrt(pi)*((b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-(b*c - a
*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x
 + a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*
c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt
(d*x + c)*sqrt(-b/d)) + (4*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d + (4*b^2*d^2*x
^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^2
 + 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x
)*cosh(b*x + a)*sinh(b*x + a) + (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^
2 + 2*(4*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d - b*d^2
)*x)*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*cosh(b
*x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx + a)}{(dx + c)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)/(d*x + c)^(7/2), x)
```

3.48 $\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=239

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sinh(2a+2bx)}{64b^3}-\frac{5d(c+dx)^{3/2}\cos}{8b^2}$$

```
[Out] (5*d*(c + d*x)^(3/2))/(16*b^2) + (c + d*x)^(7/2)/(7*d) - (5*d*(c + d*x)^(3/
2)*Cosh[a + b*x]^2)/(8*b^2) + (15*d^(5/2)*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*E
rf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(256*b^(7/2)) - (15*d^(5/2)*E^
(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])
/(256*b^(7/2)) + ((c + d*x)^(5/2)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (15*
d^2*Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(64*b^3)
```

Rubi [A] time = 0.399698, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3311, 32, 3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sinh(2a+2bx)}{64b^3}-\frac{5d(c+dx)^{3/2}\cos}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cosh[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2))/(16*b^2) + (c + d*x)^(7/2)/(7*d) - (5*d*(c + d*x)^(3/
2)*Cosh[a + b*x]^2)/(8*b^2) + (15*d^(5/2)*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*E
rf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(256*b^(7/2)) - (15*d^(5/2)*E^
(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])
/(256*b^(7/2)) + ((c + d*x)^(5/2)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (15*
d^2*Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(64*b^3)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx = -\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b}+\frac{1}{2}\int (c+dx$$

$$= \frac{(c+dx)^{7/2}}{7d}-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b}+$$

$$= \frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\cosh(a+bx)}{2b}$$

$$= \frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\cosh(a+bx)}{2b}$$

$$= \frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\cosh(a+bx)}{2b}$$

$$= \frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\cosh(a+bx)}{2b}$$

$$= \frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{15d^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{\right.}}{256b^{7/2}}$$

Mathematica [A] time = 1.15577, size = 189, normalized size = 0.79

$$\frac{\sqrt{c+dx}\left(b(c+dx)\left(7\sqrt{2}d^3\text{Gamma}\left(\frac{7}{2},\frac{2b(c+dx)}{d}\right)\left(\sinh\left(2a-\frac{2bc}{d}\right)-\cosh\left(2a-\frac{2bc}{d}\right)\right)+64b^3(c+dx)^3\sqrt{\frac{b(c+dx)}{d}}\right)-7\sqrt{2}\right.}{448b^3d^2\left(\frac{b(c+dx)}{d}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cosh[a + b*x]^2,x]
```

```
[Out] (Sqrt[c + d*x]*(-7*Sqrt[2]*d^4*Sqrt[-((b^2*(c + d*x)^2)/d^2)]*Gamma[7/2, (-
2*b*(c + d*x))/d]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]) + b*(c +
d*x)*(64*b^3*(c + d*x)^3*Sqrt[(b*(c + d*x))/d] + 7*Sqrt[2]*d^3*Gamma[7/2, (
2*b*(c + d*x))/d]*(-Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]))))/(448*
b^3*d^2*((b*(c + d*x))/d)^(3/2))
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{2}} (\cosh(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cosh(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^(5/2)*cosh(b*x+a)^2,x)
```

Maxima [A] time = 1.59023, size = 379, normalized size = 1.59

$$\frac{512\,(dx+c)^{\frac{7}{2}} - \frac{105\,\sqrt{2}\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} + \frac{105\,\sqrt{2}\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} - \frac{28\left(16\,(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{2\,bc}{d}\right)}+20\,(dx+c)^{\frac{3}{2}}bd^2e^{\cdots}\right.}{b^3 \cdots}}{3584\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3584*(512*(d*x + c)^(7/2) - 105*sqrt(2)*sqrt(pi)*d^3*erf(sqrt(2)*sqrt(d*x
 + c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^3*sqrt(-b/d)) + 105*sqrt(2)*sqrt(pi)
*d^3*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^3*sqrt(b/d)
) - 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*b*c/d) + 20*(d*x + c)^(3/2)*b*d^2*e^(
2*b*c/d) + 15*sqrt(d*x + c)*d^3*e^(2*b*c/d))*e^(-2*a - 2*(d*x + c)*b/d)/b^3
 + 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*a) - 20*(d*x + c)^(3/2)*b*d^2*e^(2*a)
+ 15*sqrt(d*x + c)*d^3*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^3)/d
```

Fricas [B] time = 2.25565, size = 2319, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3584*(105*sqrt(2)*sqrt(pi)*(d^4*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
d^4*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d) -
d^4*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(-2*
(b*c - a*d)/d) - d^4*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*s
qrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 105*sqrt(2)*sqrt(pi)*(d^4*c
osh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b*x + a)^2*sinh(-2*(b*c -
```

```
a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d) + d^4*sinh(-2*(b*c - a*d)/d))*sinh(b*
x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b*x + a)*
sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)
*sqrt(-b/d)) - 4*(112*b^3*d^3*x^2 + 112*b^3*c^2*d + 140*b^2*c*d^2 - 7*(16*b
^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^
2*d^3)*x)*cosh(b*x + a)^4 - 28*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^
2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^3
 - 7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*
d^2 - 5*b^2*d^3)*x)*sinh(b*x + a)^4 + 105*b*d^3 - 128*(b^4*d^3*x^3 + 3*b^4*
c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*cosh(b*x + a)^2 - 2*(64*b^4*d^3*x^3 +
192*b^4*c*d^2*x^2 + 192*b^4*c^2*d*x + 64*b^4*c^3 + 21*(16*b^3*d^3*x^2 + 16*
b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*cosh(b
*x + a)^2)*sinh(b*x + a)^2 + 28*(8*b^3*c*d^2 + 5*b^2*d^3)*x - 4*(7*(16*b^3*
d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d
^3)*x)*cosh(b*x + a)^3 + 64*(b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x
+ b^4*c^3)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^4*d*cosh(b*x + a
)^2 + 2*b^4*d*cosh(b*x + a)*sinh(b*x + a) + b^4*d*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cosh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{2}} \cosh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/2)*cosh(b*x + a)^2, x)
```

3.49 $\int (c+dx)^{3/2}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=211

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}-\frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)}{2b}$$

```
[Out] (3*d*Sqrt[c + d*x])/(16*b^2) + (c + d*x)^(5/2)/(5*d) - (3*d*Sqrt[c + d*x]*C
osh[a + b*x]^2)/(8*b^2) + (3*d^(3/2)*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(S
qrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(5/2)) + (3*d^(3/2)*E^(2*a -
(2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^
(5/2)) + ((c + d*x)^(3/2)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rubi [A] time = 0.303136, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3311, 32, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}-\frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cosh[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x])/(16*b^2) + (c + d*x)^(5/2)/(5*d) - (3*d*Sqrt[c + d*x]*C
osh[a + b*x]^2)/(8*b^2) + (3*d^(3/2)*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(S
qrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(5/2)) + (3*d^(3/2)*E^(2*a -
(2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^
(5/2)) + ((c + d*x)^(3/2)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (c+dx)^{3/2}\cosh^2(a+bx)\,dx = -\frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b} + \frac{1}{2}\int (c+dx)^{3/}$$

$$= \frac{(c+dx)^{5/2}}{5d} - \frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b} + \frac{(3}{}$$

$$= \frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh}{2b}$$

$$= \frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh}{2b}$$

$$= \frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh}{2b}$$

$$= \frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2} + \frac{3d^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c}}{\sqrt{d}}\right.}{64b^{5/2}}$$

Mathematica [A] time = 0.58322, size = 163, normalized size = 0.77

$$\frac{5\sqrt{2}d^3\sqrt{\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{5}{2},\frac{2b(c+dx)}{d}\right)\left(\sinh\left(2a-\frac{2bc}{d}\right)-\cosh\left(2a-\frac{2bc}{d}\right)\right)+5\sqrt{2}d^3\sqrt{-\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{5}{2},-\frac{2b(c+dx)}{d}\right)(\text{s}}{160b^3d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cosh[a + b*x]^2,x]
```

```
[Out] (32*b^3*(c + d*x)^3 + 5*Sqrt[2]*d^3*Sqrt[(b*(c + d*x))/d]*Gamma[5/2, (2*b*(
c + d*x))/d]*(-Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]) + 5*Sqrt[2]*d
^3*Sqrt[-((b*(c + d*x))/d)]*Gamma[5/2, (-2*b*(c + d*x))/d]*(Cosh[2*a - (2*b
*c)/d] + Sinh[2*a - (2*b*c)/d]))/(160*b^3*d*Sqrt[c + d*x])
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{2}} \left(\cosh\left(bx+a\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cosh(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^(3/2)*cosh(b*x+a)^2,x)
```

Maxima [A] time = 1.57011, size = 323, normalized size = 1.53

$$\frac{128\,(dx+c)^{\frac{5}{2}} + \frac{15\,\sqrt{2}\sqrt{\pi}d^2\,\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} + \frac{15\,\sqrt{2}\sqrt{\pi}d^2\,\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} - \frac{20\left(4\,(dx+c)^{\frac{3}{2}}bde^{\left(\frac{2\,bc}{d}\right)}+3\,\sqrt{dx+c}d^2e^{\left(\frac{2\,bc}{d}\right)}\right)e^{(-}}{b^2}}{640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/640*(128*(d*x + c)^(5/2) + 15*sqrt(2)*sqrt(pi)*d^2*erf(sqrt(2)*sqrt(d*x +
 c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^2*sqrt(-b/d)) + 15*sqrt(2)*sqrt(pi)*d^
2*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^2*sqrt(b/d)) -
 20*(4*(d*x + c)^(3/2)*b*d*e^(2*b*c/d) + 3*sqrt(d*x + c)*d^2*e^(2*b*c/d))*e
^(-2*a - 2*(d*x + c)*b/d)/b^2 + 20*(4*(d*x + c)^(3/2)*b*d*e^(2*a) - 3*sqrt(
d*x + c)*d^2*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^2)/d
```

Fricas [B] time = 2.21844, size = 1789, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/640*(15*sqrt(2)*sqrt(pi)*(d^3*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^
3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) - d^
3*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b
*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqr
t(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 15*sqrt(2)*sqrt(pi)*(d^3*cosh
(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d
)/d) + (d^3*cosh(-2*(b*c - a*d)/d) + d^3*sinh(-2*(b*c - a*d)/d))*sinh(b*x +
 a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)*sin
h(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sq
rt(-b/d)) - 4*(20*b^2*d^2*x - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*
x + a)^4 - 20*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)*sinh(b*x +
a)^3 - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*sinh(b*x + a)^4 + 20*b^2*c*d +
 15*b*d^2 - 32*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*x + a)^2 - 2*(1
6*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 + 15*(4*b^2*d^2*x + 4*b^2*c*d - 3
*b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 4*(5*(4*b^2*d^2*x + 4*b^2*c*d -
3*b*d^2)*cosh(b*x + a)^3 + 16*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*
x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^3*d*cosh(b*x + a)^2 + 2*b^3*d*cosh
```

```
(b*x + a)*sinh(b*x + a) + b^3*d*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^{\frac{3}{2}} \cosh^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cosh(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*cosh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{2}} \cosh(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/2)*cosh(b*x + a)^2, x)
```

3.50 $\int \sqrt{c+dx}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}$$

```
[Out] (c + d*x)^(3/2)/(3*d) + (Sqrt[d]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[
2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(2*a - (2*b*c
)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2))
 + (Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(4*b)
```

Rubi [A] time = 0.269155, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cosh[a + b*x]^2,x]
```


```
[Out] (c + d*x)^(3/2)/(3*d) + (Sqrt[d]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[
2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(2*a - (2*b*c
)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2))
 + (Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(4*b)
```


Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{c+dx}\cosh^2(a+bx)\,dx = \int \left(\frac{1}{2}\sqrt{c+dx}+\frac{1}{2}\sqrt{c+dx}\cosh(2a+2bx)\right)dx$$
$$= \frac{(c+dx)^{3/2}}{3d}+\frac{1}{2}\int \sqrt{c+dx}\cosh(2a+2bx)\,dx$$
$$= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}-\frac{d\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b}$$
$$= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}-\frac{d\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{16b}+\frac{d\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{16b}$$
$$= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}+\frac{\text{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{8b}-\text{Su}\ldots$$
$$= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{d}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+}\ldots}{}$$

Mathematica [A] time = 0.439951, size = 129, normalized size = 0.78

$$\frac{1}{48}\sqrt{c+dx}\left(\frac{3\sqrt{2}e^{2a-\frac{2bc}{d}}\text{Gamma}\left(\frac{3}{2},-\frac{2b(c+dx)}{d}\right)}{b\sqrt{-\frac{b(c+dx)}{d}}}-\frac{3\sqrt{2}e^{\frac{2bc}{d}-2a}\text{Gamma}\left(\frac{3}{2},\frac{2b(c+dx)}{d}\right)}{b\sqrt{\frac{b(c+dx)}{d}}}+\frac{16(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cosh[a + b*x]^2,x]
```

```
[Out] (Sqrt[c + d*x]*((16*(c + d*x))/d + (3*Sqrt[2]*E^(2*a - (2*b*c)/d)*Gamma[3/2
, (-2*b*(c + d*x))/d])/(b*Sqrt[-((b*(c + d*x))/d)]) - (3*Sqrt[2]*E^(-2*a +
(2*b*c)/d)*Gamma[3/2, (2*b*(c + d*x))/d])/(b*Sqrt[(b*(c + d*x))/d])))/48
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^2\sqrt{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*(d*x+c)^(1/2),x)
```

```
[Out] int(cosh(b*x+a)^2*(d*x+c)^(1/2),x)
```

Maxima [A] time = 1.54858, size = 255, normalized size = 1.54

$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{3\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}-32(dx+c)^{\frac{3}{2}}-\frac{12\sqrt{dx+c}de^{\left(2a+\frac{2(dx+c)b}{d}-\frac{2bc}{d}\right)}}{b}+\frac{12\sqrt{dx+c}de^{(-2}}{}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a -
2*b*c/d)/(b*sqrt(-b/d)) - 3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sq
rt(b/d))*e^(-2*a + 2*b*c/d)/(b*sqrt(b/d)) - 32*(d*x + c)^(3/2) - 12*sqrt(d*
x + c)*d*e^(2*a + 2*(d*x + c)*b/d - 2*b*c/d)/b + 12*sqrt(d*x + c)*d*e^(-2*a
 - 2*(d*x + c)*b/d + 2*b*c/d)/b)/d
```

Fricas [B] time = 2.21522, size = 1438, normalized size = 8.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*sqrt(pi)*(d^2*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^2*
cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^2*cosh(-2*(b*c - a*d)/d) - d^2*
sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(b*c
 - a*d)/d) - d^2*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(
b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(2)*sqrt(pi)*(d^2*cosh(b*
x + a)^2*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d
) + (d^2*cosh(-2*(b*c - a*d)/d) + d^2*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)
^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-
2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(
-b/d)) + 4*(3*b*d*cosh(b*x + a)^4 + 12*b*d*cosh(b*x + a)*sinh(b*x + a)^3 +
3*b*d*sinh(b*x + a)^4 + 8*(b^2*d*x + b^2*c)*cosh(b*x + a)^2 + 2*(4*b^2*d*x
+ 9*b*d*cosh(b*x + a)^2 + 4*b^2*c)*sinh(b*x + a)^2 - 3*b*d + 4*(3*b*d*cosh(
b*x + a)^3 + 4*(b^2*d*x + b^2*c)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c
))/(b^2*d*cosh(b*x + a)^2 + 2*b^2*d*cosh(b*x + a)*sinh(b*x + a) + b^2*d*sin
h(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c+dx}\cosh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*cosh(a + b*x)**2, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx+c}\cosh\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)*cosh(b*x + a)^2, x)
```

3.51 $\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}} \, dx$

Optimal. Leaf size=138

$$\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}$$

```
[Out] Sqrt[c + d*x]/d + (E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]) + (E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]
*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])
```

Rubi [A] time = 0.214058, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3312, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] Sqrt[c + d*x]/d + (E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]) + (E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]
*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \left(\frac{1}{2\sqrt{c+dx}}+\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}\right)dx$$
$$= \frac{\sqrt{c+dx}}{d}+\frac{1}{2}\int \frac{\cosh(2a+2bx)}{\sqrt{c+dx}}\,dx$$
$$= \frac{\sqrt{c+dx}}{d}+\frac{1}{4}\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx+\frac{1}{4}\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx$$
$$= \frac{\sqrt{c+dx}}{d}+\frac{\text{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{2d}+\frac{\text{Subst}\left(\int e^{-i\left(2ia-\frac{2ibc}{d}\right)+\frac{2bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{2d}$$
$$= \frac{\sqrt{c+dx}}{d}+\frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}$$

Mathematica [A] time = 0.118473, size = 141, normalized size = 1.02

$$\frac{e^{2a-\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{c+dx}}-\frac{e^{\frac{2bc}{d}-2a}\sqrt{\frac{b(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] Sqrt[c + d*x]/d + (E^(2*a - (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2,
(-2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[c + d*x]) - (E^(-2*a + (2*b*c)/d)*Sq
rt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[c + d*
x])
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^2 \frac{1}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] int(cosh(b*x+a)^2/(d*x+c)^(1/2),x)
```

Maxima [A] time = 1.56816, size = 144, normalized size = 1.04

$$\frac{\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{\sqrt{\frac{b}{d}}}+8\sqrt{dx+c}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a - 2*b*c/
d)/sqrt(-b/d) + sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2
*a + 2*b*c/d)/sqrt(b/d) + 8*sqrt(d*x + c))/d
```

Fricas [A] time = 2.21131, size = 369, normalized size = 2.67

$$\frac{\sqrt{2}\sqrt{\pi}\left(d\cosh\left(-\frac{2(bc-ad)}{d}\right)-d\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-\sqrt{2}\sqrt{\pi}\left(d\cosh\left(-\frac{2(bc-ad)}{d}\right)+d\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)}{8bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*sqrt(pi)*(d*cosh(-2*(b*c - a*d)/d) - d*sinh(-2*(b*c - a*d)/d))
*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(2)*sqrt(pi)*(d*cosh(
-2*(b*c - a*d)/d) + d*sinh(-2*(b*c - a*d)/d))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d
*x + c)*sqrt(-b/d)) + 8*sqrt(d*x + c)*b)/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cosh(a + b*x)**2/sqrt(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^2}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2/sqrt(d*x + c), x)
```

3.52 $\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}$$

```
[Out] (-2*Cosh[a + b*x]^2)/(d*Sqrt[c + d*x]) - (Sqrt[b]*E^(-2*a + (2*b*c)/d)*Sqrt
[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[b]*E^(
2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/
d^(3/2)
```

Rubi [A] time = 0.230148, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3313, 12, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cosh[a + b*x]^2)/(d*Sqrt[c + d*x]) - (Sqrt[b]*E^(-2*a + (2*b*c)/d)*Sqrt
[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[b]*E^(
2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/
d^(3/2)
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}+\frac{(4ib)\int -\frac{i\sinh(2a+2bx)}{2\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}+\frac{(2b)\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}+\frac{b\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{d}-\frac{b\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{(2b)\,\mathrm{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d^2}+\frac{(2b)\,\mathrm{Subst}\left(\int e^{-i\left(2ia-\frac{2ibc}{d}\right)+\frac{2bx^2}{d}}\right.}{d^2}\\
&= -\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{\sqrt{b}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{b}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}
\end{aligned}
$$

Mathematica [B] time = 2.8466, size = 570, normalized size = 4.01

$$
\frac{e^{-\frac{2b(c+dx)}{d}}\left(\sqrt{2}\sqrt{d}e^{\frac{2b(c+dx)}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\sinh(2a)\cosh\left(\frac{2bc}{d}\right)\right)+\sqrt{2}\sqrt{d}e^{\frac{2b(c+dx)}{d}}\sqrt{\frac{b(c+dx)}{d}}\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[d]*E^((2*b*(c + d*x))/d) - Sqrt[d]*Cosh[2*a]*Cosh[(2*b*c)/d] - Sqr
t[d]*E^((4*b*(c + d*x))/d)*Cosh[2*a]*Cosh[(2*b*c)/d] + Sqrt[d]*Cosh[(2*b*c)
/d]*Sinh[2*a] - Sqrt[d]*E^((4*b*(c + d*x))/d)*Cosh[(2*b*c)/d]*Sinh[2*a] + S
qrt[2]*Sqrt[d]*E^((2*b*(c + d*x))/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-
2*b*(c + d*x))/d]*(Cosh[2*a - (2*b*c)/d] + Cosh[(2*b*c)/d]*Sinh[2*a]) - Sqr
t[d]*Cosh[2*a]*Sinh[(2*b*c)/d] + Sqrt[d]*E^((4*b*(c + d*x))/d)*Cosh[2*a]*Si
nh[(2*b*c)/d] - Sqrt[b]*E^((2*b*(c + d*x))/d)*Sqrt[2*Pi]*Sqrt[c + d*x]*Cosh
[2*a]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]*Sinh[(2*b*c)/d] - Sqrt[b
]*E^((2*b*(c + d*x))/d)*Sqrt[2*Pi]*Sqrt[c + d*x]*Cosh[2*a]*Erfi[(Sqrt[2]*Sq
rt[b]*Sqrt[c + d*x])/Sqrt[d]]*Sinh[(2*b*c)/d] + Sqrt[d]*Sinh[2*a]*Sinh[(2*b
*c)/d] + Sqrt[d]*E^((4*b*(c + d*x))/d)*Sinh[2*a]*Sinh[(2*b*c)/d] + Sqrt[2]*
Sqrt[d]*E^((2*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*
x))/d]*(Cosh[2*a]*Cosh[(2*b*c)/d] - Sinh[2*a]*(Cosh[(2*b*c)/d] + Sinh[(2*b*
c)/d])))/(2*d^(3/2)*E^((2*b*(c + d*x))/d)*Sqrt[c + d*x])
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^2 (dx+c)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] int(cosh(b*x+a)^2/(d*x+c)^(3/2),x)
```

Maxima [A] time = 1.26917, size = 157, normalized size = 1.11

$$-\frac{\frac{\sqrt{2}\sqrt{\frac{(dx+c)b}{d}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{2(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{2}\sqrt{-\frac{(dx+c)b}{d}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{2(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{4}{\sqrt{dx+c}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*sqrt((d*x + c)*b/d)*e^(2*(b*c - a*d)/d)*gamma(-1/2, 2*(d*x +
c)*b/d)/sqrt(d*x + c) + sqrt(2)*sqrt(-(d*x + c)*b/d)*e^(-2*(b*c - a*d)/d)*g
amma(-1/2, -2*(d*x + c)*b/d)/sqrt(d*x + c) + 4/sqrt(d*x + c))/d
```

Fricas [B] time = 2.22597, size = 1442, normalized size = 10.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
(d*x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(b*c
- a*d)/d) - (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*x + c
)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-2*(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))
 + sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (d*
x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(b*c - a
*d)/d) + (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*x + c)*c
osh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)*sinh(-2*(b*c
- a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))
+ (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*
(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x +
 a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*sqrt(d*x + c))/((d^2*x + c*d)*cos
h(b*x + a)^2 + 2*(d^2*x + c*d)*cosh(b*x + a)*sinh(b*x + a) + (d^2*x + c*d)*
sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] Integral(cosh(a + b*x)**2/(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^2}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2/(d*x + c)^(3/2), x)
```

3.53 $\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{2\sqrt{2\pi}b^{3/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{2\pi}b^{3/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+b}{3d(c+dx)^{3/}}$$

```
[Out] (-2*Cosh[a + b*x]^2)/(3*d*(c + d*x)^(3/2)) + (2*b^(3/2)*E^(-2*a + (2*b*c)/d
)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (2
*b^(3/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[d]])/(3*d^(5/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*Sqrt[c +
 d*x])
```

Rubi [A] time = 0.308861, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3314, 32, 3312, 3307, 2180, 2204, 2205}

$$\frac{2\sqrt{2\pi}b^{3/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{2\pi}b^{3/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+b}{3d(c+dx)^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cosh[a + b*x]^2)/(3*d*(c + d*x)^(3/2)) + (2*b^(3/2)*E^(-2*a + (2*b*c)/d
)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (2
*b^(3/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[d]])/(3*d^(5/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*Sqrt[c +
 d*x])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{\left(8b^2\right)\int \frac{1}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{\left(16b^2\right)\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{16b^2\sqrt{c+dx}}{3d^3} - \frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} + \frac{\left(16b^2\right)\int \left(\frac{1}{2\sqrt{c+dx}} + \frac{\cosh(2a+2b}{2\sqrt{c+dx}}\right.}{3d^2}$$

$$= -\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} + \frac{\left(8b^2\right)\int \frac{\cosh(2a+2bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} + \frac{\left(4b^2\right)\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{\left(4b^2\right)\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} + \frac{\left(8b^2\right)\text{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx, x, \sqrt{c+dx}\right.}{3d^3}$$

$$= -\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{2b^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - 8b\,\text{c}$$

Mathematica [A] time = 1.35024, size = 156, normalized size = 0.9

$$-\frac{2e^{-2\left(a+\frac{bc}{d}\right)}\left(\sqrt{2}e^{4a}d\left(-\frac{b(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)+\sqrt{2}de^{\frac{4bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)+e^{2\left(a+\frac{bc}{d}\right)}\left(2b(c+d\right.}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(Sqrt[2]*d*E^(4*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-2*b*(c + d*x
))/d] + Sqrt[2]*d*E^((4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (2*b*(c
+ d*x))/d] + E^(2*(a + (b*c)/d))*(d*Cosh[a + b*x]^2 + 2*b*(c + d*x)*Sinh[2*
```

```
(a + b*x)]))))/(3*d^2*E^(2*(a + (b*c)/d))*(c + d*x)^(3/2))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^2 (dx+c)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^(5/2),x)

[Out] int(cosh(b*x+a)^2/(d*x+c)^(5/2),x)
```

Maxima [A] time = 1.20607, size = 159, normalized size = 0.91

$$-\frac{\frac{3\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}} e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{3\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}} e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{2}{(dx+c)^{\frac{3}{2}}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")

[Out] -1/6*(3*sqrt(2)*((d*x + c)*b/d)^(3/2)*e^(2*(b*c - a*d)/d)*gamma(-3/2, 2*(d*
x + c)*b/d)/(d*x + c)^(3/2) + 3*sqrt(2)*(-(d*x + c)*b/d)^(3/2)*e^(-2*(b*c -
 a*d)/d)*gamma(-3/2, -2*(d*x + c)*b/d)/(d*x + c)^(3/2) + 2/(d*x + c)^(3/2))
/d
```

Fricas [B] time = 2.32786, size = 2053, normalized size = 11.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")

[Out] 1/6*(4*sqrt(2)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*co
sh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh
(-2*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d
) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2
 + 2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d)
- (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sin
h(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 4*sqrt(2)*sqrt
(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d
) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d)
+ ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d) + (b*d^2*x^2 + 2*
b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b*d^2*x^2 +
2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*
c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b
/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) - ((4*b*d*x + 4*b*c + d)*cosh(b*x
 + a)^4 + 4*(4*b*d*x + 4*b*c + d)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b*d*x
```

```
+ 4*b*c + d)*sinh(b*x + a)^4 - 4*b*d*x + 2*d*cosh(b*x + a)^2 + 2*(3*(4*b*d*
x + 4*b*c + d)*cosh(b*x + a)^2 + d)*sinh(b*x + a)^2 - 4*b*c + 4*((4*b*d*x +
 4*b*c + d)*cosh(b*x + a)^3 + d*cosh(b*x + a))*sinh(b*x + a) + d)*sqrt(d*x
+ c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)^2 + 2*(d^4*x^2 + 2*c*d
^3*x + c^2*d^2)*cosh(b*x + a)*sinh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d^
2)*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Integral(cosh(a + b*x)**2/(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^2}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2/(d*x + c)^(5/2), x)
```

3.54 $\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{8\sqrt{2\pi}b^{5/2}e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8\sqrt{2\pi}b^{5/2}e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}}-\frac{8b\sinh(a+bx)\cosh(}{15d^2(c+dx)^{3/2}}$$

```
[Out] (16*b^2)/(15*d^3*Sqrt[c + d*x]) - (2*Cosh[a + b*x]^2)/(5*d*(c + d*x)^(5/2))
 - (32*b^2*Cosh[a + b*x]^2)/(15*d^3*Sqrt[c + d*x]) - (8*b^(5/2)*E^(-2*a + (
2*b*c)/d)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7
/2)) + (8*b^(5/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt
[c + d*x])/Sqrt[d]])/(15*d^(7/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(15*d
^2*(c + d*x)^(3/2))
```

Rubi [A] time = 0.311493, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3314, 32, 3313, 12, 3308, 2180, 2204, 2205}

$$-\frac{8\sqrt{2\pi}b^{5/2}e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8\sqrt{2\pi}b^{5/2}e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}}-\frac{8b\sinh(a+bx)\cosh(}{15d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^(7/2),x]
```


```
[Out] (16*b^2)/(15*d^3*Sqrt[c + d*x]) - (2*Cosh[a + b*x]^2)/(5*d*(c + d*x)^(5/2))
 - (32*b^2*Cosh[a + b*x]^2)/(15*d^3*Sqrt[c + d*x]) - (8*b^(5/2)*E^(-2*a + (
2*b*c)/d)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7
/2)) + (8*b^(5/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt
[c + d*x])/Sqrt[d]])/(15*d^(7/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(15*d
^2*(c + d*x)^(3/2))
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{\left(8b^2\right)\int \frac{1}{(c+dx)^{3/2}}\,dx}{15d^2} + \frac{\left(16b^2\right)\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2}$$

$$= \frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(64ib^3\right)\ldots}{\ldots}$$

$$= \frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(32b^3\right)\int\ldots}{\ldots}$$

$$= \frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{\left(16b^3\right)\int\ldots}{\ldots}$$

$$= \frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{\left(32b^3\right)\mathrm{S}\ldots}{\ldots}$$

$$= \frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/}\ldots}{\ldots}$$

Mathematica [B] time = 3.03441, size = 825, normalized size = 3.75

$$\frac{e^{-\frac{2b(c+dx)}{d}}\left(16\sqrt{2}d^2e^{\frac{2b(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)\left(-\frac{b(c+dx)}{d}\right)^{5/2} - 6d^2e^{\frac{2b(c+dx)}{d}} - 16b^2c^2e^{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (-6*d^2*E^((2*b*(c + d*x))/d) - 16*b^2*c^2*Cosh[2*a - (2*b*c)/d] + 4*b*c*d*
Cosh[2*a - (2*b*c)/d] - 3*d^2*Cosh[2*a - (2*b*c)/d] - 16*b^2*c^2*E^((4*b*(c
 + d*x))/d)*Cosh[2*a - (2*b*c)/d] - 4*b*c*d*E^((4*b*(c + d*x))/d)*Cosh[2*a
- (2*b*c)/d] - 3*d^2*E^((4*b*(c + d*x))/d)*Cosh[2*a - (2*b*c)/d] - 32*b^2*c
*d*x*Cosh[2*a - (2*b*c)/d] + 4*b*d^2*x*Cosh[2*a - (2*b*c)/d] - 32*b^2*c*d*E
^((4*b*(c + d*x))/d)*x*Cosh[2*a - (2*b*c)/d] - 4*b*d^2*E^((4*b*(c + d*x))/d
)*x*Cosh[2*a - (2*b*c)/d] - 16*b^2*d^2*x^2*Cosh[2*a - (2*b*c)/d] - 16*b^2*d
^2*E^((4*b*(c + d*x))/d)*x^2*Cosh[2*a - (2*b*c)/d] + 16*Sqrt[2]*d^2*E^((2*b
*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma[1/2, (2*b*(c + d*x))/d]*(Cosh[
2*a - (2*b*c)/d] - Sinh[2*a - (2*b*c)/d]) + 16*b^2*c^2*Sinh[2*a - (2*b*c)/d
] - 4*b*c*d*Sinh[2*a - (2*b*c)/d] + 3*d^2*Sinh[2*a - (2*b*c)/d] - 16*b^2*c^
2*E^((4*b*(c + d*x))/d)*Sinh[2*a - (2*b*c)/d] - 4*b*c*d*E^((4*b*(c + d*x))/
d)*Sinh[2*a - (2*b*c)/d] - 3*d^2*E^((4*b*(c + d*x))/d)*Sinh[2*a - (2*b*c)/d
] + 32*b^2*c*d*x*Sinh[2*a - (2*b*c)/d] - 4*b*d^2*x*Sinh[2*a - (2*b*c)/d] -
32*b^2*c*d*E^((4*b*(c + d*x))/d)*x*Sinh[2*a - (2*b*c)/d] - 4*b*d^2*E^((4*b*
(c + d*x))/d)*x*Sinh[2*a - (2*b*c)/d] + 16*b^2*d^2*x^2*Sinh[2*a - (2*b*c)/d
] - 16*b^2*d^2*E^((4*b*(c + d*x))/d)*x^2*Sinh[2*a - (2*b*c)/d] + 16*Sqrt[2]
*d^2*E^((2*b*(c + d*x))/d)*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, (-2*b*(c +
 d*x))/d]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]))/(30*d^3*E^((2*b*
(c + d*x))/d)*(c + d*x)^(5/2))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^2 (dx+c)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^(7/2),x)
```

```
[Out] int(cosh(b*x+a)^2/(d*x+c)^(7/2),x)
```

Maxima [A] time = 1.20105, size = 157, normalized size = 0.71

$$-\frac{\frac{5\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}} e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} + \frac{5\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}} e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} + \frac{1}{(dx+c)^{\frac{5}{2}}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(5*sqrt(2)*((d*x + c)*b/d)^(5/2)*e^(2*(b*c - a*d)/d)*gamma(-5/2, 2*(d*
x + c)*b/d)/(d*x + c)^(5/2) + 5*sqrt(2)*(-(d*x + c)*b/d)^(5/2)*e^(-2*(b*c -
 a*d)/d)*gamma(-5/2, -2*(d*x + c)*b/d)/(d*x + c)^(5/2) + 1/(d*x + c)^(5/2))
/d
```

Fricas [B] time = 2.38752, size = 2974, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/30*(16*sqrt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*
d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) +
 ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*c -
a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-2
*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b*c - a*d
)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 16*sq
rt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*c
osh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*c - a*d)/d) + (b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-2*(b*c - a*d)
/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b
*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) + (16*b^2*d^2*x^2
 + (16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x
)*cosh(b*x + a)^4 + 4*(16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8
*b^2*c*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a)^3 + (16*b^2*d^2*x^2 + 16*b
^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^4 + 16*b^
2*c^2 + 6*d^2*cosh(b*x + a)^2 - 4*b*c*d + 6*((16*b^2*d^2*x^2 + 16*b^2*c^2 +
 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^2 + d^2)*sinh(b*x
 + a)^2 + 3*d^2 + 4*(8*b^2*c*d - b*d^2)*x + 4*((16*b^2*d^2*x^2 + 16*b^2*c^2
 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^3 + 3*d^2*cosh(
b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4
*x + c^3*d^3)*cosh(b*x + a)^2 + 2*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^
3*d^3)*cosh(b*x + a)*sinh(b*x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x +
 c^3*d^3)*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(bx+a\right)^2}{\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2/(d*x + c)^(7/2), x)
```

3.55 $\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx$

Optimal. Leaf size=251

$$\frac{32\sqrt{2\pi}b^{7/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}+\frac{32\sqrt{2\pi}b^{7/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}-\frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}}-\frac{128b^3\sinh(a+bx)\cosh}{105d^4\sqrt{c+dx}}$$

```
[Out] (16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (2*Cosh[a + b*x]^2)/(7*d*(c + d*x)^(7/
2)) - (32*b^2*Cosh[a + b*x]^2)/(105*d^3*(c + d*x)^(3/2)) + (32*b^(7/2)*E^(-
2*a + (2*b*c)/d)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(
105*d^(9/2)) + (32*b^(7/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqr
t[b]*Sqrt[c + d*x])/Sqrt[d]])/(105*d^(9/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b
*x])/(35*d^2*(c + d*x)^(5/2)) - (128*b^3*Cosh[a + b*x]*Sinh[a + b*x])/(105*
d^4*Sqrt[c + d*x])
```

Rubi [A] time = 0.37924, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3314, 32, 3312, 3307, 2180, 2204, 2205}

$$\frac{32\sqrt{2\pi}b^{7/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}+\frac{32\sqrt{2\pi}b^{7/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}-\frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}}-\frac{128b^3\sinh(a+bx)\cosh}{105d^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (2*Cosh[a + b*x]^2)/(7*d*(c + d*x)^(7/
2)) - (32*b^2*Cosh[a + b*x]^2)/(105*d^3*(c + d*x)^(3/2)) + (32*b^(7/2)*E^(-
2*a + (2*b*c)/d)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(
105*d^(9/2)) + (32*b^(7/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqr
t[b]*Sqrt[c + d*x])/Sqrt[d]])/(105*d^(9/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b
*x])/(35*d^2*(c + d*x)^(5/2)) - (128*b^3*Cosh[a + b*x]*Sinh[a + b*x])/(105*
d^4*Sqrt[c + d*x])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{\left(8b^2\right)\int \frac{1}{(c+dx)^{5/2}}\,dx}{35d^2} + \frac{\left(16b^2\right)\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}}{35d^2}$$

$$= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - 1$$

$$= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{256b^4\sqrt{c+dx}}{105d^5} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)}{35d^2(c+}$$

$$= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - 1$$

$$= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - 1$$

$$= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - 1$$

$$= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{32b^{7/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right.}{105d^{9/2}}$$

Mathematica [A] time = 0.828249, size = 222, normalized size = 0.88

$$\frac{2\left(16\sqrt{2}b^3(c+dx)^3e^{2a-\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right) - 16\sqrt{2}b^3(c+dx)^3e^{\frac{2bc}{d}-2a}\sqrt{\frac{b(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right) - 3}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (2*(8*b^2*d*(c + d*x)^2 - 15*d^3*Cosh[a + b*x]^2 - 16*b^2*d*(c + d*x)^2*Cos
h[a + b*x]^2 + 16*Sqrt[2]*b^3*E^(2*a - (2*b*c)/d)*(c + d*x)^3*Sqrt[-((b*(c
+ d*x))/d)]*Gamma[1/2, (-2*b*(c + d*x))/d] - 16*Sqrt[2]*b^3*E^(-2*a + (2*b*
c)/d)*(c + d*x)^3*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d] - 6*b
*d^2*(c + d*x)*Sinh[2*(a + b*x)] - 32*b^3*(c + d*x)^3*Sinh[2*(a + b*x)]))/(
105*d^4*(c + d*x)^(7/2))
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int (\cosh (bx + a))^2 (dx + c)^{-\frac{9}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^(9/2),x)
```

```
[Out] int(cosh(b*x+a)^2/(d*x+c)^(9/2),x)
```

Maxima [A] time = 1.2126, size = 157, normalized size = 0.63

$$-\frac{\frac{14\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{7}{2}} e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2}, \frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}} + \frac{14\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{7}{2}} e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2}, -\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}} + \frac{1}{(dx+c)^{\frac{7}{2}}}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7*(14*sqrt(2)*((d*x + c)*b/d)^(7/2)*e^(2*(b*c - a*d)/d)*gamma(-7/2, 2*(d
*x + c)*b/d)/(d*x + c)^(7/2) + 14*sqrt(2)*(-(d*x + c)*b/d)^(7/2)*e^(-2*(b*c
 - a*d)/d)*gamma(-7/2, -2*(d*x + c)*b/d)/(d*x + c)^(7/2) + 1/(d*x + c)^(7/2
))/d
```

Fricas [B] time = 2.29016, size = 3951, normalized size = 15.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/210*(64*sqrt(2)*sqrt(pi)*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*
x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (b^
3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*
cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(-2*(b*c - a*d)/d) - (b^3*
d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*si
nh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*cosh(-2*(b*c -
```

```
a*d)/d) - (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*
x + b^3*c^4)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)
*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 64*sqrt(2)*sqrt(pi)*((b^3*d^4*x^4 +
 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x +
a)^2*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^
2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (
(b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^
4)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*
x^2 + 4*b^3*c^3*d*x + b^3*c^4)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*
((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c
^4)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*sinh(-2*(b*c -
a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) +
(64*b^3*d^3*x^3 + 64*b^3*c^3 - 16*b^2*c^2*d - 30*d^3*cosh(b*x + a)^2 - (64*
b^3*d^3*x^3 + 64*b^3*c^3 + 16*b^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^3*
c*d^2 + b^2*d^3)*x^2 + 4*(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*cosh(b*x
 + a)^4 - 4*(64*b^3*d^3*x^3 + 64*b^3*c^3 + 16*b^2*c^2*d + 12*b*c*d^2 + 15*d
^3 + 16*(12*b^3*c*d^2 + b^2*d^3)*x^2 + 4*(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*
d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^3 - (64*b^3*d^3*x^3 + 64*b^3*c^3 + 16*b
^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^3*c*d^2 + b^2*d^3)*x^2 + 4*(48*b^
3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*sinh(b*x + a)^4 + 12*b*c*d^2 - 15*d^3 +
 16*(12*b^3*c*d^2 - b^2*d^3)*x^2 - 6*(5*d^3 + (64*b^3*d^3*x^3 + 64*b^3*c^3
+ 16*b^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^3*c*d^2 + b^2*d^3)*x^2 + 4*
(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*cosh(b*x + a)^2)*sinh(b*x + a)^2
+ 4*(48*b^3*c^2*d - 8*b^2*c*d^2 + 3*b*d^3)*x - 4*(15*d^3*cosh(b*x + a) + (6
4*b^3*d^3*x^3 + 64*b^3*c^3 + 16*b^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^
3*c*d^2 + b^2*d^3)*x^2 + 4*(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*cosh(b
*x + a)^3)*sinh(b*x + a))*sqrt(d*x + c))/((d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^
6*x^2 + 4*c^3*d^5*x + c^4*d^4)*cosh(b*x + a)^2 + 2*(d^8*x^4 + 4*c*d^7*x^3 +
 6*c^2*d^6*x^2 + 4*c^3*d^5*x + c^4*d^4)*cosh(b*x + a)*sinh(b*x + a) + (d^8*
x^4 + 4*c*d^7*x^3 + 6*c^2*d^6*x^2 + 4*c^3*d^5*x + c^4*d^4)*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(bx+a\right)^2}{\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2/(d*x + c)^(9/2), x)
```

3.56 $\int (c+dx)^{5/2} \cosh^3(a+bx)\,dx$

Optimal. Leaf size=381

$$\frac{45\sqrt{\pi}d^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}+\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}-\frac{45\sqrt{\pi}d^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\right.}{576b^{7/2}}$$

```
[Out] (-5*d*(c + d*x)^(3/2)*Cosh[a + b*x])/(3*b^2) - (5*d*(c + d*x)^(3/2)*Cosh[a
+ b*x]^3)/(18*b^2) + (45*d^(5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]])/(64*b^(7/2)) + (5*d^(5/2)*E^(-3*a + (3*b*c)/d)*Sqrt[P
i/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (45*d^(5
/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(
7/2)) - (5*d^(5/2)*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (45*d^2*Sqrt[c + d*x]*Sinh[a + b*x])/
(16*b^3) + (2*(c + d*x)^(5/2)*Sinh[a + b*x])/(3*b) + ((c + d*x)^(5/2)*Cosh[
a + b*x]^2*Sinh[a + b*x])/(3*b) + (5*d^2*Sqrt[c + d*x]*Sinh[3*a + 3*b*x])/(
144*b^3)
```

Rubi [A] time = 0.906379, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3311, 3296, 3308, 2180, 2204, 2205, 3312}

$$\frac{45\sqrt{\pi}d^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}+\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}-\frac{45\sqrt{\pi}d^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\right.}{576b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cosh[a + b*x]^3,x]
```


```
[Out] (-5*d*(c + d*x)^(3/2)*Cosh[a + b*x])/(3*b^2) - (5*d*(c + d*x)^(3/2)*Cosh[a
+ b*x]^3)/(18*b^2) + (45*d^(5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]])/(64*b^(7/2)) + (5*d^(5/2)*E^(-3*a + (3*b*c)/d)*Sqrt[P
i/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (45*d^(5
/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(
7/2)) - (5*d^(5/2)*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (45*d^2*Sqrt[c + d*x]*Sinh[a + b*x])/
(16*b^3) + (2*(c + d*x)^(5/2)*Sinh[a + b*x])/(3*b) + ((c + d*x)^(5/2)*Cosh[
a + b*x]^2*Sinh[a + b*x])/(3*b) + (5*d^2*Sqrt[c + d*x]*Sinh[3*a + 3*b*x])/(
144*b^3)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx = -\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\cosh^2(a+bx)\sinh(a+bx)}{3b}+\frac{2}{3}\int (c+d$$

$$= -\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{2(c+dx)^{5/2}\sinh(a+bx)}{3b}+\frac{(c+dx)^{5/2}\cosh^2(a+bx}{3b}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2}-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{2(c+dx)^{5/2}\sinh(a+b}{3b}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2}-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{45d^2\sqrt{c+dx}\sinh(a+}{16b^3}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2}-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{45d^2\sqrt{c+dx}\sinh(a+}{16b^3}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2}-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{45d^2\sqrt{c+dx}\sinh(a+}{16b^3}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2}-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{45d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\mathrm{erf}\left(\frac{\sqrt{b}}{}\right.}{64b^{7/2}}$$

Mathematica [A] time = 3.73641, size = 243, normalized size = 0.64

$$-\frac{d^3\left(\sqrt{3}\sqrt{-\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{7}{2},-\frac{3b(c+dx)}{d}\right)\left(\sinh\left(3a-\frac{3bc}{d}\right)+\cosh\left(3a-\frac{3bc}{d}\right)\right)+\left(\cosh\left(a-\frac{bc}{d}\right)-\sinh\left(a-\frac{bc}{d}\right)\right)\left(\sqrt{\frac{b(c}{}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cosh[a + b*x]^3,x]
```

```
[Out] -(d^3*(Sqrt[3]*Sqrt[-((b*(c + d*x))/d)]*Gamma[7/2, (-3*b*(c + d*x))/d]*(Cos
h[3*a - (3*b*c)/d] + Sinh[3*a - (3*b*c)/d]) + (Sqrt[(b*(c + d*x))/d]*(243*G
amma[7/2, (b*(c + d*x))/d] + Sqrt[3]*Gamma[7/2, (3*b*(c + d*x))/d]*(Cosh[2*
a - (2*b*c)/d] - Sinh[2*a - (2*b*c)/d])) + 243*Sqrt[-((b*(c + d*x))/d)]*Gam
ma[7/2, -((b*(c + d*x))/d)]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d])
)*(Cosh[a - (b*c)/d] - Sinh[a - (b*c)/d])))/(648*b^4*Sqrt[c + d*x])
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{2}} (\cosh(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cosh(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^(5/2)*cosh(b*x+a)^3,x)
```

Maxima [A] time = 1.66634, size = 693, normalized size = 1.82

$$-\frac{\frac{5\sqrt{3}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}-\frac{5\sqrt{3}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}+\frac{1215\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}-\frac{1215\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)}{b^3\sqrt{\frac{b}{d}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/1728*(5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*
a - 3*b*c/d)/(b^3*sqrt(-b/d)) - 5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x
 + c)*sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^3*sqrt(b/d)) + 1215*sqrt(pi)*d^3*erf
(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) - 1215*sqrt(pi)*d
^3*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) + 162*(4*(d*
x + c)^(5/2)*b^2*d*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt
(d*x + c)*d^3*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^3 + 6*(12*(d*x + c)^(5/2)
*b^2*d*e^(3*b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(3*b*c/d) + 5*sqrt(d*x + c)
*d^3*e^(3*b*c/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^3 - 6*(12*(d*x + c)^(5/2)*b^
2*d*e^(3*a) - 10*(d*x + c)^(3/2)*b*d^2*e^(3*a) + 5*sqrt(d*x + c)*d^3*e^(3*a
))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^3 - 162*(4*(d*x + c)^(5/2)*b^2*d*e^a - 1
0*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d -
b*c/d)/b^3)/d
```

Fricas [B] time = 2.24555, size = 4815, normalized size = 12.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1728*(5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d^
3*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/d) - d^
3*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-3*(b
*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 +
3*(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-3
*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/
d)) + 5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d^3*
cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/d) + d^3*
sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-3*(b*c
 - a*d)/d) + d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*
(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^3*cosh(b*x + a)^2*sinh(-3*(
b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/
d)) + 1215*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d^3*cosh(b*
x + a)^3*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) - d^3*sinh(-(b*c
- a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^
3*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^3*cosh(b*x + a
)^2*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b
*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 1215*sqrt(pi)*(d^3*cosh(b
*x + a)^3*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) +
 (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*
(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-(b*c - a*
d)/d))*sinh(b*x + a)^2 + 3*(d^3*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) + d^3*
cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*
x + c)*sqrt(-b/d)) - 6*(12*b^3*d^2*x^2 - (12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*
b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^6 - 6*(12*b
^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)
*x)*cosh(b*x + a)*sinh(b*x + a)^5 - (12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c
*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*sinh(b*x + a)^6 + 12*b^3*c^2 -
 27*(4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b
^2*d^2)*x)*cosh(b*x + a)^4 - 3*(36*b^3*d^2*x^2 + 36*b^3*c^2 - 90*b^2*c*d +
135*b*d^2 + 5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*b
^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^2 + 18*(4*b^3*c*d - 5*b^2*d^2)*x)*sinh
(b*x + a)^4 + 10*b^2*c*d - 4*(5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d +
 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^3 + 27*(4*b^3*d^2*x^
2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b
*x + a))*sinh(b*x + a)^3 + 5*b*d^2 + 27*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 10*b^2
*c*d + 15*b*d^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*x)*cosh(b*x + a)^2 + 3*(36*b^3*
d^2*x^2 + 36*b^3*c^2 - 5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^
2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^4 + 90*b^2*c*d + 135*b*d^2
- 54*(4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*
b^2*d^2)*x)*cosh(b*x + a)^2 + 18*(4*b^3*c*d + 5*b^2*d^2)*x)*sinh(b*x + a)^2
 + 2*(12*b^3*c*d + 5*b^2*d^2)*x - 6*((12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*
c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^5 + 18*(4*b^3*d
^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*c
osh(b*x + a)^3 - 9*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 10*b^2*c*d + 15*b*d^2 + 2*(
4*b^3*c*d + 5*b^2*d^2)*x)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^4
*cosh(b*x + a)^3 + 3*b^4*cosh(b*x + a)^2*sinh(b*x + a) + 3*b^4*cosh(b*x + a
)*sinh(b*x + a)^2 + b^4*sinh(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cosh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{\frac{5}{2}} \cosh(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/2)*cosh(b*x + a)^3, x)
```

3.57 $\int (c+dx)^{3/2}\cosh^3(a+bx)\,dx$

Optimal. Leaf size=326

$$\frac{9\sqrt{\pi}d^{3/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}d^{3/2}e^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{9\sqrt{\pi}d^{3/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}d^{3/2}e^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\sqrt{3}\cdots\right.}{96b^{5/2}}$$

```
[Out] -((d*Sqrt[c + d*x]*Cosh[a + b*x])/b^2) - (d*Sqrt[c + d*x]*Cosh[a + b*x]^3)/
(6*b^2) + (9*d^(3/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]])/(32*b^(5/2)) + (d^(3/2)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt
[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (9*d^(3/2)*E^(a - (b*c)
/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(32*b^(5/2)) + (d^(3/2)
*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d
]])/(96*b^(5/2)) + (2*(c + d*x)^(3/2)*Sinh[a + b*x])/(3*b) + ((c + d*x)^(3/
2)*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b)
```

Rubi [A] time = 0.712373, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3311, 3296, 3307, 2180, 2204, 2205, 3312}

$$\frac{9\sqrt{\pi}d^{3/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}d^{3/2}e^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{9\sqrt{\pi}d^{3/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}d^{3/2}e^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\sqrt{3}\cdots\right.}{96b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cosh[a + b*x]^3,x]
```

```
[Out] -((d*Sqrt[c + d*x]*Cosh[a + b*x])/b^2) - (d*Sqrt[c + d*x]*Cosh[a + b*x]^3)/
(6*b^2) + (9*d^(3/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]])/(32*b^(5/2)) + (d^(3/2)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt
[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (9*d^(3/2)*E^(a - (b*c)
/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(32*b^(5/2)) + (d^(3/2)
*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d
]])/(96*b^(5/2)) + (2*(c + d*x)^(3/2)*Sinh[a + b*x])/(3*b) + ((c + d*x)^(3/
2)*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{3/2}\cosh^3(a+bx)\,dx = -\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\cosh^2(a+bx)\sinh(a+bx)}{3b}+\frac{2}{3}\int (c+dx)^{3/2}\text{co}$$

$$= -\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{2(c+dx)^{3/2}\sinh(a+bx)}{3b}+\frac{(c+dx)^{3/2}\cosh^2(a+bx)\sinh(}{3b}$$

$$= -\frac{d\sqrt{c+dx}\cosh(a+bx)}{b^2}-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{2(c+dx)^{3/2}\sinh(a+bx)}{3b}+\frac{(c+}{}$$

$$= -\frac{d\sqrt{c+dx}\cosh(a+bx)}{b^2}-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{2(c+dx)^{3/2}\sinh(a+bx)}{3b}+\frac{(c+}{}$$

$$= -\frac{d\sqrt{c+dx}\cosh(a+bx)}{b^2}-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{2(c+dx)^{3/2}\sinh(a+bx)}{3b}+\frac{(c+}{}$$

$$= -\frac{d\sqrt{c+dx}\cosh(a+bx)}{b^2}-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{9d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}+\frac{d}{}$$

Mathematica [A] time = 1.9289, size = 243, normalized size = 0.75

$$\frac{d^2\left(\sqrt{3}\sqrt{-\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{5}{2},-\frac{3b(c+dx)}{d}\right)\left(\sinh\left(3a-\frac{3bc}{d}\right)+\cosh\left(3a-\frac{3bc}{d}\right)\right)+\left(\cosh\left(a-\frac{bc}{d}\right)-\sinh\left(a-\frac{bc}{d}\right)\right)\left(81\sqrt{-\frac{b(c-}{}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cosh[a + b*x]^3,x]
```

```
[Out] (d^2*(Sqrt[3]*Sqrt[-((b*(c + d*x))/d)]*Gamma[5/2, (-3*b*(c + d*x))/d]*(Cosh
[3*a - (3*b*c)/d] + Sinh[3*a - (3*b*c)/d]) + (81*Sqrt[-((b*(c + d*x))/d)]*G
amma[5/2, -((b*(c + d*x))/d)]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d
]) + Sqrt[(b*(c + d*x))/d]*(-81*Gamma[5/2, (b*(c + d*x))/d] + Sqrt[3]*Gamma
[5/2, (3*b*(c + d*x))/d]*(-Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d])))
*(Cosh[a - (b*c)/d] - Sinh[a - (b*c)/d])))/(216*b^3*Sqrt[c + d*x])
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{2}} (\cosh (bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cosh(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^(3/2)*cosh(b*x+a)^3,x)
```

Maxima [A] time = 1.86017, size = 579, normalized size = 1.78

$$\frac{\frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} + \frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} + \frac{81\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} + \frac{81\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\right.}}{b^2\sqrt{\frac{b}{d}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/288*(sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a -
3*b*c/d)/(b^2*sqrt(-b/d)) + sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*
sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^2*sqrt(b/d)) + 81*sqrt(pi)*d^2*erf(sqrt(d*
x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) + 81*sqrt(pi)*d^2*erf(sqr
t(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) - 54*(2*(d*x + c)^(3/2
)*b*d*e^(b*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^2
 - 6*(2*(d*x + c)^(3/2)*b*d*e^(3*b*c/d) + sqrt(d*x + c)*d^2*e^(3*b*c/d))*e^
(-3*a - 3*(d*x + c)*b/d)/b^2 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*a) - sqrt(d*x
+ c)*d^2*e^(3*a))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^2 + 54*(2*(d*x + c)^(3/2)
*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*c/d)/b^2)/d
```

Fricas [B] time = 2.01765, size = 3621, normalized size = 11.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/288*(sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d^2*c
osh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) - d^2*s
inh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(b*c
- a*d)/d) - d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(
```

```
d^2*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-3*(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))
 - sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d^2*cosh(
b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) + d^2*sinh(
-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(b*c - a*
d)/d) + d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*
cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-3*(b*c -
 a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) +
 81*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)^
3*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh(-(b*c - a*d)/
d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^2*cosh(
b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*cosh(b*x + a)^2*cos
h(-(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)
)*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 81*sqrt(pi)*(d^2*cosh(b*x + a)^3
*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d^2*cos
h(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh
(b*x + a)*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*si
nh(b*x + a)^2 + 3*(d^2*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x
+ a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sq
rt(-b/d)) + 6*((2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^6 + 6*(2*b^2*d*x +
 2*b^2*c - b*d)*cosh(b*x + a)*sinh(b*x + a)^5 + (2*b^2*d*x + 2*b^2*c - b*d)
*sinh(b*x + a)^6 + 9*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a)^4 + 3*(6*b
^2*d*x + 6*b^2*c + 5*(2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^2 - 9*b*d)*s
inh(b*x + a)^4 - 2*b^2*d*x + 4*(5*(2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)
^3 + 9*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a))*sinh(b*x + a)^3 - 2*b^2
*c - 9*(2*b^2*d*x + 2*b^2*c + 3*b*d)*cosh(b*x + a)^2 + 3*(5*(2*b^2*d*x + 2*
b^2*c - b*d)*cosh(b*x + a)^4 - 6*b^2*d*x - 6*b^2*c + 18*(2*b^2*d*x + 2*b^2*
c - 3*b*d)*cosh(b*x + a)^2 - 9*b*d)*sinh(b*x + a)^2 - b*d + 6*((2*b^2*d*x +
 2*b^2*c - b*d)*cosh(b*x + a)^5 + 6*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x
+ a)^3 - 3*(2*b^2*d*x + 2*b^2*c + 3*b*d)*cosh(b*x + a))*sinh(b*x + a))*sqrt
(d*x + c))/(b^3*cosh(b*x + a)^3 + 3*b^3*cosh(b*x + a)^2*sinh(b*x + a) + 3*b
^3*cosh(b*x + a)*sinh(b*x + a)^2 + b^3*sinh(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cosh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{2}} \cosh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/2)*cosh(b*x + a)^3, x)
```

3.58 $\int \sqrt{c+dx}\cosh^3(a+bx)\,dx$

Optimal. Leaf size=275

$$\frac{3\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{3\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\ldots}{\ldots}\right)}{48b^{3/2}}$$

```
[Out] (3*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/
(16*b^(3/2)) + (Sqrt[d]*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b
]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (3*Sqrt[d]*E^(a - (b*c)/d)*Sqrt[P
i]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(3*a -
(3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^
(3/2)) + (3*Sqrt[c + d*x]*Sinh[a + b*x])/(4*b) + (Sqrt[c + d*x]*Sinh[3*a +
3*b*x])/(12*b)
```

Rubi [A] time = 0.483521, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{3\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\ldots}{\ldots}\right)}{48b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cosh[a + b*x]^3,x]
```

```
[Out] (3*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/
(16*b^(3/2)) + (Sqrt[d]*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b
]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (3*Sqrt[d]*E^(a - (b*c)/d)*Sqrt[P
i]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(3*a -
(3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^
(3/2)) + (3*Sqrt[c + d*x]*Sinh[a + b*x])/(4*b) + (Sqrt[c + d*x]*Sinh[3*a +
3*b*x])/(12*b)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c+dx}\cosh^3(a+bx)\,dx &= \int \left(\frac{3}{4}\sqrt{c+dx}\cosh(a+bx)+\frac{1}{4}\sqrt{c+dx}\cosh(3a+3bx)\right)dx\\
&= \frac{1}{4}\int \sqrt{c+dx}\cosh(3a+3bx)\,dx+\frac{3}{4}\int \sqrt{c+dx}\cosh(a+bx)\,dx\\
&= \frac{3\sqrt{c+dx}\sinh(a+bx)}{4b}+\frac{\sqrt{c+dx}\sinh(3a+3bx)}{12b}-\frac{d\int \frac{\sinh(3a+3bx)}{\sqrt{c+dx}}\,dx}{24b}-\frac{(3d)\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}}{8b}\\
&= \frac{3\sqrt{c+dx}\sinh(a+bx)}{4b}+\frac{\sqrt{c+dx}\sinh(3a+3bx)}{12b}-\frac{d\int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx}{48b}+\frac{d\int \frac{e^{i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx}{48b}-\\
&= \frac{3\sqrt{c+dx}\sinh(a+bx)}{4b}+\frac{\sqrt{c+dx}\sinh(3a+3bx)}{12b}+\frac{\text{Subst}\left(\int e^{i\left(3ia-\frac{3ibc}{d}\right)-\frac{3bx^2}{d}}\,dx,x,\sqrt{c+}\right.}{24b}\\
&= \frac{3\sqrt{d}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{d}e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{3\sqrt{d}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+}}{\sqrt{d}}\right.}{16b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.286851, size = 210, normalized size = 0.76

$$\frac{\sqrt{c+dx}\,e^{-3\left(a+\frac{bc}{d}\right)}\left(\sqrt{3}e^{6a}\sqrt{\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{3}{2},-\frac{3b(c+dx)}{d}\right)+27e^{4a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)-e^{\frac{4bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\left(27e^{2}\right.\right.}{72b\sqrt{-\frac{b^2(c+dx)^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cosh[a + b*x]^3,x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[3]*E^(6*a)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, (-3*b*(c +
 d*x))/d] + 27*E^(4*a + (2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, -((b*(c
 + d*x))/d)] - E^((4*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*(27*E^(2*a)*Gamma[3/2
, (b*(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[3/2, (3*b*(c + d*x))/d])))
/(72*b*E^(3*(a + (b*c)/d))*Sqrt[-((b^2*(c + d*x)^2)/d^2)])
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^3\sqrt{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*(d*x+c)^(1/2),x)

[Out] int(cosh(b*x+a)^3*(d*x+c)^(1/2),x)
```

Maxima [A] time = 1.70376, size = 451, normalized size = 1.64

$$-\frac{\frac{\sqrt{3}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{\sqrt{3}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}+\frac{27\sqrt{\pi}d\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{27\sqrt{\pi}d\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right.}}{b\sqrt{\frac{b}{d}}}}{144\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="maxima")

[Out] -1/144*(sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3
*b*c/d)/(b*sqrt(-b/d)) - sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(
b/d))*e^(-3*a + 3*b*c/d)/(b*sqrt(b/d)) + 27*sqrt(pi)*d*erf(sqrt(d*x + c)*sq
rt(-b/d))*e^(a - b*c/d)/(b*sqrt(-b/d)) - 27*sqrt(pi)*d*erf(sqrt(d*x + c)*sq
rt(b/d))*e^(-a + b*c/d)/(b*sqrt(b/d)) - 6*sqrt(d*x + c)*d*e^(3*a + 3*(d*x +
 c)*b/d - 3*b*c/d)/b - 54*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d - b*c/d)/b +
 54*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b + 6*sqrt(d*x + c)*d*e^
(-3*a - 3*(d*x + c)*b/d + 3*b*c/d)/b)/d
```

Fricas [B] time = 1.8717, size = 2965, normalized size = 10.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/144*(sqrt(3)*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d*cosh(
b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) - d*sinh(-3*(
b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d)
- d*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x +
 a)^2*cosh(-3*(b*c - a*d)/d) - d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*si
nh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) + sqrt(3)*sqrt(
pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)^3*sinh(-3*(
b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) + d*sinh(-3*(b*c - a*d)/d))*sinh(
b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)*si
nh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-3*(b*c -
 a*d)/d) + d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b
/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) + 27*sqrt(pi)*(d*cosh(b*x + a)^3*
cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d*cosh(-(b
*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)
*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)
^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^2*sinh(-(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 27*sq
rt(pi)*(d*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)^3*sinh(-(b
*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) + d*sinh(-(b*c - a*d)/d))*sinh(b*x +
 a)^3 + 3*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-(b*
c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) +
d*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(
```

```
d*x + c)*sqrt(-b/d)) + 6*(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x +
a)^5 + b*sinh(b*x + a)^6 + 9*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + 3
*b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 9*b*cosh(b*x + a))*sinh(b*x
+ a)^3 - 9*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 18*b*cosh(b*x + a)^
2 - 3*b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 + 6*b*cosh(b*x + a)^3 - 3*b
*cosh(b*x + a))*sinh(b*x + a) - b)*sqrt(d*x + c))/(b^2*cosh(b*x + a)^3 + 3*
b^2*cosh(b*x + a)^2*sinh(b*x + a) + 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b
^2*sinh(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + dx} \cosh^3{(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*cosh(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx + c} \cosh\left(bx + a\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)*cosh(b*x + a)^3, x)
```

3.59 $\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=228

$$\frac{3\sqrt{\pi}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\pi}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

```
[Out] (3*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[
b]*Sqrt[d]) + (E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c
+ d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (3*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(3*a - (3*b*c)/d)*
Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d
])
```

Rubi [A] time = 0.373702, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\pi}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/Sqrt[c + d*x],x]
```


```
[Out] (3*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[
b]*Sqrt[d]) + (E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c
+ d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (3*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(3*a - (3*b*c)/d)*
Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d
])
```


Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx &= \int \left(\frac{3\cosh(a+bx)}{4\sqrt{c+dx}}+\frac{\cosh(3a+3bx)}{4\sqrt{c+dx}}\right)dx\\ &= \frac{1}{4}\int \frac{\cosh(3a+3bx)}{\sqrt{c+dx}}\,dx+\frac{3}{4}\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx\\ &= \frac{1}{8}\int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx+\frac{1}{8}\int \frac{e^{i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx+\frac{3}{8}\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx+\frac{3}{8}\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx\\ &= \frac{\text{Subst}\left(\int e^{i\left(3ia-\frac{3ibc}{d}\right)-\frac{3bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4d}+\frac{\text{Subst}\left(\int e^{-i\left(3ia-\frac{3ibc}{d}\right)+\frac{3bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4d}+\frac{3\,\text{Subst}\left(\right.}{}\\ &= \frac{3e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\text{e}}{8}\end{aligned}$$

Mathematica [A] time = 0.206166, size = 192, normalized size = 0.84

$$\frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(\sqrt{3}e^{6a}\sqrt{-\frac{b(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)+9e^{4a+\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)-e^{\frac{4bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\left(9e^{2a}\text{Gamma}\right.\right.}{24b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[3]*E^(6*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*x))/d] +
9*E^(4*a + (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))/d
)] - E^((4*b*c)/d)*Sqrt[(b*(c + d*x))/d]*(9*E^(2*a)*Gamma[1/2, (b*(c + d*x)
)/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[1/2, (3*b*(c + d*x))/d]))/(24*b*E^(3*(a
+ (b*c)/d))*Sqrt[c + d*x])
```

Maple [F] time = 0.123, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^3 \frac{1}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] int(cosh(b*x+a)^3/(d*x+c)^(1/2),x)
```

Maxima [A] time = 1.62049, size = 239, normalized size = 1.05

$$\frac{\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{\sqrt{\frac{b}{d}}}+\frac{9\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{9\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3*b*c
/d)/sqrt(-b/d) + sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))*e^(-
3*a + 3*b*c/d)/sqrt(b/d) + 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a -
b*c/d)/sqrt(-b/d) + 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/
sqrt(b/d))/d
```

Fricas [A] time = 2.08736, size = 594, normalized size = 2.61

$$\sqrt{3}\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{3\,(bc-ad)}{d}\right)-\sinh\left(-\frac{3\,(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-\sqrt{3}\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{3\,(bc-ad)}{d}\right)+\sinh\left(-\frac{3\,(bc-ad}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(3)*sqrt(pi)*sqrt(b/d)*(cosh(-3*(b*c - a*d)/d) - sinh(-3*(b*c - a
*d)/d))*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(3)*sqrt(pi)*sqrt(-b/d)*
(cosh(-3*(b*c - a*d)/d) + sinh(-3*(b*c - a*d)/d))*erf(sqrt(3)*sqrt(d*x + c)
*sqrt(-b/d)) + 9*sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c - a*
d)/d))*erf(sqrt(d*x + c)*sqrt(b/d)) - 9*sqrt(pi)*sqrt(-b/d)*(cosh(-(b*c - a
*d)/d) + sinh(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cosh(a + b*x)**3/sqrt(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^3}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(cosh(b*x + a)^3/sqrt(d*x + c), x)
```

3.60 $\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=246

$$-\frac{3\sqrt{\pi}\sqrt{b}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}-\frac{\sqrt{3\pi}\sqrt{b}e^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{3\sqrt{\pi}\sqrt{b}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{\sqrt{3\pi}\sqrt{b}e^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{}}{}\right.}{4d^{3/2}}$$

```
[Out] (-2*Cosh[a + b*x]^3)/(d*Sqrt[c + d*x]) - (3*Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[P
i]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) - (Sqrt[b]*E^(-3*a + (
3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/
2)) + (3*Sqrt[b]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]])/(4*d^(3/2)) + (Sqrt[b]*E^(3*a - (3*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2))
```

Rubi [A] time = 0.421196, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3313, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}\sqrt{b}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}-\frac{\sqrt{3\pi}\sqrt{b}e^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{3\sqrt{\pi}\sqrt{b}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{\sqrt{3\pi}\sqrt{b}e^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{}}{}\right.}{4d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cosh[a + b*x]^3)/(d*Sqrt[c + d*x]) - (3*Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[P
i]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) - (Sqrt[b]*E^(-3*a + (
3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/
2)) + (3*Sqrt[b]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]])/(4*d^(3/2)) + (Sqrt[b]*E^(3*a - (3*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2))
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}} + \frac{(6ib)\int \left(-\frac{i\sinh(a+bx)}{4\sqrt{c+dx}} - \frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}$$

$$= -\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}} + \frac{(3b)\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{2d} + \frac{(3b)\int \frac{\sinh(3a+3bx)}{\sqrt{c+dx}}\,dx}{2d}$$

$$= -\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}} + \frac{(3b)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4d} - \frac{(3b)\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4d} + \frac{(3b)\int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx}{4d} - \frac{(3b)\int \frac{e^{i(3ia+3ibx)}}{\sqrt{c+dx}}}{4d}$$

$$= -\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}} - \frac{(3b)\,\mathrm{Subst}\left(\int e^{i\left(3ia-\frac{3ibc}{d}\right)-\frac{3bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{2d^2} - \frac{(3b)\,\mathrm{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx,\right.}{2d^2}$$

$$= -\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}} - \frac{3\sqrt{b}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{\sqrt{b}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{3\sqrt{b}e^{a-\frac{bc}{d}}\sqrt{}}{4}$$

Mathematica [B] time = 2.73577, size = 717, normalized size = 2.91

$$\frac{e^{-\frac{3b(c+dx)}{d}}\left(\sqrt{3}\sqrt{d}e^{\frac{3b(c+dx)}{d}}\sqrt{-\frac{b(c+dx)}{d}}\cosh\left(3a-\frac{3bc}{d}\right)\mathrm{Gamma}\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)+\sqrt{3}\sqrt{d}e^{\frac{3b(c+dx)}{d}}\sqrt{\frac{b(c+dx)}{d}}\cosh\left(3a-\frac{3bc}{d}\right)\mathrm{Gamm}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (-(Sqrt[d]*Cosh[3*a - (3*b*c)/d]) - Sqrt[d]*E^((6*b*(c + d*x))/d)*Cosh[3*a
- (3*b*c)/d] - 3*Sqrt[d]*E^((2*b*(c + d*x))/d)*Cosh[a - (b*c)/d] - 3*Sqrt[d
]*E^((4*b*(c + d*x))/d)*Cosh[a - (b*c)/d] + Sqrt[3]*Sqrt[d]*E^((3*b*(c + d*
x))/d)*Sqrt[-((b*(c + d*x))/d)]*Cosh[3*a - (3*b*c)/d]*Gamma[1/2, (-3*b*(c +
 d*x))/d] + 3*Sqrt[d]*E^((3*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/d]*Cosh[a -
(b*c)/d]*Gamma[1/2, (b*(c + d*x))/d] + Sqrt[3]*Sqrt[d]*E^((3*b*(c + d*x))/d
)*Sqrt[(b*(c + d*x))/d]*Cosh[3*a - (3*b*c)/d]*Gamma[1/2, (3*b*(c + d*x))/d]
 + Sqrt[d]*Sinh[3*a - (3*b*c)/d] - Sqrt[d]*E^((6*b*(c + d*x))/d)*Sinh[3*a -
 (3*b*c)/d] + Sqrt[b]*E^((3*b*(c + d*x))/d)*Sqrt[3*Pi]*Sqrt[c + d*x]*Erf[(S
qrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]*Sinh[3*a - (3*b*c)/d] + Sqrt[b]*E^((
3*b*(c + d*x))/d)*Sqrt[3*Pi]*Sqrt[c + d*x]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d
*x])/Sqrt[d]]*Sinh[3*a - (3*b*c)/d] + 3*Sqrt[d]*E^((2*b*(c + d*x))/d)*Sinh[
a - (b*c)/d] - 3*Sqrt[d]*E^((4*b*(c + d*x))/d)*Sinh[a - (b*c)/d] - 3*Sqrt[d
]*E^((3*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]*S
inh[a - (b*c)/d] + 3*Sqrt[d]*E^((3*b*(c + d*x))/d)*Sqrt[-((b*(c + d*x))/d)]
*Gamma[1/2, -((b*(c + d*x))/d)]*(Cosh[a - (b*c)/d] + Sinh[a - (b*c)/d]))/(4
*d^(3/2)*E^((3*b*(c + d*x))/d)*Sqrt[c + d*x])
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^3 (dx+c)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] int(cosh(b*x+a)^3/(d*x+c)^(3/2),x)
```

Maxima [A] time = 1.36259, size = 265, normalized size = 1.08

$$-\frac{\frac{\sqrt{3}\sqrt{\frac{(dx+c)b}{d}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{3(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{3}\sqrt{-\frac{(dx+c)b}{d}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{3(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{3\sqrt{\frac{(dx+c)b}{d}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{3\sqrt{-\frac{(dx+c)b}{d}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2}\right.}{\sqrt{dx+c}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(3)*sqrt((d*x + c)*b/d)*e^(3*(b*c - a*d)/d)*gamma(-1/2, 3*(d*x +
c)*b/d)/sqrt(d*x + c) + sqrt(3)*sqrt(-(d*x + c)*b/d)*e^(-3*(b*c - a*d)/d)*g
amma(-1/2, -3*(d*x + c)*b/d)/sqrt(d*x + c) + 3*sqrt((d*x + c)*b/d)*e^(-a +
b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d*x + c) + 3*sqrt(-(d*x + c)*b/d)*e^
(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sqrt(d*x + c))/d
```

Fricas [B] time = 2.37642, size = 3313, normalized size = 13.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(3)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) -
(d*x + c)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c
- a*d)/d) - (d*x + c)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c
)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-3*(b
*c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b*c -
 a*d)/d) - (d*x + c)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))
*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) + sqrt(3)*sqrt(pi)*((d*x +
c)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^3*sinh(
-3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c - a*d)/d) + (d*x + c)*sinh(-3*(
b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c)*cosh(b*x + a)*cosh(-3*(b*c -
a*d)/d) + (d*x + c)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 +
 3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x +
 a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*
x + c)*sqrt(-b/d)) + 3*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cosh(-(b*c - a*d
)/d) - (d*x + c)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b
*c - a*d)/d) - (d*x + c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x +
c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-(b*c
```

```
- a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-(b*c - a*d)
/d) - (d*x + c)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b
/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*co
sh(-(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((d*x
 + c)*cosh(-(b*c - a*d)/d) + (d*x + c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^
3 + 3*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + (d*x + c)*cosh(b*x +
a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cos
h(-(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*
x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (cosh(b*x + a)^6 + 6*cos
h(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a))
*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x +
 a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + cosh(b
*x + a))*sinh(b*x + a) + 1)*sqrt(d*x + c))/((d^2*x + c*d)*cosh(b*x + a)^3 +
 3*(d^2*x + c*d)*cosh(b*x + a)^2*sinh(b*x + a) + 3*(d^2*x + c*d)*cosh(b*x +
 a)*sinh(b*x + a)^2 + (d^2*x + c*d)*sinh(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] Integral(cosh(a + b*x)**3/(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^3}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3/(d*x + c)^(3/2), x)
```

3.61 $\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=277

$$\frac{\sqrt{\pi}b^{3/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{\sqrt{3\pi}b^{3/2}e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{\sqrt{\pi}b^{3/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{\sqrt{3\pi}b^{3/2}e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\right.}{2d^{5/2}}$$

```
[Out] (-2*Cosh[a + b*x]^3)/(3*d*(c + d*x)^(3/2)) + (b^(3/2)*E^(-a + (b*c)/d)*Sqrt
[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) + (b^(3/2)*E^(-3*a +
 (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(
5/2)) + (b^(3/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]])/(2*d^(5/2)) + (b^(3/2)*E^(3*a - (3*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) - (4*b*Cosh[a + b*x]^2*Sinh[a +
 b*x])/(d^2*Sqrt[c + d*x])
```

Rubi [A] time = 0.604716, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3314, 3307, 2180, 2204, 2205, 3312}

$$\frac{\sqrt{\pi}b^{3/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{\sqrt{3\pi}b^{3/2}e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{\sqrt{\pi}b^{3/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{\sqrt{3\pi}b^{3/2}e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\right.}{2d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cosh[a + b*x]^3)/(3*d*(c + d*x)^(3/2)) + (b^(3/2)*E^(-a + (b*c)/d)*Sqrt
[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) + (b^(3/2)*E^(-3*a +
 (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(
5/2)) + (b^(3/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]])/(2*d^(5/2)) + (b^(3/2)*E^(3*a - (3*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) - (4*b*Cosh[a + b*x]^2*Sinh[a +
 b*x])/(d^2*Sqrt[c + d*x])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{d^2\sqrt{c+dx}} - \frac{\left(8b^2\right)\int\frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} + \frac{\left(12b^2\right)\int\frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,d}{d^2}$$

$$= -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{d^2\sqrt{c+dx}} - \frac{\left(4b^2\right)\int\frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{d^2} - \frac{\left(4b^2\right)\int\frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{d^2} + $$

$$= -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{d^2\sqrt{c+dx}} - \frac{\left(8b^2\right)\operatorname{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{4b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{4b\cosh^2(a+b}{d^2\sqrt{c}}$$

$$= -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{4b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{4b\cosh^2(a+b}{d^2\sqrt{c}}$$

$$= -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{b^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}}{2}$$

Mathematica [A] time = 2.90672, size = 253, normalized size = 0.91

$$\frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(-3\sqrt{3}e^{6a}d\left(-\frac{b(c+dx)}{d}\right)^{3/2}\operatorname{Gamma}\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right) - 3de^{4a+\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\operatorname{Gamma}\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right) - 3de^{2a+\frac{4bc}{d}}\left(\frac{b(c+d}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[3]*d*E^(6*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-3*b*(c + d*x)
)/d] - 3*d*E^(4*a + (2*b*c)/d)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(
c + d*x))/d)] - 3*d*E^(2*a + (4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2,
(b*(c + d*x))/d] - 3*Sqrt[3]*d*E^((6*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[
```

```
1/2, (3*b*(c + d*x))/d] - 4*E^(3*(a + (b*c)/d))*Cosh[a + b*x]^2*(d*Cosh[a +
 b*x] + 6*b*(c + d*x)*Sinh[a + b*x]))/(6*d^2*E^(3*(a + (b*c)/d))*(c + d*x)^
(3/2))
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int (\cosh (bx + a))^3 (dx + c)^{-\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] int(cosh(b*x+a)^3/(d*x+c)^(5/2),x)
```

Maxima [A] time = 1.35984, size = 262, normalized size = 0.95

$$-\frac{3\left(\frac{\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(a-\cdots\right.}\cdots}{(dx+\cdots}\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/8*(sqrt(3)*((d*x + c)*b/d)^(3/2)*e^(3*(b*c - a*d)/d)*gamma(-3/2, 3*(d*x
+ c)*b/d)/(d*x + c)^(3/2) + sqrt(3)*(-(d*x + c)*b/d)^(3/2)*e^(-3*(b*c - a*d
)/d)*gamma(-3/2, -3*(d*x + c)*b/d)/(d*x + c)^(3/2) + ((d*x + c)*b/d)^(3/2)*
e^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*x + c)^(3/2) + (-(d*x + c)*b/d
)^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)*b/d)/(d*x + c)^(3/2))/d
```

Fricas [B] time = 2.36515, size = 4825, normalized size = 17.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*c
osh(-3*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*sin
h(-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b*c - a*d)/
d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^
3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d)
 - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*si
nh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-3*
(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-3*(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))
 - 6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*cosh
(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*sinh(-
3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b*c - a*d)/d)
```

```
+ (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 +
 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) +
(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(
b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-3*(b*
c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-3*(b*c
- a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))
+ 6*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*cosh(-(b*c -
a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*sinh(-(b*c - a*d)
/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 +
2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b*d^2*x^2 +
2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*
d*x + b*c^2)*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - (b*d^2*x^
2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))
*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 6*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*
x + b*c^2)*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x +
b*c^2)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c
^2)*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d
)/d))*sinh(b*x + a)^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*co
sh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-(b
*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x
 + a)^2*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a
)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(
-b/d)) - ((6*b*d*x + 6*b*c + d)*cosh(b*x + a)^6 + 6*(6*b*d*x + 6*b*c + d)*c
osh(b*x + a)*sinh(b*x + a)^5 + (6*b*d*x + 6*b*c + d)*sinh(b*x + a)^6 + 3*(2
*b*d*x + 2*b*c + d)*cosh(b*x + a)^4 + 3*(2*b*d*x + 5*(6*b*d*x + 6*b*c + d)*
cosh(b*x + a)^2 + 2*b*c + d)*sinh(b*x + a)^4 + 4*(5*(6*b*d*x + 6*b*c + d)*c
osh(b*x + a)^3 + 3*(2*b*d*x + 2*b*c + d)*cosh(b*x + a))*sinh(b*x + a)^3 - 6
*b*d*x - 3*(2*b*d*x + 2*b*c - d)*cosh(b*x + a)^2 + 3*(5*(6*b*d*x + 6*b*c +
d)*cosh(b*x + a)^4 - 2*b*d*x + 6*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)^2 - 2*
b*c + d)*sinh(b*x + a)^2 - 6*b*c + 6*((6*b*d*x + 6*b*c + d)*cosh(b*x + a)^5
 + 2*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)^3 - (2*b*d*x + 2*b*c - d)*cosh(b*x
 + a))*sinh(b*x + a) + d)*sqrt(d*x + c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*c
osh(b*x + a)^3 + 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)^2*sinh(b*x
 + a) + 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)*sinh(b*x + a)^2 + (
d^4*x^2 + 2*c*d^3*x + c^2*d^2)*sinh(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Integral(cosh(a + b*x)**3/(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^3}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3/(d*x + c)^(5/2), x)
```

3.62 $\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{\sqrt{\pi}b^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{3\sqrt{3\pi}b^{5/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{\sqrt{\pi}b^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{3\sqrt{3\pi}b^{5/2}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\right.}{5d^{7/2}}$$

```
[Out] (16*b^2*Cosh[a + b*x])/(5*d^3*Sqrt[c + d*x]) - (2*Cosh[a + b*x]^3)/(5*d*(c
+ d*x)^(5/2)) - (24*b^2*Cosh[a + b*x]^3)/(5*d^3*Sqrt[c + d*x]) - (b^(5/2)*E
^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2))
- (3*b^(5/2)*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c +
d*x])/Sqrt[d]])/(5*d^(7/2)) + (b^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[
b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) + (3*b^(5/2)*E^(3*a - (3*b*c)/d)*Sq
rt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) - (4*b*
Cosh[a + b*x]^2*Sinh[a + b*x])/(5*d^2*(c + d*x)^(3/2))
```

Rubi [A] time = 0.680456, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3314, 3297, 3308, 2180, 2204, 2205, 3313}

$$-\frac{\sqrt{\pi}b^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{3\sqrt{3\pi}b^{5/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{\sqrt{\pi}b^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{3\sqrt{3\pi}b^{5/2}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\right.}{5d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/(c + d*x)^(7/2),x]
```


```
[Out] (16*b^2*Cosh[a + b*x])/(5*d^3*Sqrt[c + d*x]) - (2*Cosh[a + b*x]^3)/(5*d*(c
+ d*x)^(5/2)) - (24*b^2*Cosh[a + b*x]^3)/(5*d^3*Sqrt[c + d*x]) - (b^(5/2)*E
^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2))
- (3*b^(5/2)*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c +
d*x])/Sqrt[d]])/(5*d^(7/2)) + (b^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[
b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) + (3*b^(5/2)*E^(3*a - (3*b*c)/d)*Sq
rt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) - (4*b*
Cosh[a + b*x]^2*Sinh[a + b*x])/(5*d^2*(c + d*x)^(3/2))
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{\left(8b^2\right)\int\frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx}{5d^2} + \frac{\left(12b^2\right)\int\frac{\cosh^3(a+\cdots}{(c+dx)^{3/\cdots}}}{5d^2}$$

$$= \frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{5d^2(c+dx)^{3/2}} + \cdots$$

$$= \frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{5d^2(c+dx)^{3/2}} - \cdots$$

$$= \frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{5d^2(c+dx)^{3/2}} + \cdots$$

$$= \frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{8b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \cdots$$

$$= \frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}} - \frac{b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{3b\cdots}{\cdots}$$

Mathematica [B] time = 6.32461, size = 3211, normalized size = 9.7

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (3*(Sinh[a]*(-((-2*E^((b*(c + d*x))/d)*(3*d^2 + 2*b*d*(c + d*x) + 4*b^2*(c
+ d*x)^2) + 8*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, -((b*(c + d*x))/d)]
 + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*b*d*E^((b*(c + d*x))/d
)*(c + d*x)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d])/E^((b*(c +
 d*x))/d))*Sinh[(b*c)/d])/(30*d^3*(c + d*x)^(5/2)) + (2*Cosh[(b*c)/d]*(-(b*
(c + d*x)*(2*E^((b*(c + d*x))/d)*(d + 2*b*(c + d*x)) + 4*d*(-((b*(c + d*x))
/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)] + (2*(d - 2*b*(c + d*x) + 2*d*E^(
(b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d]))/E^((
b*(c + d*x))/d)))/2 - 3*d^2*Sinh[(b*(c + d*x))/d]))/(15*d^3*(c + d*x)^(5/2)
)) + Cosh[a]*((Cosh[(b*c)/d]*(-2*E^((b*(c + d*x))/d)*(3*d^2 + 2*b*d*(c + d*
x) + 4*b^2*(c + d*x)^2) + 8*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, -((b*
(c + d*x))/d)] + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*b*d*E^((
b*(c + d*x))/d)*(c + d*x)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/
d])/E^((b*(c + d*x))/d)))/(30*d^3*(c + d*x)^(5/2)) - (2*Sinh[(b*c)/d]*(-(b*
(c + d*x)*(2*E^((b*(c + d*x))/d)*(d + 2*b*(c + d*x)) + 4*d*(-((b*(c + d*x))
/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)] + (2*(d - 2*b*(c + d*x) + 2*d*E^(
(b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d]))/E^((
b*(c + d*x))/d)))/2 - 3*d^2*Sinh[(b*(c + d*x))/d]))/(15*d^3*(c + d*x)^(5/2)
))))/4 + (Sinh[3*a]*(-((1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d
^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x)
)/d))^(5/2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24
*b^2*(c + d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(
5/2)*Gamma[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d))*Sinh[(b*c)/d])/(
10*d^3*(c + d*x)^(5/2)) - (2*Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-6*b^(
5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]
] - 6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x
])/Sqrt[d]] + Sqrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*
b^2*(c + d*x)^2)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))) +
Cosh[3*a]*((Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d
)*(d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c +
d*x))/d))^(5/2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x)
- 24*b^2*(c + d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/
d)^(5/2)*Gamma[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d)))/(10*d^3*(c
+ d*x)^(5/2)) + (2*(1 + 2*Cosh[(2*b*c)/d])*Sinh[(b*c)/d]*(-6*b^(5/2)*Sqrt[3
*Pi]*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/
2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]
 + Sqrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*
x)^2)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))))/4 + (-(Cosh[
3*a]*(-((1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d^2 + 2*b*d*(c +
 d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x))/d))^(5/2)*Gam
ma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^
2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma[1/2,
 (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d))*Sinh[(b*c)/d])/(10*d^3*(c + d*x
)^(5/2)) - (2*Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-6*b^(5/2)*Sqrt[3*Pi]
*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/2)*S
qrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + S
qrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2
)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2)))) - Sinh[3*a]*((Co
sh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d^2 + 2*b*d
*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x))/d))^(5/2
)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c +
d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma
[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d)))/(10*d^3*(c + d*x)^(5/2))
+ (2*(1 + 2*Cosh[(2*b*c)/d])*Sinh[(b*c)/d]*(-6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)
^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/2)*Sqrt[3*Pi]*
(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + Sqrt[d]*(2*
b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2)*Sinh[(3*
```

```
b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))))/8 + (Cosh[3*a]*(-((1 + 2*C
osh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d^2 + 2*b*d*(c + d*x) + 12*b^2*(
c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, (-3*b*(c
 + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2 + 24*Sqrt[3]*d
^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma[1/2, (3*b*(c + d*x))
/d])/E^((3*b*(c + d*x))/d))*Sinh[(b*c)/d])/(10*d^3*(c + d*x)^(5/2)) - (2*Co
sh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)
*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/2)*Sqrt[3*Pi]*(c + d
*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + Sqrt[d]*(2*b*d*(c
 + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2)*Sinh[(3*b*(c +
 d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))) + Sinh[3*a]*((Cosh[(b*c)/d]*(-1 +
 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d^2 + 2*b*d*(c + d*x) + 12*b
^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, (-3*
b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2 + 24*Sqrt[
3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma[1/2, (3*b*(c + d
*x))/d])/E^((3*b*(c + d*x))/d)))/(10*d^3*(c + d*x)^(5/2)) + (2*(1 + 2*Cosh[
(2*b*c)/d])*Sinh[(b*c)/d]*(-6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erf[(Sqrt[
3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*E
rfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + Sqrt[d]*(2*b*d*(c + d*x)*Cos
h[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2)*Sinh[(3*b*(c + d*x))/d]))
)/(5*d^(7/2)*(c + d*x)^(5/2))))/8
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int (\cosh(bx+a))^3 (dx+c)^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c)^(7/2),x)
```

```
[Out] int(cosh(b*x+a)^3/(d*x+c)^(7/2),x)
```

Maxima [A] time = 1.34055, size = 265, normalized size = 0.8

$$-\frac{3\left(\frac{3\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{3\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{(\cdots}}{(\cdots}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -3/8*(3*sqrt(3)*((d*x + c)*b/d)^(5/2)*e^(3*(b*c - a*d)/d)*gamma(-5/2, 3*(d*
x + c)*b/d)/(d*x + c)^(5/2) + 3*sqrt(3)*(-(d*x + c)*b/d)^(5/2)*e^(-3*(b*c -
 a*d)/d)*gamma(-5/2, -3*(d*x + c)*b/d)/(d*x + c)^(5/2) + ((d*x + c)*b/d)^(5
/2)*e^(-a + b*c/d)*gamma(-5/2, (d*x + c)*b/d)/(d*x + c)^(5/2) + (-(d*x + c)
*b/d)^(5/2)*e^(a - b*c/d)*gamma(-5/2, -(d*x + c)*b/d)/(d*x + c)^(5/2))/d
```

Fricas [B] time = 2.47978, size = 7121, normalized size = 21.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/20*(12*sqrt(3)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*
d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) +
 ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-3*(b*c -
a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-3
*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-3*(b*c - a*d
)/d))*sinh(b*x + a)^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*s
inh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) + 12*sqrt(3)*s
qrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x
 + a)^3*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*
x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-3*(b*c - a*d)/d))*si
nh(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)
^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x
+ a)^2*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(
-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) + 4*sqrt(pi)*((b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^3*cosh(-(b*c - a*d)/
d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a
)^3*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-(b*c - a*d)/
d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a
)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - (b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-(b*
c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 4*sqrt
(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x +
a)^3*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)
^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x
+ a)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*cosh(-(b
*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*co
sh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x
+ c)*sqrt(-b/d)) + ((12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^
2*c*d + b*d^2)*x)*cosh(b*x + a)^6 + 6*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*
d + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a)^5 + (12*b^2
*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*sinh(b*x
+ a)^6 + 12*b^2*d^2*x^2 + (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*
(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 + (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c
*d + 15*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^
2)*x)*cosh(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^4 +
12*b^2*c^2 + 4*(5*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*
```

```
c*d + b*d^2)*x)*cosh(b*x + a)^3 + (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*
d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a))*sinh(b*x + a)^3 - 2*b*c*d + (
4*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*cosh
(b*x + a)^2 + (4*b^2*d^2*x^2 + 15*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d +
d^2 + 2*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 + 4*b^2*c^2 - 2*b*c*d + 6*(
4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh
(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*sinh(b*x + a)^2 + d^2 + 2*(1
2*b^2*c*d - b*d^2)*x + 2*(3*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 +
2*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^5 + 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 +
2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^3 + (4*b^2*d^2*x^2
 + 4*b^2*c^2 - 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*cosh(b*x + a))*si
nh(b*x + a))*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3
)*cosh(b*x + a)^3 + 3*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*cosh(
b*x + a)^2*sinh(b*x + a) + 3*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3
)*cosh(b*x + a)*sinh(b*x + a)^2 + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^
3*d^3)*sinh(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c)**(7/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)^3}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate(cosh(b*x + a)^3/(d*x + c)^(7/2), x)
```

3.63 $\int (dx)^{3/2} \cosh(fx)\, dx$

Optimal. Leaf size=111

$$\frac{3\sqrt{\pi}d^{3/2}\text{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}+\frac{3\sqrt{\pi}d^{3/2}\text{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}-\frac{3d\sqrt{dx}\cosh(fx)}{2f^2}+\frac{(dx)^{3/2}\sinh(fx)}{f}$$

```
[Out] (-3*d*Sqrt[d*x]*Cosh[f*x])/(2*f^2) + (3*d^(3/2)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[
d*x])/Sqrt[d]])/(8*f^(5/2)) + (3*d^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/
Sqrt[d]])/(8*f^(5/2)) + ((d*x)^(3/2)*Sinh[f*x])/f
```

Rubi [A] time = 0.15545, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3296, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}d^{3/2}\text{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}+\frac{3\sqrt{\pi}d^{3/2}\text{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}-\frac{3d\sqrt{dx}\cosh(fx)}{2f^2}+\frac{(dx)^{3/2}\sinh(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Cosh[f*x],x]
```

```
[Out] (-3*d*Sqrt[d*x]*Cosh[f*x])/(2*f^2) + (3*d^(3/2)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[
d*x])/Sqrt[d]])/(8*f^(5/2)) + (3*d^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/
Sqrt[d]])/(8*f^(5/2)) + ((d*x)^(3/2)*Sinh[f*x])/f
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (dx)^{3/2}\cosh(fx)\,dx = \frac{(dx)^{3/2}\sinh(fx)}{f} - \frac{(3d)\int \sqrt{dx}\sinh(fx)\,dx}{2f}$$
$$= -\frac{3d\sqrt{dx}\cosh(fx)}{2f^2} + \frac{(dx)^{3/2}\sinh(fx)}{f} + \frac{\left(3d^2\right)\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx}{4f^2}$$
$$= -\frac{3d\sqrt{dx}\cosh(fx)}{2f^2} + \frac{(dx)^{3/2}\sinh(fx)}{f} + \frac{\left(3d^2\right)\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{8f^2} + \frac{\left(3d^2\right)\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{8f^2}$$
$$= -\frac{3d\sqrt{dx}\cosh(fx)}{2f^2} + \frac{(dx)^{3/2}\sinh(fx)}{f} + \frac{(3d)\,\text{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{4f^2} + \frac{(3d)\,\text{Subst}\left(\int e^{\frac{f}{}}\right.}{4f^2}$$
$$= -\frac{3d\sqrt{dx}\cosh(fx)}{2f^2} + \frac{3d^{3/2}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{3d^{3/2}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{(dx)^{3/2}\sinh(fx)}{f}$$

Mathematica [A] time = 0.0133719, size = 51, normalized size = 0.46

$$\frac{d^2\left(\sqrt{-fx}\,\text{Gamma}\left(\frac{5}{2},-fx\right) - \sqrt{fx}\,\text{Gamma}\left(\frac{5}{2},fx\right)\right)}{2f^3\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*Cosh[f*x],x]
```

```
[Out] (d^2*(Sqrt[-(f*x)]*Gamma[5/2, -(f*x)] - Sqrt[f*x]*Gamma[5/2, f*x]))/(2*f^3*
Sqrt[d*x])
```

Maple [C] time = 0.027, size = 133, normalized size = 1.2

$$\frac{-2i\sqrt{2}\sqrt{\pi}}{f}(dx)^{\frac{3}{2}}\left(-\frac{\sqrt{2}\left(10fx+15\right)e^{-fx}}{80\sqrt{\pi}f^2}\sqrt{x}\left(if\right)^{\frac{5}{2}} - \frac{\sqrt{2}\left(-10fx+15\right)e^{fx}}{80\sqrt{\pi}f^2}\sqrt{x}\left(if\right)^{\frac{5}{2}} + \frac{3\sqrt{2}}{32}\left(if\right)^{\frac{5}{2}}\text{Erf}\left(\sqrt{x}\sqrt{f}\right)f^{-\frac{5}{2}} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*cosh(f*x),x)
```

```
[Out] -2*I*(d*x)^(3/2)/x^(3/2)*2^(1/2)/(I*f)^(3/2)*Pi^(1/2)/f*(-1/80/Pi^(1/2)*x^(
1/2)*2^(1/2)*(I*f)^(5/2)*(10*f*x+15)/f^2*exp(-f*x)-1/80/Pi^(1/2)*x^(1/2)*2^
(1/2)*(I*f)^(5/2)*(-10*f*x+15)/f^2*exp(f*x)+3/32*(I*f)^(5/2)*2^(1/2)/f^(5/2
)*erf(x^(1/2)*f^(1/2))+3/32*(I*f)^(5/2)*2^(1/2)/f^(5/2)*erfi(x^(1/2)*f^(1/2
)))
```

Maxima [B] time = 1.06195, size = 235, normalized size = 2.12

$$\frac{16\,(dx)^{\frac{5}{2}}\cosh\left(fx\right) + \frac{f\left(\frac{15\,\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f^3\sqrt{\frac{f}{d}}}+\frac{15\,\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f^3\sqrt{-\frac{f}{d}}}-\frac{2\left(4\,(dx)^{\frac{5}{2}}df^2-10\,(dx)^{\frac{3}{2}}d^2f+15\,\sqrt{dx}d^3\right)e^{(fx)}}{f^3}-\frac{2\left(4\,(dx)^{\frac{5}{2}}df^2+10\,(dx)^{\frac{3}{2}}d^2f+15\,\sqrt{dx}d^3\right)e^{(-fx)}}{f^3}\right)}{d}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*cosh(f*x),x, algorithm="maxima")
```

```
[Out] 1/40*(16*(d*x)^(5/2)*cosh(f*x) + f*(15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(f/d)
)/(f^3*sqrt(f/d)) + 15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(-f/d))/(f^3*sqrt(-f/
d)) - 2*(4*(d*x)^(5/2)*d*f^2 - 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)*d^3)*e^(
f*x)/f^3 - 2*(4*(d*x)^(5/2)*d*f^2 + 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)*d^3
)*e^(-f*x)/f^3)/d)/d
```

Fricas [B] time = 1.83915, size = 466, normalized size = 4.2

$$\frac{3\,\sqrt{\pi}\left(d^2\cosh\left(fx\right)+d^2\sinh\left(fx\right)\right)\sqrt{\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)-3\,\sqrt{\pi}\left(d^2\cosh\left(fx\right)+d^2\sinh\left(fx\right)\right)\sqrt{-\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)-2\left(2\,df\right.}{8\left(f^3\cosh\left(fx\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*cosh(f*x),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqr
t(f/d)) - 3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*
x)*sqrt(-f/d)) - 2*(2*d*f^2*x - (2*d*f^2*x - 3*d*f)*cosh(f*x)^2 - 2*(2*d*f^
2*x - 3*d*f)*cosh(f*x)*sinh(f*x) - (2*d*f^2*x - 3*d*f)*sinh(f*x)^2 + 3*d*f)
*sqrt(d*x))/(f^3*cosh(f*x) + f^3*sinh(f*x))
```

Sympy [C] time = 142.406, size = 131, normalized size = 1.18

$$\frac{5d^{\frac{3}{2}}x^{\frac{3}{2}}\sinh\left(fx\right)\Gamma\left(\frac{5}{4}\right)}{4f\Gamma\left(\frac{9}{4}\right)}-\frac{15d^{\frac{3}{2}}\sqrt{x}\cosh\left(fx\right)\Gamma\left(\frac{5}{4}\right)}{8f^2\Gamma\left(\frac{9}{4}\right)}+\frac{15\sqrt{2}\sqrt{\pi}d^{\frac{3}{2}}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{16f^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*cosh(f*x),x)
```

```
[Out] 5*d**(3/2)*x**(3/2)*sinh(f*x)*gamma(5/4)/(4*f*gamma(9/4)) - 15*d**(3/2)*sqr
t(x)*cosh(f*x)*gamma(5/4)/(8*f**2*gamma(9/4)) + 15*sqrt(2)*sqrt(pi)*d**(3/2
)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(pi))*gamma
(5/4)/(16*f**(5/2)*gamma(9/4))
```

Giac [A] time = 1.23246, size = 194, normalized size = 1.75

$$-\frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f^2}+\frac{2\left(2\sqrt{dx}d^2fx+3\sqrt{dx}d^2\right)e^{(-fx)}}{f^2}}{8d}-\frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f^2}-\frac{2\left(2\sqrt{dx}d^2fx-3\sqrt{dx}d^2\right)e^{(fx)}}{f^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*cosh(f*x),x, algorithm="giac")
```

```
[Out] -1/8*(3*sqrt(pi)*d^3*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f^2) + 2*(2*sqr
t(d*x)*d^2*f*x + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d - 1/8*(3*sqrt(pi)*d^3*erf
(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*f)*f^2) - 2*(2*sqrt(d*x)*d^2*f*x - 3*sqr
t(d*x)*d^2)*e^(f*x)/f^2)/d
```

3.64 $\int \sqrt{dx} \cosh(fx) \, dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\pi}\sqrt{d}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}+\frac{\sqrt{dx}\sinh(fx)}{f}$$

```
[Out] (Sqrt[d]*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(4*f^(3/2)) - (Sqrt[d]*
Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(4*f^(3/2)) + (Sqrt[d*x]*Sinh[f
*x])/f
```

Rubi [A] time = 0.106476, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3296, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{d}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}+\frac{\sqrt{dx}\sinh(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Cosh[f*x],x]
```

```
[Out] (Sqrt[d]*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(4*f^(3/2)) - (Sqrt[d]*
Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(4*f^(3/2)) + (Sqrt[d*x]*Sinh[f
*x])/f
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{dx}\cosh(fx)\,dx &= \frac{\sqrt{dx}\sinh(fx)}{f} - \frac{d\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx}{2f} \\
&= \frac{\sqrt{dx}\sinh(fx)}{f} + \frac{d\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{4f} - \frac{d\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{4f} \\
&= \frac{\sqrt{dx}\sinh(fx)}{f} + \frac{\text{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{2f} - \frac{\text{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{2f} \\
&= \frac{\sqrt{d}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} + \frac{\sqrt{dx}\sinh(fx)}{f}
\end{aligned}
$$

Mathematica [A] time = 0.0111476, size = 48, normalized size = 0.52

$$
-\frac{d\left(\sqrt{-fx}\,\text{Gamma}\left(\frac{3}{2},-fx\right)+\sqrt{fx}\,\text{Gamma}\left(\frac{3}{2},fx\right)\right)}{2f^2\sqrt{dx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*Cosh[f*x],x]
```

```
[Out] -(d*(Sqrt[-(f*x)]*Gamma[3/2, -(f*x)] + Sqrt[f*x]*Gamma[3/2, f*x]))/(2*f^2*S
qrt[d*x])
```

Maple [C] time = 0.019, size = 121, normalized size = 1.3

$$
\frac{-i\sqrt{\pi}\sqrt{2}}{f}\sqrt{dx}\left(\frac{\sqrt{2}\mathrm{e}^{fx}}{4\sqrt{\pi}f}\sqrt{x}\left(if\right)^{\frac{3}{2}}-\frac{\sqrt{2}\mathrm{e}^{-fx}}{4\sqrt{\pi}f}\sqrt{x}\left(if\right)^{\frac{3}{2}}+\frac{\sqrt{2}}{8}\left(if\right)^{\frac{3}{2}}\mathrm{Erf}\left(\sqrt{x}\sqrt{f}\right)f^{-\frac{3}{2}}-\frac{\sqrt{2}}{8}\left(if\right)^{\frac{3}{2}}\mathrm{erfi}\left(\sqrt{x}\sqrt{f}\right)f^{-\frac{3}{2}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{i}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)*(d*x)^(1/2),x)
```

```
[Out] -I*Pi^(1/2)*(d*x)^(1/2)/x^(1/2)*2^(1/2)/(I*f)^(1/2)/f*(1/4/Pi^(1/2)*x^(1/2)
*2^(1/2)*(I*f)^(3/2)/f*exp(f*x)-1/4/Pi^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(3/2)/f*
exp(-f*x)+1/8*(I*f)^(3/2)*2^(1/2)/f^(3/2)*erf(x^(1/2)*f^(1/2))-1/8*(I*f)^(3
/2)*2^(1/2)/f^(3/2)*erfi(x^(1/2)*f^(1/2)))
```

Maxima [B] time = 1.1235, size = 200, normalized size = 2.17

$$
\frac{8\,(dx)^{\frac{3}{2}}\cosh\left(fx\right)+\frac{f\left(\frac{3\sqrt{\pi}d^2\,\text{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f^2\sqrt{\frac{f}{d}}}-\frac{3\sqrt{\pi}d^2\,\text{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f^2\sqrt{-\frac{f}{d}}}-\frac{2\left(2\,(dx)^{\frac{3}{2}}df-3\sqrt{dx}d^2\right)e^{(fx)}}{f^2}-\frac{2\left(2\,(dx)^{\frac{3}{2}}df+3\sqrt{dx}d^2\right)e^{(-fx)}}{f^2}\right)}{d}}{12\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(8*(d*x)^(3/2)*cosh(f*x) + f*(3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(f/d))/
(f^2*sqrt(f/d)) - 3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(-f/d))/(f^2*sqrt(-f/d))
 - 2*(2*(d*x)^(3/2)*d*f - 3*sqrt(d*x)*d^2)*e^(f*x)/f^2 - 2*(2*(d*x)^(3/2)*d
*f + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d)/d
```

Fricas [B] time = 1.84988, size = 355, normalized size = 3.86

$$\frac{\sqrt{\pi}\left(d\cosh\left(fx\right)+d\sinh\left(fx\right)\right)\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)+\sqrt{\pi}\left(d\cosh\left(fx\right)+d\sinh\left(fx\right)\right)\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)+2\left(f\cosh\left(fx\right.\right.}{4\left(f^{2}\cosh\left(fx\right)+f^{2}\sinh\left(fx\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d)
) + sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqrt(-f/d
)) + 2*(f*cosh(f*x)^2 + 2*f*cosh(f*x)*sinh(f*x) + f*sinh(f*x)^2 - f)*sqrt(d
*x))/(f^2*cosh(f*x) + f^2*sinh(f*x))
```

Sympy [C] time = 3.5449, size = 100, normalized size = 1.09

$$\frac{3\sqrt{d}\sqrt{x}\sinh\left(fx\right)\Gamma\left(\frac{3}{4}\right)}{4f\Gamma\left(\frac{7}{4}\right)}-\frac{3\sqrt{2}\sqrt{\pi}\sqrt{d}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{8f^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)*(d*x)**(1/2),x)
```

```
[Out] 3*sqrt(d)*sqrt(x)*sinh(f*x)*gamma(3/4)/(4*f*gamma(7/4)) - 3*sqrt(2)*sqrt(pi
)*sqrt(d)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(
pi))*gamma(3/4)/(8*f**(3/2)*gamma(7/4))
```

Giac [A] time = 1.21625, size = 138, normalized size = 1.5

$$-\frac{\frac{\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f}-\frac{\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f}-\frac{2\sqrt{dx}de^{\left(fx\right)}}{f}+\frac{2\sqrt{dx}de^{\left(-fx\right)}}{f}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(pi)*d^2*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f) - sqrt(pi)*d^2
*erf(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*f)*f) - 2*sqrt(d*x)*d*e^(f*x)/f + 2*
sqrt(d*x)*d*e^(-f*x)/f)/d
```

3.65 $\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{\pi}\text{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\text{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] (Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f]) + (Sqrt[Pi]
*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.0774998, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\text{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\text{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[f*x]/Sqrt[d*x],x]
```

```
[Out] (Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f]) + (Sqrt[Pi]
*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx = \frac{1}{2}\int \frac{e^{-fx}}{\sqrt{dx}}\,dx + \frac{1}{2}\int \frac{e^{fx}}{\sqrt{dx}}\,dx$$
$$= \frac{\operatorname{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{d} + \frac{\operatorname{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{d}$$
$$= \frac{\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] time = 0.0077995, size = 48, normalized size = 0.62

$$\frac{\sqrt{-fx}\operatorname{Gamma}\left(\frac{1}{2}, -fx\right) - \sqrt{fx}\operatorname{Gamma}\left(\frac{1}{2}, fx\right)}{2f\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[f*x]/Sqrt[d*x],x]
```

```
[Out] (Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] - Sqrt[f*x]*Gamma[1/2, f*x])/(2*f*Sqrt[d*x
])
```

Maple [C] time = 0.022, size = 72, normalized size = 0.9

$$\frac{-\frac{i}{2}\sqrt{\pi}\sqrt{2}}{f}\sqrt{x}\sqrt{if}\left(\frac{\sqrt{2}}{2}\sqrt{if}\operatorname{Erf}\left(\sqrt{x}\sqrt{f}\right)\frac{1}{\sqrt{f}} + \frac{\sqrt{2}}{2}\sqrt{if}\operatorname{erfi}\left(\sqrt{x}\sqrt{f}\right)\frac{1}{\sqrt{f}}\right)\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)/(d*x)^(1/2),x)
```

```
[Out] -1/2*I*Pi^(1/2)/(d*x)^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(1/2)/f*(1/2*(I*f)^(1/2)*
2^(1/2)/f^(1/2)*erf(x^(1/2)*f^(1/2))+1/2*(I*f)^(1/2)*2^(1/2)/f^(1/2)*erfi(x
^(1/2)*f^(1/2)))
```

Maxima [B] time = 1.05695, size = 158, normalized size = 2.05

$$\frac{4\sqrt{dx}\cosh\left(fx\right) - \frac{\left(\frac{2\sqrt{dx}de^{(fx)}}{f} + \frac{2\sqrt{dx}de^{(-fx)}}{f} - \frac{\sqrt{\pi}d\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f\sqrt{\frac{f}{d}}} - \frac{\sqrt{\pi}d\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f\sqrt{-\frac{f}{d}}}\right)f}{d}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(d*x)*cosh(f*x) - (2*sqrt(d*x)*d*e^(f*x)/f + 2*sqrt(d*x)*d*e^(-f
*x)/f - sqrt(pi)*d*erf(sqrt(d*x)*sqrt(f/d))/(f*sqrt(f/d)) - sqrt(pi)*d*erf(
sqrt(d*x)*sqrt(-f/d))/(f*sqrt(-f/d)))*f/d)/d
```

Fricas [A] time = 1.83252, size = 136, normalized size = 1.77

$$\frac{\sqrt{\pi}\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)-\sqrt{\pi}\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(pi)*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d)) - sqrt(pi)*sqrt(-f/d)*erf(
sqrt(d*x)*sqrt(-f/d)))/f
```

Sympy [C] time = 1.64281, size = 66, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{\pi}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{4\sqrt{d}\sqrt{f}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/
sqrt(pi))*gamma(1/4)/(4*sqrt(d)*sqrt(f)*gamma(5/4))
```

Giac [A] time = 1.29224, size = 81, normalized size = 1.05

$$-\frac{\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}}+\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(pi)*d*erf(-sqrt(d*f)*sqrt(d*x)/d)/sqrt(d*f) + sqrt(pi)*d*erf(-sq
rt(-d*f)*sqrt(d*x)/d)/sqrt(-d*f))/d
```

3.66 $\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{\sqrt{\pi}\sqrt{f}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\pi}\sqrt{f}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cosh(fx)}{d\sqrt{dx}}$$

```
[Out] (-2*Cosh[f*x])/(d*Sqrt[d*x]) - (Sqrt[f]*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sq
rt[d]])/d^(3/2) + (Sqrt[f]*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/d^(3
/2)
```

Rubi [A] time = 0.112532, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3297, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{f}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\pi}\sqrt{f}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cosh(fx)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[f*x]/(d*x)^(3/2),x]
```

```
[Out] (-2*Cosh[f*x])/(d*Sqrt[d*x]) - (Sqrt[f]*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sq
rt[d]])/d^(3/2) + (Sqrt[f]*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/d^(3
/2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx &= -\frac{2\cosh(fx)}{d\sqrt{dx}}+\frac{(2f)\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx}{d}\\
&= -\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{f\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{d}+\frac{f\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{d}\\
&= -\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{(2f)\,\mathrm{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx,x,\sqrt{dx}\right)}{d^2}+\frac{(2f)\,\mathrm{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx,x,\sqrt{dx}\right)}{d^2}\\
&= -\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{\sqrt{f}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{f}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0342088, size = 67, normalized size = 0.76

$$\frac{xe^{-fx}\left(e^{fx}\sqrt{-fx}\,\mathrm{Gamma}\left(\frac{1}{2},-fx\right)+e^{fx}\sqrt{fx}\,\mathrm{Gamma}\left(\frac{1}{2},fx\right)-e^{2fx}-1\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[f*x]/(d*x)^(3/2),x]
```

```
[Out] (x*(-1 - E^(2*f*x) + E^(f*x)*Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] + E^(f*x)*Sqrt
[f*x]*Gamma[1/2, f*x]))/(E^(f*x)*(d*x)^(3/2))
```

Maple [C] time = 0.022, size = 115, normalized size = 1.3

$$\frac{-\frac{i}{4}\sqrt{\pi}\sqrt{2}}{f}x^{\frac{3}{2}}\left(if\right)^{\frac{3}{2}}\left(-2\frac{\sqrt{2}e^{fx}}{\sqrt{\pi}\sqrt{x}\sqrt{if}}-2\frac{\sqrt{2}e^{-fx}}{\sqrt{\pi}\sqrt{x}\sqrt{if}}-2\frac{\sqrt{2}\sqrt{f}\,\mathrm{Erf}\left(\sqrt{x}\sqrt{f}\right)}{\sqrt{if}}+2\frac{\sqrt{2}\sqrt{f}\,\mathrm{erfi}\left(\sqrt{x}\sqrt{f}\right)}{\sqrt{if}}\right)(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)/(d*x)^(3/2),x)
```

```
[Out] -1/4*I*Pi^(1/2)/(d*x)^(3/2)*x^(3/2)*2^(1/2)*(I*f)^(3/2)/f*(-2/Pi^(1/2)/x^(1
/2)*2^(1/2)/(I*f)^(1/2)*exp(f*x)-2/Pi^(1/2)/x^(1/2)*2^(1/2)/(I*f)^(1/2)*exp
(-f*x)-2/(I*f)^(1/2)*2^(1/2)*f^(1/2)*erf(x^(1/2)*f^(1/2))+2/(I*f)^(1/2)*2^(
1/2)*f^(1/2)*erfi(x^(1/2)*f^(1/2)))
```

Maxima [A] time = 1.04295, size = 103, normalized size = 1.17

$$-\frac{\frac{f\left(\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{\sqrt{\frac{f}{d}}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{\sqrt{-\frac{f}{d}}}\right)}{d}+\frac{2\cosh(fx)}{\sqrt{dx}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -(f*(sqrt(pi)*erf(sqrt(d*x)*sqrt(f/d))/sqrt(f/d) - sqrt(pi)*erf(sqrt(d*x)*s
qrt(-f/d))/sqrt(-f/d))/d + 2*cosh(f*x)/sqrt(d*x))/d
```

Fricas [B] time = 1.79208, size = 356, normalized size = 4.05

$$-\frac{\sqrt{\pi}\left(dx\cosh\left(fx\right)+dx\sinh\left(fx\right)\right)\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)+\sqrt{\pi}\left(dx\cosh\left(fx\right)+dx\sinh\left(fx\right)\right)\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)+\sqrt{dx}\left(\mathrm{co}\right.}{d^2x\cosh\left(fx\right)+d^2x\sinh\left(fx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d
)) + sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqrt
(-f/d)) + sqrt(d*x)*(cosh(f*x)^2 + 2*cosh(f*x)*sinh(f*x) + sinh(f*x)^2 + 1)
)/(d^2*x*cosh(f*x) + d^2*x*sinh(f*x))
```

Sympy [C] time = 7.4263, size = 99, normalized size = 1.12

$$-\frac{\sqrt{2}\sqrt{\pi}\sqrt{f}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{2d^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{\cosh\left(fx\right)\Gamma\left(-\frac{1}{4}\right)}{2d^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)**(3/2),x)
```

```
[Out] -sqrt(2)*sqrt(pi)*sqrt(f)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*e
xp(I*pi/4)/sqrt(pi))*gamma(-1/4)/(2*d**(3/2)*gamma(3/4)) + cosh(f*x)*gamma(
-1/4)/(2*d**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(fx\right)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x)/(d*x)^(3/2), x)
```

3.67 $\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2\sqrt{\pi}f^{3/2}\text{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{\pi}f^{3/2}\text{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4f\sinh(fx)}{3d^2\sqrt{dx}}-\frac{2\cosh(fx)}{3d(dx)^{3/2}}$$

```
[Out] (-2*Cosh[f*x])/(3*d*(d*x)^(3/2)) + (2*f^(3/2)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*
x])/Sqrt[d]])/(3*d^(5/2)) + (2*f^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sq
rt[d]])/(3*d^(5/2)) - (4*f*Sinh[f*x])/(3*d^2*Sqrt[d*x])
```

Rubi [A] time = 0.14846, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3297, 3307, 2180, 2204, 2205}

$$\frac{2\sqrt{\pi}f^{3/2}\text{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{\pi}f^{3/2}\text{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4f\sinh(fx)}{3d^2\sqrt{dx}}-\frac{2\cosh(fx)}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[f*x]/(d*x)^(5/2),x]
```

```
[Out] (-2*Cosh[f*x])/(3*d*(d*x)^(3/2)) + (2*f^(3/2)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*
x])/Sqrt[d]])/(3*d^(5/2)) + (2*f^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sq
rt[d]])/(3*d^(5/2)) - (4*f*Sinh[f*x])/(3*d^2*Sqrt[d*x])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx &= -\frac{2\cosh(fx)}{3d(dx)^{3/2}}+\frac{(2f)\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx}{3d}\\
&= -\frac{2\cosh(fx)}{3d(dx)^{3/2}}-\frac{4f\sinh(fx)}{3d^2\sqrt{dx}}+\frac{\left(4f^2\right)\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx}{3d^2}\\
&= -\frac{2\cosh(fx)}{3d(dx)^{3/2}}-\frac{4f\sinh(fx)}{3d^2\sqrt{dx}}+\frac{\left(2f^2\right)\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{3d^2}+\frac{\left(2f^2\right)\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{3d^2}\\
&= -\frac{2\cosh(fx)}{3d(dx)^{3/2}}-\frac{4f\sinh(fx)}{3d^2\sqrt{dx}}+\frac{\left(4f^2\right)\operatorname{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx,x,\sqrt{dx}\right)}{3d^3}+\frac{\left(4f^2\right)\operatorname{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx,x,\sqrt{dx}\right)}{3d^3}\\
&= -\frac{2\cosh(fx)}{3d(dx)^{3/2}}+\frac{2f^{3/2}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2f^{3/2}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4f\sinh(fx)}{3d^2\sqrt{dx}}
\end{aligned}$$

Mathematica [A] time = 0.0930811, size = 78, normalized size = 0.68

$$\frac{x\left(-4(-fx)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},-fx\right)+e^{-fx}\left(-4e^{fx}(fx)^{3/2}\mathrm{Gamma}\left(\frac{1}{2},fx\right)+4fx-2\right)-2e^{fx}(2fx+1)\right)}{6(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[f*x]/(d*x)^(5/2),x]
```

```
[Out] (x*(-2*E^(f*x)*(1 + 2*f*x) - 4*(-(f*x))^(3/2)*Gamma[1/2, -(f*x)] + (-2 + 4*
f*x - 4*E^(f*x)*(f*x)^(3/2)*Gamma[1/2, f*x])/E^(f*x)))/(6*(d*x)^(5/2))
```

Maple [C] time = 0.027, size = 126, normalized size = 1.1

$$\frac{-\frac{i}{8}\sqrt{\pi}\sqrt{2}}{f}x^{\frac{5}{2}}\left(if\right)^{\frac{5}{2}}\left(-\frac{8\sqrt{2}\mathrm{e}^{-fx}}{3\sqrt{\pi}}\left(-fx+\frac{1}{2}\right)x^{-\frac{3}{2}}\left(if\right)^{-\frac{3}{2}}-\frac{8\sqrt{2}\mathrm{e}^{fx}}{3\sqrt{\pi}}\left(fx+\frac{1}{2}\right)x^{-\frac{3}{2}}\left(if\right)^{-\frac{3}{2}}+\frac{8\sqrt{2}}{3}f^{\frac{3}{2}}\mathrm{Erf}\left(\sqrt{x}\sqrt{f}\right)\left(if\right)^{-\frac{3}{2}}+\frac{8}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)/(d*x)^(5/2),x)
```

```
[Out] -1/8*I*Pi^(1/2)/(d*x)^(5/2)*x^(5/2)*2^(1/2)*(I*f)^(5/2)/f*(-8/3/Pi^(1/2)/x^
(3/2)*2^(1/2)/(I*f)^(3/2)*(-f*x+1/2)*exp(-f*x)-8/3/Pi^(1/2)/x^(3/2)*2^(1/2)
/(I*f)^(3/2)*(f*x+1/2)*exp(f*x)+8/3/(I*f)^(3/2)*2^(1/2)*f^(3/2)*erf(x^(1/2)
*f^(1/2))+8/3/(I*f)^(3/2)*2^(1/2)*f^(3/2)*erfi(x^(1/2)*f^(1/2)))
```

Maxima [A] time = 1.26112, size = 78, normalized size = 0.68

$$\frac{\frac{f\left(\frac{\sqrt{fx}\,\Gamma\left(-\frac{1}{2},fx\right)}{\sqrt{dx}}-\frac{\sqrt{-fx}\,\Gamma\left(-\frac{1}{2},-fx\right)}{\sqrt{dx}}\right)}{d}-\frac{2\cosh\left(fx\right)}{\left(dx\right)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(f*(sqrt(f*x)*gamma(-1/2, f*x)/sqrt(d*x) - sqrt(-f*x)*gamma(-1/2, -f*x)
/sqrt(d*x))/d - 2*cosh(f*x)/(d*x)^(3/2))/d
```

Fricas [B] time = 1.85452, size = 452, normalized size = 3.96

$$\frac{2\sqrt{\pi}\left(dfx^2\cosh\left(fx\right)+dfx^2\sinh\left(fx\right)\right)\sqrt{\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)-2\sqrt{\pi}\left(dfx^2\cosh\left(fx\right)+dfx^2\sinh\left(fx\right)\right)\sqrt{-\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\cdots}\right.}{3\left(d^3x^2\cosh\left(fx\right)+d^3x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqrt(f/d)*erf(sqrt(
d*x)*sqrt(f/d)) - 2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqrt(-
f/d)*erf(sqrt(d*x)*sqrt(-f/d)) - ((2*f*x + 1)*cosh(f*x)^2 + 2*(2*f*x + 1)*c
osh(f*x)*sinh(f*x) + (2*f*x + 1)*sinh(f*x)^2 - 2*f*x + 1)*sqrt(d*x))/(d^3*x
^2*cosh(f*x) + d^3*x^2*sinh(f*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx\right)}{\left(dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x)/(d*x)^(5/2), x)
```

3.68 $\int \sqrt{c+dx}\,\mathrm{sech}(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\sqrt{c+dx}\,\text{sech}(a+bx),x\right)$$

```
[Out] Unintegrable[Sqrt[c + d*x]*Sech[a + b*x], x]
```

Rubi [A] time = 0.0283211, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{c+dx}\,\text{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + d*x]*Sech[a + b*x],x]
```

```
[Out] Defer[Int][Sqrt[c + d*x]*Sech[a + b*x], x]
```

Rubi steps

$$\int \sqrt{c+dx}\,\text{sech}(a+bx)\,dx = \int \sqrt{c+dx}\,\text{sech}(a+bx)\,dx$$

Mathematica [A] time = 11.1617, size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\,\text{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x]*Sech[a + b*x],x]
```

```
[Out] Integrate[Sqrt[c + d*x]*Sech[a + b*x], x]
```

Maple [A] time = 0.068, size = 0, normalized size = 0.

$$\int \text{sech}\,(bx+a)\,\sqrt{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] int(sech(b*x+a)*(d*x+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx+c}\,\mathrm{sech}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x + c)*sech(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{dx+c}\,\mathrm{sech}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x + c)*sech(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\,\mathrm{sech}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sech(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx+c}\,\mathrm{sech}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)*sech(b*x + a), x)
```

3.69 $\int \frac{\mathrm{sech}(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\text{sech}(a+bx)}{\sqrt{c+dx}},x\right)$$

```
[Out] Unintegrable[Sech[a + b*x]/Sqrt[c + d*x], x]
```

Rubi [A] time = 0.0295777, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{sech}(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] Defer[Int][Sech[a + b*x]/Sqrt[c + d*x], x]
```

Rubi steps

$$\int \frac{\text{sech}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\text{sech}(a+bx)}{\sqrt{c+dx}}\,dx$$

Mathematica [A] time = 9.8447, size = 0, normalized size = 0.

$$\int \frac{\text{sech}(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] Integrate[Sech[a + b*x]/Sqrt[c + d*x], x]
```

Maple [A] time = 0.064, size = 0, normalized size = 0.

$$\int \text{sech}\,(bx+a)\,\frac{1}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] int(sech(b*x+a)/(d*x+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)/sqrt(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)}{\sqrt{dx+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)/sqrt(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sech(a + b*x)/sqrt(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(bx+a)}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)/sqrt(d*x + c), x)
```

3.70 $\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx$

Optimal. Leaf size=61

$$\frac{9}{8}\text{Unintegrable}\left(\frac{\cosh^{\frac{3}{2}}(x)}{x},x\right)-\frac{3}{8}\text{Unintegrable}\left(\frac{1}{x\sqrt{\cosh(x)}},x\right)-\frac{\cosh^{\frac{3}{2}}(x)}{2x^2}-\frac{3\sinh(x)\sqrt{\cosh(x)}}{4x}$$

```
[Out] -Cosh[x]^(3/2)/(2*x^2) - (3*Sqrt[Cosh[x]]*Sinh[x])/(4*x) - (3*Unintegrable[
1/(x*Sqrt[Cosh[x]]), x])/8 + (9*Unintegrable[Cosh[x]^(3/2)/x, x])/8
```

Rubi [A] time = 0.0853537, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cosh[x]^(3/2)/x^3,x]
```

```
[Out] -Cosh[x]^(3/2)/(2*x^2) - (3*Sqrt[Cosh[x]]*Sinh[x])/(4*x) - (3*Defer[Int][1/
(x*Sqrt[Cosh[x]]), x])/8 + (9*Defer[Int][Cosh[x]^(3/2)/x, x])/8
```

Rubi steps

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx=-\frac{\cosh^{\frac{3}{2}}(x)}{2x^2}-\frac{3\sqrt{\cosh(x)}\sinh(x)}{4x}-\frac{3}{8}\int \frac{1}{x\sqrt{\cosh(x)}}\,dx+\frac{9}{8}\int \frac{\cosh^{\frac{3}{2}}(x)}{x}\,dx$$

Mathematica [A] time = 3.84929, size = 0, normalized size = 0.

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[x]^(3/2)/x^3,x]
```

```
[Out] Integrate[Cosh[x]^(3/2)/x^3, x]
```

Maple [A] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}(\cosh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^(3/2)/x^3,x)
```

```
[Out] int(cosh(x)^(3/2)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)^(3/2)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(x)^(3/2)/x^3, x)
```

3.71 $\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)dx$

Optimal. Leaf size=20

$$\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}-4\sqrt{\cosh(x)}$$

```
[Out] -4*Sqrt[Cosh[x]] + (2*x*Sinh[x])/Sqrt[Cosh[x]]
```

Rubi [A] time = 0.047739, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3315}

$$\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}-4\sqrt{\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

```
[Out] -4*Sqrt[Cosh[x]] + (2*x*Sinh[x])/Sqrt[Cosh[x]]
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)dx &= \int \frac{x}{\cosh^{\frac{3}{2}}(x)}\,dx+\int x\sqrt{\cosh(x)}\,dx\\ &= -4\sqrt{\cosh(x)}+\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}\end{aligned}$$

Mathematica [B] time = 0.372303, size = 46, normalized size = 2.3

$$\frac{2\sinh(x)\left(x-\frac{2\sinh(x)\cosh(x)\sqrt{\tanh^2\left(\frac{x}{2}\right)}}{(\cosh(x)-1)^{3/2}\sqrt{\cosh(x)+1}}\right)}{\sqrt{\cosh(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

```
[Out] (2*Sinh[x]*(x - (2*Cosh[x]*Sinh[x]*Sqrt[Tanh[x/2]^2])/((-1 + Cosh[x])^(3/2)
*Sqrt[1 + Cosh[x]])))/Sqrt[Cosh[x]]
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int x\left(\cosh\left(x\right)\right)^{-\frac{3}{2}}+x\sqrt{\cosh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cosh\left(x\right)}+\frac{x}{\cosh\left(x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(\cosh^{2}\left(x\right)+1\right)}{\cosh^{\frac{3}{2}}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(3/2)+x*cosh(x)**(1/2),x)
```

```
[Out] Integral(x*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cosh\left(x\right)}+\frac{x}{\cosh\left(x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

3.72 $\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right) dx$

Optimal. Leaf size=24

$$\frac{4}{3\sqrt{\cosh(x)}}+\frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

```
[Out] 4/(3*Sqrt[Cosh[x]]) + (2*x*Sinh[x])/(3*Cosh[x]^(3/2))
```

Rubi [A] time = 0.0499309, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3315}

$$\frac{4}{3\sqrt{\cosh(x)}}+\frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

```
[Out] 4/(3*Sqrt[Cosh[x]]) + (2*x*Sinh[x])/(3*Cosh[x]^(3/2))
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right) dx &= -\left(\frac{1}{3}\int \frac{x}{\sqrt{\cosh(x)}}\,dx\right)+\int \frac{x}{\cosh^{\frac{5}{2}}(x)}\,dx\\ &= \frac{4}{3\sqrt{\cosh(x)}}+\frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A] time = 0.074049, size = 16, normalized size = 0.67

$$\frac{2(x\tanh(x)+2)}{3\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

```
[Out] (2*(2 + x*Tanh[x]))/(3*Sqrt[Cosh[x]])
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x(\cosh(x))^{-\frac{5}{2}} - \frac{x}{3}\frac{1}{\sqrt{\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{3\sqrt{\cosh(x)}} + \frac{x}{\cosh(x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

Fricas [B] time = 1.75972, size = 374, normalized size = 15.58

$$\frac{4\left((x+2)\cosh(x)^3 + 3(x+2)\cosh(x)\sinh(x)^2 + (x+2)\sinh(x)^3 - (x-2)\cosh(x) + \left(3(x+2)\cosh(x)^2 - x + 2\right)\sin\right.}{3\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 + 1\right)\sinh(x)^2 + 2\cosh(x)^2 + 4\left(\cosh(x)^3 + \cosh(x)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3 -
(x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 - x + 2)*sinh(x))*sqrt(cosh(x))/(cos
h(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 +
2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(5/2)-1/3*x/cosh(x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{3\sqrt{\cosh(x)}} + \frac{x}{\cosh(x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

3.73 $\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\cosh(x)} \right) dx$

Optimal. Leaf size=47

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)} - \frac{12\sqrt{\cosh(x)}}{5} + \frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)} + \frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

```
[Out] 4/(15*Cosh[x]^(3/2)) - (12*Sqrt[Cosh[x]])/5 + (2*x*Sinh[x])/(5*Cosh[x]^(5/2
)) + (6*x*Sinh[x])/(5*Sqrt[Cosh[x]])
```

Rubi [A] time = 0.0657676, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3315}

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)} - \frac{12\sqrt{\cosh(x)}}{5} + \frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)} + \frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

```
[Out] 4/(15*Cosh[x]^(3/2)) - (12*Sqrt[Cosh[x]])/5 + (2*x*Sinh[x])/(5*Cosh[x]^(5/2
)) + (6*x*Sinh[x])/(5*Sqrt[Cosh[x]])
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned} \int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\cosh(x)} \right) dx &= \frac{3}{5}\int x\sqrt{\cosh(x)}\,dx + \int \frac{x}{\cosh^{\frac{7}{2}}(x)}\,dx \\ &= \frac{4}{15\cosh^{\frac{3}{2}}(x)} + \frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)} + \frac{3}{5}\int \frac{x}{\cosh^{\frac{3}{2}}(x)}\,dx + \frac{3}{5}\int x\sqrt{\cosh(x)}\,dx \\ &= \frac{4}{15\cosh^{\frac{3}{2}}(x)} - \frac{12\sqrt{\cosh(x)}}{5} + \frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)} + \frac{6x\sinh(x)}{5\sqrt{\cosh(x)}} \end{aligned}$$

Mathematica [A] time = 0.61568, size = 64, normalized size = 1.36

$$\frac{1}{5}\sqrt{\cosh(x)}\left(6x\tanh(x) + \left(2x\tanh(x) + \frac{4}{3}\right)\operatorname{sech}^2(x) - \frac{12\sinh^2(x)}{\sqrt{\cosh(x)-1}(\cosh(x)+1)^{3/2}\sqrt{\tanh^2\left(\frac{x}{2}\right)}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

```
[Out] (Sqrt[Cosh[x]]*((-12*Sinh[x]^2)/(Sqrt[-1 + Cosh[x]]*(1 + Cosh[x])^(3/2)*Sqr
t[Tanh[x/2]^2]) + 6*x*Tanh[x] + Sech[x]^2*(4/3 + 2*x*Tanh[x])))/5
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int x (\cosh (x))^{-\frac{7}{2}} + \frac{3 x}{5} \sqrt{\cosh (x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3}{5} x \sqrt{\cosh (x)} + \frac{x}{\cosh (x)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(7/2)+3/5*x*cosh(x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3}{5}x\sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

3.74 $\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right) dx$

Optimal. Leaf size=36

$$\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)$$

```
[Out] -8*x*Sqrt[Cosh[x]] - (16*I)*EllipticE[(I/2)*x, 2] + (2*x^2*Sinh[x])/Sqrt[Co
sh[x]]
```

Rubi [A] time = 0.0918835, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3316, 2639}

$$\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

```
[Out] -8*x*Sqrt[Cosh[x]] - (16*I)*EllipticE[(I/2)*x, 2] + (2*x^2*Sinh[x])/Sqrt[Co
sh[x]]
```

Rule 3316

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[((c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1
)), x] + (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n +
 2), x], x] + Dist[(d^2*m*(m - 1))/(b^2*f^2*(n + 1)*(n + 2)), Int[(c + d*x)
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] - Simp[(d*m*(c + d*x)^(m - 1)*(b*
Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; FreeQ[{b, c, d, e,
 f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right) dx &= \int \frac{x^2}{\cosh^{\frac{3}{2}}(x)}\,dx+\int x^2\sqrt{\cosh(x)}\,dx\\ &= -8x\sqrt{\cosh(x)}+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}+8\int \sqrt{\cosh(x)}\,dx\\ &= -8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}\end{aligned}$$

Mathematica [C] time = 0.946524, size = 76, normalized size = 2.11

$$\frac{4\sqrt{\cosh(x)}(\sinh(x)+\cosh(x))\left(8\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2x}\right)(\sinh(x)-\cosh(x))\sqrt{\sinh(2x)+\cosh(2x)+1}+x^2\sinh(x)-4(\right.}{e^{2x}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

```
[Out] (4*Sqrt[Cosh[x]]*(Cosh[x] + Sinh[x])*(-4*(-2 + x)*Cosh[x] + x^2*Sinh[x] + 8
*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(2*x)]*(-Cosh[x] + Sinh[x])*Sqrt[1 +
Cosh[2*x] + Sinh[2*x]]))/(1 + E^(2*x))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int x^{2} \left(\cosh \left(x\right)\right)^{-\frac{3}{2}}+x^{2} \sqrt{\cosh \left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

```
[Out] int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2} \sqrt{\cosh \left(x\right)} + \frac{x^{2}}{\cosh \left(x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2} \left(\cosh^{2}{\left(x \right)} + 1\right)}{\cosh^{\frac{3}{2}}{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/cosh(x)**(3/2)+x**2*cosh(x)**(1/2),x)
```

```
[Out] Integral(x**2*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\cosh(x)} + \frac{x^2}{\cosh(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

3.75 $\int (c+dx)^m (b\cosh(e+fx))^n \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((c+dx)^m(b\cosh(e+fx))^n, x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*(b*Cosh[e + f*x])^n, x]
```

Rubi [A] time = 0.0458665, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m (b\cosh(e+fx))^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(b*Cosh[e + f*x])^n,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(b*Cosh[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m (b\cosh(e+fx))^n \, dx = \int (c+dx)^m (b\cosh(e+fx))^n \, dx$$

Mathematica [A] time = 2.78959, size = 0, normalized size = 0.

$$\int (c+dx)^m (b\cosh(e+fx))^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(b*Cosh[e + f*x])^n,x]
```

```
[Out] Integrate[(c + d*x)^m*(b*Cosh[e + f*x])^n, x]
```

Maple [A] time = 0.072, size = 0, normalized size = 0.

$$\int (dx+c)^m \left(b\cosh\left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(b*cosh(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(b*cosh(f*x+e))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \left(b\cosh\left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*cosh(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*(b*cosh(f*x + e))^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(dx+c\right)^m \left(b\cosh\left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*cosh(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*(b*cosh(f*x + e))^n, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cosh\left(e+fx\right)\right)^n (c+dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(b*cosh(f*x+e))**n,x)
```

```
[Out] Integral((b*cosh(e + f*x))**n*(c + d*x)**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \left(b\cosh\left(fx+e\right)\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*cosh(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*(b*cosh(f*x + e))^n, x)
```

3.76 $\int (c+dx)^m \cosh^3(a+bx)\,dx$

Optimal. Leaf size=237

$$\frac{3^{-m-1}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3b(c+dx)}{d}\right)}{8b}+\frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+dx)}{d}\right)}{8b}$$

```
[Out] (3^(-1 - m)*E^(3*a - (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (-3*b*(c + d*x))/d
])/(8*b*(-((b*(c + d*x))/d))^m) + (3*E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 +
m, -((b*(c + d*x))/d)])/(8*b*(-((b*(c + d*x))/d))^m) - (3*E^(-a + (b*c)/d)*
(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(8*b*((b*(c + d*x))/d)^m) - (3^(
-1 - m)*E^(-3*a + (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (3*b*(c + d*x))/d])/(
8*b*((b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.28378, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3312, 3307, 2181}

$$\frac{3^{-m-1}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3b(c+dx)}{d}\right)}{8b}+\frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+dx)}{d}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cosh[a + b*x]^3,x]
```

```
[Out] (3^(-1 - m)*E^(3*a - (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (-3*b*(c + d*x))/d
])/(8*b*(-((b*(c + d*x))/d))^m) + (3*E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 +
m, -((b*(c + d*x))/d)])/(8*b*(-((b*(c + d*x))/d))^m) - (3*E^(-a + (b*c)/d)*
(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(8*b*((b*(c + d*x))/d)^m) - (3^(
-1 - m)*E^(-3*a + (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (3*b*(c + d*x))/d])/(
8*b*((b*(c + d*x))/d)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \cosh^3(a+bx)\,dx = \int \left(\frac{3}{4}(c+dx)^m \cosh(a+bx) + \frac{1}{4}(c+dx)^m \cosh(3a+3bx)\right) dx$$
$$= \frac{1}{4}\int (c+dx)^m \cosh(3a+3bx)\,dx + \frac{3}{4}\int (c+dx)^m \cosh(a+bx)\,dx$$
$$= \frac{1}{8}\int e^{-i(3ia+3ibx)}(c+dx)^m\,dx + \frac{1}{8}\int e^{i(3ia+3ibx)}(c+dx)^m\,dx + \frac{3}{8}\int e^{-i(ia+ibx)}(c+dx)^m\,d$$
$$= \frac{3^{-1-m}e^{3a-\frac{3bc}{d}}(c+dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)}{8b} + \frac{3e^{a-\frac{bc}{d}}(c+dx)^m \left(-\frac{b(c+dx)}{d}\right)}{8b}$$

Mathematica [A] time = 0.188269, size = 205, normalized size = 0.86

$$\frac{3^{-m-1}e^{-3\left(a+\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{b^2(c+dx)^2}{d^2}\right)^{-m} \left(e^{6a}\left(b\left(\frac{c}{d}+x\right)\right)^m \text{Gamma}\left(m+1,-\frac{3b(c+dx)}{d}\right) + 3^{m+2}e^{4a+\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right)\right)^m \text{Gamm}\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cosh[a + b*x]^3,x]
```

```
[Out] (3^(-1 - m)*(c + d*x)^m*(E^(6*a)*(b*(c/d + x))^m*Gamma[1 + m, (-3*b*(c + d*
x))/d] + 3^(2 + m)*E^(4*a + (2*b*c)/d)*(b*(c/d + x))^m*Gamma[1 + m, -((b*(c
 + d*x))/d)] - E^((4*b*c)/d)*(-((b*(c + d*x))/d))^m*(3^(2 + m)*E^(2*a)*Gamm
a[1 + m, (b*(c + d*x))/d] + E^((2*b*c)/d)*Gamma[1 + m, (3*b*(c + d*x))/d]))
)/(8*b*E^(3*(a + (b*c)/d))*(-((b^2*(c + d*x)^2)/d^2))^m)
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cosh(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cosh(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cosh(b*x+a)^3,x)
```

Maxima [A] time = 1.27982, size = 217, normalized size = 0.92

$$-\frac{(dx+c)^{m+1}e^{\left(-3a+\frac{3bc}{d}\right)}E_{-m}\left(\frac{3(dx+c)b}{d}\right)}{8d} - \frac{3(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{8d} - \frac{3(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{8d} - \frac{(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(d*x + c)^(m + 1)*e^(-3*a + 3*b*c/d)*exp_integral_e(-m, 3*(d*x + c)*b/
d)/d - 3/8*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b/
d)/d - 3/8*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*b/
d)/d - 1/8*(d*x + c)^(m + 1)*e^(3*a - 3*b*c/d)*exp_integral_e(-m, -3*(d*x +
```

```
c)*b/d)/d
```

Fricas [A] time = 1.94804, size = 801, normalized size = 3.38

$$-\frac{\cosh\left(\frac{dm\log\left(\frac{3b}{d}\right)-3bc+3ad}{d}\right)\Gamma\left(m+1,\frac{3(bdx+bc)}{d}\right)+9\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)-9\cosh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)\Gamma\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(cosh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d)*gamma(m + 1, 3*(b*d*x + b*c
)/d) + 9*cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) -
 9*cosh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) - cos
h((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d)*gamma(m + 1, -3*(b*d*x + b*c)/d) - g
amma(m + 1, 3*(b*d*x + b*c)/d)*sinh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d) - 9
*gamma(m + 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) + 9*gamma
(m + 1, -(b*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d) + gamma(m + 1
, -3*(b*d*x + b*c)/d)*sinh((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cosh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cosh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cosh(b*x + a)^3, x)
```

3.77 $\int (c+dx)^m \cosh^2(a+bx)\,dx$

Optimal. Leaf size=144

$$\frac{2^{-m-3}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2b(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{\frac{2bc}{d}-2a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\right.}{b}$$

```
[Out] (c + d*x)^(1 + m)/(2*d*(1 + m)) + (2^(-3 - m)*E^(2*a - (2*b*c)/d)*(c + d*x)
^m*Gamma[1 + m, (-2*b*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) - (2^(-3 -
m)*E^(-2*a + (2*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (2*b*(c + d*x))/d])/(b*((b
*(c + d*x))/d)^m)
```

Rubi [A] time = 0.184035, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3312, 3307, 2181}

$$\frac{2^{-m-3}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2b(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{\frac{2bc}{d}-2a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cosh[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(2*d*(1 + m)) + (2^(-3 - m)*E^(2*a - (2*b*c)/d)*(c + d*x)
^m*Gamma[1 + m, (-2*b*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) - (2^(-3 -
m)*E^(-2*a + (2*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (2*b*(c + d*x))/d])/(b*((b
*(c + d*x))/d)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \cosh^2(a+bx)\,dx = \int \left(\frac{1}{2}(c+dx)^m + \frac{1}{2}(c+dx)^m \cosh(2a+2bx)\right) dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{2}\int (c+dx)^m \cosh(2a+2bx)\,dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{4}\int e^{-i(2ia+2ibx)}(c+dx)^m\,dx + \frac{1}{4}\int e^{i(2ia+2ibx)}(c+dx)^m\,dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-3-m}e^{-2a+\frac{2bc}{d}}(c}{}$$

Mathematica [A] time = 0.201685, size = 132, normalized size = 0.92

$$\frac{1}{8}(c+dx)^m\left(\frac{2^{-m}e^{2a-\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-m}e^{\frac{2bc}{d}-2a}\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2b(c+dx)}{d}\right)}{b} + \frac{4c}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cosh[a + b*x]^2,x]
```

```
[Out] ((c + d*x)^m*((4*c + 4*d*x)/(d + d*m) + (E^(2*a - (2*b*c)/d)*Gamma[1 + m, (
-2*b*(c + d*x))/d])/(2^m*b*(-((b*(c + d*x))/d))^m) - (E^(-2*a + (2*b*c)/d)*
Gamma[1 + m, (2*b*(c + d*x))/d])/(2^m*b*((b*(c + d*x))/d)^m)))/8
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cosh(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cosh(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cosh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79576, size = 597, normalized size = 4.15

$$-\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{2b}{d}\right)-2bc+2ad}{d}\right)\Gamma\left(m+1,\frac{2(bdx+bc)}{d}\right) - (dm+d)\cosh\left(\frac{dm\log\left(-\frac{2b}{d}\right)+2bc-2ad}{d}\right)\Gamma\left(m+1,-\frac{2(bdx+bc)}{d}\right) - (dm}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((d*m + d)*cosh((d*m*log(2*b/d) - 2*b*c + 2*a*d)/d)*gamma(m + 1, 2*(b*
d*x + b*c)/d) - (d*m + d)*cosh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d)*gamma(m
 + 1, -2*(b*d*x + b*c)/d) - (d*m + d)*gamma(m + 1, 2*(b*d*x + b*c)/d)*sinh(
(d*m*log(2*b/d) - 2*b*c + 2*a*d)/d) + (d*m + d)*gamma(m + 1, -2*(b*d*x + b*
c)/d)*sinh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d) - 4*(b*d*x + b*c)*cosh(m*lo
g(d*x + c)) - 4*(b*d*x + b*c)*sinh(m*log(d*x + c)))/(b*d*m + b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \cosh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cosh(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*cosh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cosh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cosh(b*x + a)^2, x)
```

3.78 $\int (c+dx)^m \cosh(a+bx)\,dx$

Optimal. Leaf size=110

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+dx)}{d}\right)}{2b}-\frac{e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{b(c+dx)}{d}\right)}{2b}$$

```
[Out] (E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 + m, -((b*(c + d*x))/d)])/(2*b*(-((b*(
c + d*x))/d))^m) - (E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))
/d])/(2*b*((b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.0913823, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3307, 2181}

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+dx)}{d}\right)}{2b}-\frac{e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{b(c+dx)}{d}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cosh[a + b*x],x]
```

```
[Out] (E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 + m, -((b*(c + d*x))/d)])/(2*b*(-((b*(
c + d*x))/d))^m) - (E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))
/d])/(2*b*((b*(c + d*x))/d)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m\cosh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}(c+dx)^m\,dx+\frac{1}{2}\int e^{i(ia+ibx)}(c+dx)^m\,dx\\ &= \frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)}{2b}-\frac{e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{b(c\right.}{}}{2b}\end{aligned}$$

Mathematica [A] time = 0.0558259, size = 102, normalized size = 0.93

$$\frac{e^{-a-\frac{bc}{d}}(c+dx)^m\left(e^{2a}\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+dx)}{d}\right)-e^{\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right)\right)^{-m}\text{Gamma}\left(m+1,\frac{b(c+dx)}{d}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cosh[a + b*x],x]
```

```
[Out] (E^(-a - (b*c)/d)*(c + d*x)^m*((E^(2*a)*Gamma[1 + m, -((b*(c + d*x))/d)])/(
-((b*(c + d*x))/d))^m - (E^((2*b*c)/d)*Gamma[1 + m, (b*(c + d*x))/d])/(b*(c
/d + x))^m))/(2*b)
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int (dx+c)^m \cosh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cosh(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cosh(b*x+a),x)
```

Maxima [A] time = 1.24449, size = 107, normalized size = 0.97

$$-\frac{(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{2\,d}-\frac{(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b/d)/d -
 1/2*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*b/d)/d
```

Fricas [A] time = 1.90513, size = 378, normalized size = 3.44

$$-\frac{\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)-\cosh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)-\Gamma\left(m+1,\frac{bdx+bc}{d}\right)\sinh\left(\frac{dm\log\left(\cdots\right.}{\cdots}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) - co
sh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) - gamma(m
+ 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) + gamma(m + 1, -(b
*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cosh(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cosh(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cosh(b*x + a), x)
```

3.79 $\int (c+dx)^m \mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\text{sech}(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Sech[a + b*x], x]
```

Rubi [A] time = 0.0202649, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \text{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sech[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Sech[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \text{sech}(a+bx)\,dx = \int (c+dx)^m \text{sech}(a+bx)\,dx$$

Mathematica [A] time = 5.65431, size = 0, normalized size = 0.

$$\int (c+dx)^m \text{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sech[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Sech[a + b*x], x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int (dx+c)^m \,\text{sech}\,(bx+a)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sech(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sech(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \,\text{sech}\,(bx+a)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sech(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(dx+c\right)^{m}\operatorname{sech}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sech(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\left(c+dx\right)^{m}\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sech(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*sech(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(dx+c\right)^{m}\operatorname{sech}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sech(b*x + a), x)
```

3.80 $\int (c+dx)^m \text{sech}^2(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\text{sech}^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Sech[a + b*x]^2, x]
```

Rubi [A] time = 0.0358342, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \text{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sech[a + b*x]^2,x]

[Out] Defer[Int][(c + d*x)^m*Sech[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \text{sech}^2(a+bx)\,dx = \int (c+dx)^m \text{sech}^2(a+bx)\,dx$$

Mathematica [A] time = 3.29465, size = 0, normalized size = 0.

$$\int (c+dx)^m \text{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sech[a + b*x]^2,x]

[Out] Integrate[(c + d*x)^m*Sech[a + b*x]^2, x]
```

Maple [A] time = 0.052, size = 0, normalized size = 0.

$$\int (dx+c)^m (\text{sech}\,(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sech(b*x+a)^2,x)

[Out] int((d*x+c)^m*sech(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \,\text{sech}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sech(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(dx+c\right)^{m}\operatorname{sech}\left(bx+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sech(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c+dx\right)^{m}\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sech(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*sech(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(dx+c\right)^{m}\operatorname{sech}\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sech(b*x + a)^2, x)
```

3.81 $\int x^{3+m} \cosh(a+bx)\,dx$

Optimal. Leaf size=59

$$-\frac{e^a x^m (-bx)^{-m} \text{Gamma}(m+4,-bx)}{2b^4}-\frac{e^{-a} x^m (bx)^{-m} \text{Gamma}(m+4,bx)}{2b^4}$$

```
[Out] -(E^a*x^m*Gamma[4 + m, -(b*x)])/(2*b^4*(-(b*x))^m) - (x^m*Gamma[4 + m, b*x]
)/(2*b^4*E^a*(b*x)^m)
```

Rubi [A] time = 0.0738787, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3307, 2181}

$$-\frac{e^a x^m (-bx)^{-m} \text{Gamma}(m+4,-bx)}{2b^4}-\frac{e^{-a} x^m (bx)^{-m} \text{Gamma}(m+4,bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Cosh[a + b*x],x]
```

```
[Out] -(E^a*x^m*Gamma[4 + m, -(b*x)])/(2*b^4*(-(b*x))^m) - (x^m*Gamma[4 + m, b*x]
)/(2*b^4*E^a*(b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{3+m} \cosh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)} x^{3+m}\,dx+\frac{1}{2}\int e^{i(ia+ibx)} x^{3+m}\,dx\\
&= -\frac{e^a x^m (-bx)^{-m} \Gamma(4+m,-bx)}{2b^4}-\frac{e^{-a} x^m (bx)^{-m} \Gamma(4+m,bx)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.0380859, size = 54, normalized size = 0.92

$$-\frac{e^a x^m (-bx)^{-m} \text{Gamma}(m+4,-bx)+e^{-a} x^m (bx)^{-m} \text{Gamma}(m+4,bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Cosh[a + b*x],x]
```

```
[Out] -((E^a*x^m*Gamma[4 + m, -(b*x)])/(-(b*x))^m + (x^m*Gamma[4 + m, b*x])/(E^a*
(b*x)^m))/(2*b^4)
```

Maple [C] time = 0.033, size = 73, normalized size = 1.2

$$\frac{x^{4+m}\cosh(a)}{4+m}\,{}_1F_2(2+\frac{m}{2};\frac{1}{2},3+\frac{m}{2};\frac{x^2b^2}{4})+\frac{bx^{5+m}\sinh(a)}{5+m}\,{}_1F_2(\frac{5}{2}+\frac{m}{2};\frac{3}{2},\frac{7}{2}+\frac{m}{2};\frac{x^2b^2}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3+m)*cosh(b*x+a),x)
```

```
[Out] 1/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[1/2,3+1/2*m],1/4*x^2*b^2)*cosh(a)+b/(5
+m)*x^(5+m)*hypergeom([5/2+1/2*m],[3/2,7/2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Maxima [A] time = 1.26048, size = 74, normalized size = 1.25

$$-\frac{1}{2}\,(bx)^{-m-4}\,x^{m+4}e^{(-a)}\Gamma\left(m+4,bx\right)-\frac{1}{2}\,(-bx)^{-m-4}\,x^{m+4}e^{a}\Gamma\left(m+4,-bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x)^(-m - 4)*x^(m + 4)*e^(-a)*gamma(m + 4, b*x) - 1/2*(-b*x)^(-m - 4
)*x^(m + 4)*e^a*gamma(m + 4, -b*x)
```

Fricas [A] time = 1.81808, size = 259, normalized size = 4.39

$$-\frac{\cosh\left((m+3)\log(b)+a\right)\Gamma\left(m+4,bx\right)-\cosh\left((m+3)\log(-b)-a\right)\Gamma\left(m+4,-bx\right)+\Gamma\left(m+4,-bx\right)\sinh\left((m+3)\log(\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((m + 3)*log(b) + a)*gamma(m + 4, b*x) - cosh((m + 3)*log(-b) - a
)*gamma(m + 4, -b*x) + gamma(m + 4, -b*x)*sinh((m + 3)*log(-b) - a) - gamma
(m + 4, b*x)*sinh((m + 3)*log(b) + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*cosh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+3} \cosh(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*cosh(b*x + a), x)
```

3.82 $\int x^{2+m} \cosh(a + bx)\, dx$

Optimal. Leaf size=59

$$\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m + 3, -bx)}{2b^3} - \frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m + 3, bx)}{2b^3}$$

```
[Out] (E^a*x^m*Gamma[3 + m, -(b*x)])/(2*b^3*(-(b*x))^m) - (x^m*Gamma[3 + m, b*x])
/(2*b^3*E^a*(b*x)^m)
```

Rubi [A] time = 0.0721975, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3307, 2181}

$$\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m + 3, -bx)}{2b^3} - \frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m + 3, bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Cosh[a + b*x],x]
```

```
[Out] (E^a*x^m*Gamma[3 + m, -(b*x)])/(2*b^3*(-(b*x))^m) - (x^m*Gamma[3 + m, b*x])
/(2*b^3*E^a*(b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{2+m} \cosh(a + bx)\, dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{2+m}\, dx + \frac{1}{2}\int e^{i(ia+ibx)}x^{2+m}\, dx \\
&= \frac{e^a x^m(-bx)^{-m}\Gamma(3 + m, -bx)}{2b^3} - \frac{e^{-a}x^m(bx)^{-m}\Gamma(3 + m, bx)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.0194083, size = 54, normalized size = 0.92

$$\frac{e^{-a}x^m\left(e^{2a}(-bx)^{-m}\text{Gamma}(m + 3, -bx) - (bx)^{-m}\text{Gamma}(m + 3, bx)\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Cosh[a + b*x],x]
```

```
[Out] (x^m*((E^(2*a)*Gamma[3 + m, -(b*x)])/(-(b*x))^m - Gamma[3 + m, b*x]/(b*x)^m
))/(2*b^3*E^a)
```

Maple [C] time = 0.055, size = 73, normalized size = 1.2

$$\frac{x^{3+m}\cosh(a)}{3+m}\,{}_1F_2\left(\frac{3}{2}+\frac{m}{2};\frac{1}{2},\frac{5}{2}+\frac{m}{2};\frac{x^2b^2}{4}\right)+\frac{bx^{4+m}\sinh(a)}{4+m}\,{}_1F_2\left(2+\frac{m}{2};\frac{3}{2},3+\frac{m}{2};\frac{x^2b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*cosh(b*x+a),x)
```

```
[Out] 1/(3+m)*x^(3+m)*hypergeom([3/2+1/2*m],[1/2,5/2+1/2*m],1/4*x^2*b^2)*cosh(a)+
b/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[3/2,3+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Maxima [A] time = 1.14434, size = 74, normalized size = 1.25

$$-\frac{1}{2}\,(bx)^{-m-3}\,x^{m+3}e^{(-a)}\Gamma(m+3,bx)-\frac{1}{2}\,(-bx)^{-m-3}\,x^{m+3}e^{a}\Gamma(m+3,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x)^(-m - 3)*x^(m + 3)*e^(-a)*gamma(m + 3, b*x) - 1/2*(-b*x)^(-m - 3
)*x^(m + 3)*e^a*gamma(m + 3, -b*x)
```

Fricas [A] time = 1.89764, size = 259, normalized size = 4.39

$$-\frac{\cosh((m+2)\log(b)+a)\,\Gamma(m+3,bx)-\cosh((m+2)\log(-b)-a)\,\Gamma(m+3,-bx)+\Gamma(m+3,-bx)\sinh((m+2)\,\mathrm{l}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((m + 2)*log(b) + a)*gamma(m + 3, b*x) - cosh((m + 2)*log(-b) - a
)*gamma(m + 3, -b*x) + gamma(m + 3, -b*x)*sinh((m + 2)*log(-b) - a) - gamma
(m + 3, b*x)*sinh((m + 2)*log(b) + a))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*cosh(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+2} \cosh{(bx+a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*cosh(b*x + a), x)
```

3.83 $\int x^{1+m}\cosh(a+bx)\,dx$

Optimal. Leaf size=59

$$-\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+2,-bx)}{2b^2}-\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+2,bx)}{2b^2}$$

```
[Out] -(E^a*x^m*Gamma[2 + m, -(b*x)])/(2*b^2*(-(b*x))^m) - (x^m*Gamma[2 + m, b*x]
)/(2*b^2*E^a*(b*x)^m)
```

Rubi [A] time = 0.0711949, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3307, 2181}

$$-\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+2,-bx)}{2b^2}-\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+2,bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Cosh[a + b*x],x]
```

```
[Out] -(E^a*x^m*Gamma[2 + m, -(b*x)])/(2*b^2*(-(b*x))^m) - (x^m*Gamma[2 + m, b*x]
)/(2*b^2*E^a*(b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{1+m}\cosh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{1+m}\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^{1+m}\,dx\\ &= -\frac{e^a x^m(-bx)^{-m}\Gamma(2+m,-bx)}{2b^2}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(2+m,bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0320544, size = 54, normalized size = 0.92

$$-\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+2,-bx)+e^{-a}x^m(bx)^{-m}\text{Gamma}(m+2,bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Cosh[a + b*x],x]
```

```
[Out] -((E^a*x^m*Gamma[2 + m, -(b*x)])/(-(b*x))^m + (x^m*Gamma[2 + m, b*x])/(E^a*
(b*x)^m))/(2*b^2)
```

Maple [C] time = 0.048, size = 73, normalized size = 1.2

$$\frac{x^{2+m}\cosh(a)}{2+m}\,{}_1F_2\left(1+\frac{m}{2};\frac{1}{2},2+\frac{m}{2};\frac{x^2b^2}{4}\right)+\frac{bx^{3+m}\sinh(a)}{3+m}\,{}_1F_2\left(\frac{3}{2}+\frac{m}{2};\frac{3}{2},\frac{5}{2}+\frac{m}{2};\frac{x^2b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*cosh(b*x+a),x)
```

```
[Out] 1/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[1/2,2+1/2*m],1/4*x^2*b^2)*cosh(a)+b/(3
+m)*x^(3+m)*hypergeom([3/2+1/2*m],[3/2,5/2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Maxima [A] time = 1.15272, size = 74, normalized size = 1.25

$$-\frac{1}{2}\,(bx)^{-m-2}\,x^{m+2}e^{(-a)}\Gamma\left(m+2,bx\right)-\frac{1}{2}\,(-bx)^{-m-2}\,x^{m+2}e^{a}\Gamma\left(m+2,-bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x)^(-m - 2)*x^(m + 2)*e^(-a)*gamma(m + 2, b*x) - 1/2*(-b*x)^(-m - 2
)*x^(m + 2)*e^a*gamma(m + 2, -b*x)
```

Fricas [A] time = 1.7484, size = 259, normalized size = 4.39

$$-\frac{\cosh\left((m+1)\log(b)+a\right)\Gamma\left(m+2,bx\right)-\cosh\left((m+1)\log(-b)-a\right)\Gamma\left(m+2,-bx\right)+\Gamma\left(m+2,-bx\right)\sinh\left((m+1)\log(\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((m + 1)*log(b) + a)*gamma(m + 2, b*x) - cosh((m + 1)*log(-b) - a
)*gamma(m + 2, -b*x) + gamma(m + 2, -b*x)*sinh((m + 1)*log(-b) - a) - gamma
(m + 2, b*x)*sinh((m + 1)*log(b) + a))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*cosh(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1}\cosh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*cosh(b*x + a), x)
```

3.84 $\int x^m \cosh(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^a x^m (-bx)^{-m}\text{Gamma}(m+1,-bx)}{2b}-\frac{e^{-a} x^m (bx)^{-m}\text{Gamma}(m+1,bx)}{2b}$$

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m)
```

Rubi [A] time = 0.0687868, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3307, 2181}

$$\frac{e^a x^m (-bx)^{-m}\text{Gamma}(m+1,-bx)}{2b}-\frac{e^{-a} x^m (bx)^{-m}\text{Gamma}(m+1,bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x],x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^m \cosh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx\\ &= \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0177996, size = 54, normalized size = 0.92

$$\frac{e^{-a}x^m\left(e^{2a}(-bx)^{-m}\text{Gamma}(m+1,-bx)-(bx)^{-m}\text{Gamma}(m+1,bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x],x]
```

```
[Out] (x^m*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m - Gamma[1 + m, b*x]/(b*x)^m
))/(2*b*E^a)
```

Maple [C] time = 0.039, size = 73, normalized size = 1.2

$$\frac{x^{1+m}\cosh(a)}{1+m}{}_1F_2(\frac{1}{2}+\frac{m}{2};\frac{1}{2},\frac{3}{2}+\frac{m}{2};\frac{x^2b^2}{4})+\frac{bx^{2+m}\sinh(a)}{2+m}{}_1F_2(1+\frac{m}{2};\frac{3}{2},2+\frac{m}{2};\frac{x^2b^2}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a),x)
```

```
[Out] 1/(1+m)*x^(1+m)*hypergeom([1/2+1/2*m],[1/2,3/2+1/2*m],1/4*x^2*b^2)*cosh(a)+
b/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[3/2,2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Maxima [A] time = 1.15347, size = 74, normalized size = 1.25

$$-\frac{1}{2}\left(bx\right)^{-m-1}x^{m+1}e^{(-a)}\Gamma\left(m+1,bx\right)-\frac{1}{2}\left(-bx\right)^{-m-1}x^{m+1}e^{a}\Gamma\left(m+1,-bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x)^(-m - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) - 1/2*(-b*x)^(-m - 1
)*x^(m + 1)*e^a*gamma(m + 1, -b*x)
```

Fricas [A] time = 1.92761, size = 227, normalized size = 3.85

$$-\frac{\cosh\left(m\log\left(b\right)+a\right)\Gamma\left(m+1,bx\right)-\cosh\left(m\log\left(-b\right)-a\right)\Gamma\left(m+1,-bx\right)+\Gamma\left(m+1,-bx\right)\sinh\left(m\log\left(-b\right)-a\right)-\Gamma\left(r\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh(m*log(b) + a)*gamma(m + 1, b*x) - cosh(m*log(-b) - a)*gamma(m +
1, -b*x) + gamma(m + 1, -b*x)*sinh(m*log(-b) - a) - gamma(m + 1, b*x)*sinh(
m*log(b) + a))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a), x)
```

3.85 $\int x^{-1+m}\cosh(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{1}{2}e^a x^m(-bx)^{-m}\text{Gamma}(m,-bx)-\frac{1}{2}e^{-a}x^m(bx)^{-m}\text{Gamma}(m,bx)$$

```
[Out] -(E^a*x^m*Gamma[m, -(b*x)])/(2*(-(b*x))^m) - (x^m*Gamma[m, b*x])/(2*E^a*(b*
x)^m)
```

Rubi [A] time = 0.0690003, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3307, 2181}

$$-\frac{1}{2}e^a x^m(-bx)^{-m}\text{Gamma}(m,-bx)-\frac{1}{2}e^{-a}x^m(bx)^{-m}\text{Gamma}(m,bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Cosh[a + b*x],x]
```

```
[Out] -(E^a*x^m*Gamma[m, -(b*x)])/(2*(-(b*x))^m) - (x^m*Gamma[m, b*x])/(2*E^a*(b*
x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{-1+m}\cosh(a+bx)\,dx&=\frac{1}{2}\int e^{-i(ia+ibx)}x^{-1+m}\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^{-1+m}\,dx\\&=-\frac{1}{2}e^a x^m(-bx)^{-m}\Gamma(m,-bx)-\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m,bx)\end{aligned}$$

Mathematica [A] time = 0.0198166, size = 49, normalized size = 1.

$$-\frac{1}{2}e^a x^m(-bx)^{-m}\text{Gamma}(m,-bx)-\frac{1}{2}e^{-a}x^m(bx)^{-m}\text{Gamma}(m,bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Cosh[a + b*x],x]
```

```
[Out] -(E^a*x^m*Gamma[m, -(b*x)])/(2*(-(b*x))^m) - (x^m*Gamma[m, b*x])/(2*E^a*(b*
x)^m)
```

Maple [C] time = 0.046, size = 67, normalized size = 1.4

$$\frac{x^m \cosh(a)}{m} {}_1F_2(\frac{m}{2}; \frac{1}{2}, 1+\frac{m}{2}; \frac{x^2b^2}{4}) + \frac{bx^{1+m}\sinh(a)}{1+m} {}_1F_2(\frac{1}{2}+\frac{m}{2}; \frac{3}{2}, \frac{3}{2}+\frac{m}{2}; \frac{x^2b^2}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*cosh(b*x+a),x)
```

```
[Out] 1/m*x^m*hypergeom([1/2*m],[1/2,1+1/2*m],1/4*x^2*b^2)*cosh(a)+b/(1+m)*x^(1+m
)*hypergeom([1/2+1/2*m],[3/2,3/2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Maxima [A] time = 1.17037, size = 58, normalized size = 1.18

$$-\frac{x^m e^{(-a)}\Gamma(m, bx)}{2(bx)^m} - \frac{x^m e^a \Gamma(m, -bx)}{2(-bx)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^m*e^(-a)*gamma(m, b*x)/(b*x)^m - 1/2*x^m*e^a*gamma(m, -b*x)/(-b*x)^m
```

Fricas [A] time = 1.97397, size = 238, normalized size = 4.86

$$-\frac{\cosh((m-1)\log(b)+a)\Gamma(m, bx) - \cosh((m-1)\log(-b)-a)\Gamma(m, -bx) + \Gamma(m, -bx)\sinh((m-1)\log(-b)-a) - \Gamma}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((m - 1)*log(b) + a)*gamma(m, b*x) - cosh((m - 1)*log(-b) - a)*ga
mma(m, -b*x) + gamma(m, -b*x)*sinh((m - 1)*log(-b) - a) - gamma(m, b*x)*sin
h((m - 1)*log(b) + a))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*cosh(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1} \cosh (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*cosh(b*x + a), x)
```

3.86 $\int x^{-2+m}\cosh(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}e^a b x^m(-bx)^{-m}\text{Gamma}(m-1,-bx)-\frac{1}{2}e^{-a}bx^m(bx)^{-m}\text{Gamma}(m-1,bx)$$

```
[Out] (b*E^a*x^m*Gamma[-1 + m, -(b*x)])/(2*(-(b*x))^m) - (b*x^m*Gamma[-1 + m, b*x
])/(2*E^a*(b*x)^m)
```

Rubi [A] time = 0.0691807, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3307, 2181}

$$\frac{1}{2}e^a b x^m(-bx)^{-m}\text{Gamma}(m-1,-bx)-\frac{1}{2}e^{-a}bx^m(bx)^{-m}\text{Gamma}(m-1,bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Cosh[a + b*x],x]
```

```
[Out] (b*E^a*x^m*Gamma[-1 + m, -(b*x)])/(2*(-(b*x))^m) - (b*x^m*Gamma[-1 + m, b*x
])/(2*E^a*(b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{-2+m}\cosh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{-2+m}\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^{-2+m}\,dx\\ &= \frac{1}{2}be^a x^m(-bx)^{-m}\Gamma(-1+m,-bx)-\frac{1}{2}be^{-a}x^m(bx)^{-m}\Gamma(-1+m,bx)\end{aligned}$$

Mathematica [A] time = 0.0186507, size = 52, normalized size = 0.95

$$\frac{1}{2}e^{-a}bx^m\left(e^{2a}(-bx)^{-m}\text{Gamma}(m-1,-bx)-(bx)^{-m}\text{Gamma}(m-1,bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Cosh[a + b*x],x]
```

```
[Out] (b*x^m*((E^(2*a)*Gamma[-1 + m, -(b*x)])/(-(b*x))^m - Gamma[-1 + m, b*x]/(b*
x)^m))/(2*E^a)
```

Maple [C] time = 0.055, size = 67, normalized size = 1.2

$$\frac{x^{-1+m}\cosh(a)}{-1+m}\,{}_1F_2\!\left(-\frac{1}{2}+\frac{m}{2};\frac{1}{2},\frac{1}{2}+\frac{m}{2};\frac{x^2b^2}{4}\right)+\frac{bx^m\sinh(a)}{m}\,{}_1F_2\!\left(\frac{m}{2};\frac{3}{2},1+\frac{m}{2};\frac{x^2b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)*cosh(b*x+a),x)
```

```
[Out] 1/(-1+m)*x^(-1+m)*hypergeom([-1/2+1/2*m],[1/2,1/2+1/2*m],1/4*x^2*b^2)*cosh(
a)+b/m*x^m*hypergeom([1/2*m],[3/2,1+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Maxima [A] time = 1.15493, size = 74, normalized size = 1.35

$$-\frac{1}{2}\,(bx)^{-m+1}\,x^{m-1}e^{(-a)}\Gamma(m-1,bx)-\frac{1}{2}\,(-bx)^{-m+1}\,x^{m-1}e^{a}\Gamma(m-1,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x)^(-m + 1)*x^(m - 1)*e^(-a)*gamma(m - 1, b*x) - 1/2*(-b*x)^(-m + 1
)*x^(m - 1)*e^a*gamma(m - 1, -b*x)
```

Fricas [A] time = 1.84898, size = 259, normalized size = 4.71

$$-\frac{\cosh((m-2)\log(b)+a)\,\Gamma(m-1,bx)-\cosh((m-2)\log(-b)-a)\,\Gamma(m-1,-bx)+\Gamma(m-1,-bx)\sinh((m-2)\log}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((m - 2)*log(b) + a)*gamma(m - 1, b*x) - cosh((m - 2)*log(-b) - a
)*gamma(m - 1, -b*x) + gamma(m - 1, -b*x)*sinh((m - 2)*log(-b) - a) - gamma
(m - 1, b*x)*sinh((m - 2)*log(b) + a))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*cosh(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-2} \cosh(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*cosh(b*x + a), x)
```

3.87 $\int x^{-3+m} \cosh(a + bx)\,dx$

Optimal. Leaf size=59

$$-\frac{1}{2}e^{a}b^{2}x^{m}(-bx)^{-m}\text{Gamma}(m-2,-bx)-\frac{1}{2}e^{-a}b^{2}x^{m}(bx)^{-m}\text{Gamma}(m-2,bx)$$

```
[Out] -(b^2*E^a*x^m*Gamma[-2 + m, -(b*x)])/(2*(-(b*x))^m) - (b^2*x^m*Gamma[-2 + m
, b*x])/(2*E^a*(b*x)^m)
```

Rubi [A] time = 0.0704377, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3307, 2181}

$$-\frac{1}{2}e^{a}b^{2}x^{m}(-bx)^{-m}\text{Gamma}(m-2,-bx)-\frac{1}{2}e^{-a}b^{2}x^{m}(bx)^{-m}\text{Gamma}(m-2,bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Cosh[a + b*x],x]
```

```
[Out] -(b^2*E^a*x^m*Gamma[-2 + m, -(b*x)])/(2*(-(b*x))^m) - (b^2*x^m*Gamma[-2 + m
, b*x])/(2*E^a*(b*x)^m)
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{-3+m}\cosh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{-3+m}\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^{-3+m}\,dx\\ &= -\frac{1}{2}b^{2}e^{a}x^{m}(-bx)^{-m}\Gamma(-2+m,-bx)-\frac{1}{2}b^{2}e^{-a}x^{m}(bx)^{-m}\Gamma(-2+m,bx)\end{aligned}$$

Mathematica [A] time = 0.0221103, size = 55, normalized size = 0.93

$$\frac{1}{2}e^{-a}b^{2}x^{m}\left(-e^{2a}(-bx)^{-m}\text{Gamma}(m-2,-bx)-(bx)^{-m}\text{Gamma}(m-2,bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Cosh[a + b*x],x]
```

```
[Out] (b^2*x^m*(-((E^(2*a)*Gamma[-2 + m, -(b*x)])/(-(b*x))^m) - Gamma[-2 + m, b*x
]/(b*x)^m))/(2*E^a)
```

Maple [C] time = 0.032, size = 71, normalized size = 1.2

$$\frac{x^{-2+m}\cosh(a)}{-2+m}\,{}_1F_2(-1+\frac{m}{2};\frac{1}{2},\frac{m}{2};\frac{x^2b^2}{4})+\frac{bx^{-1+m}\sinh(a)}{-1+m}\,{}_1F_2(-\frac{1}{2}+\frac{m}{2};\frac{3}{2},\frac{1}{2}+\frac{m}{2};\frac{x^2b^2}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)*cosh(b*x+a),x)
```

```
[Out] 1/(-2+m)*x^(-2+m)*hypergeom([-1+1/2*m],[1/2,1/2*m],1/4*x^2*b^2)*cosh(a)+b/(
-1+m)*x^(-1+m)*hypergeom([-1/2+1/2*m],[3/2,1/2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Maxima [A] time = 1.16083, size = 74, normalized size = 1.25

$$-\frac{1}{2}\,(bx)^{-m+2}\,x^{m-2}e^{(-a)}\Gamma(m-2,bx)-\frac{1}{2}\,(-bx)^{-m+2}\,x^{m-2}e^{a}\Gamma(m-2,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x)^(-m + 2)*x^(m - 2)*e^(-a)*gamma(m - 2, b*x) - 1/2*(-b*x)^(-m + 2
)*x^(m - 2)*e^a*gamma(m - 2, -b*x)
```

Fricas [A] time = 1.89121, size = 259, normalized size = 4.39

$$-\frac{\cosh((m-3)\log(b)+a)\,\Gamma(m-2,bx)-\cosh((m-3)\log(-b)-a)\,\Gamma(m-2,-bx)+\Gamma(m-2,-bx)\sinh((m-3)\log(-}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((m - 3)*log(b) + a)*gamma(m - 2, b*x) - cosh((m - 3)*log(-b) - a
)*gamma(m - 2, -b*x) + gamma(m - 2, -b*x)*sinh((m - 3)*log(-b) - a) - gamma
(m - 2, b*x)*sinh((m - 3)*log(b) + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*cosh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-3} \cosh(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)*cosh(b*x + a), x)
```

3.88 $\int x^{3+m}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=86

$$-\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\text{Gamma}(m+4,-2bx)}{b^4}-\frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\text{Gamma}(m+4,2bx)}{b^4}+\frac{x^{m+4}}{2(m+4)}$$

```
[Out] x^(4 + m)/(2*(4 + m)) - (2^(-6 - m)*E^(2*a)*x^m*Gamma[4 + m, -2*b*x])/(b^4*
(-(b*x))^m) - (2^(-6 - m)*x^m*Gamma[4 + m, 2*b*x])/(b^4*E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.150147, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$-\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\text{Gamma}(m+4,-2bx)}{b^4}-\frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\text{Gamma}(m+4,2bx)}{b^4}+\frac{x^{m+4}}{2(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] x^(4 + m)/(2*(4 + m)) - (2^(-6 - m)*E^(2*a)*x^m*Gamma[4 + m, -2*b*x])/(b^4*
(-(b*x))^m) - (2^(-6 - m)*x^m*Gamma[4 + m, 2*b*x])/(b^4*E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{3+m}\cosh^2(a+bx)\,dx &= \int \left(\frac{x^{3+m}}{2}+\frac{1}{2}x^{3+m}\cosh(2a+2bx)\right)dx\\
&= \frac{x^{4+m}}{2(4+m)}+\frac{1}{2}\int x^{3+m}\cosh(2a+2bx)\,dx\\
&= \frac{x^{4+m}}{2(4+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{3+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{3+m}\,dx\\
&= \frac{x^{4+m}}{2(4+m)}-\frac{2^{-6-m}e^{2a}x^m(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4}-\frac{2^{-6-m}e^{-2a}x^m(bx)^{-m}\Gamma(4+m,2bx)}{b^4}
\end{aligned}$$

Mathematica [A] time = 0.105598, size = 79, normalized size = 0.92

$$\frac{1}{64}x^m\left(-\frac{e^{2a}2^{-m}(-bx)^{-m}\text{Gamma}(m+4,-2bx)}{b^4}-\frac{e^{-2a}2^{-m}(bx)^{-m}\text{Gamma}(m+4,2bx)}{b^4}+\frac{32x^4}{m+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] (x^m*((32*x^4)/(4 + m) - (E^(2*a)*Gamma[4 + m, -2*b*x])/(2^m*b^4*(-(b*x))^m
) - Gamma[4 + m, 2*b*x]/(2^m*b^4*E^(2*a)*(b*x)^m)))/64
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int x^{3+m}\left(\cosh\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3+m)*cosh(b*x+a)^2,x)
```

```
[Out] int(x^(3+m)*cosh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87783, size = 425, normalized size = 4.94

$$\frac{4\,bx\cosh\left((m+3)\log\left(x\right)\right)-(m+4)\cosh\left((m+3)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m+4,2\,bx\right)+(m+4)\cosh\left((m+3)\log\left(-2\,b\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh((m + 3)*log(x)) - (m + 4)*cosh((m + 3)*log(2*b) + 2*a)*gamm
a(m + 4, 2*b*x) + (m + 4)*cosh((m + 3)*log(-2*b) - 2*a)*gamma(m + 4, -2*b*x
) + (m + 4)*gamma(m + 4, 2*b*x)*sinh((m + 3)*log(2*b) + 2*a) - (m + 4)*gamm
a(m + 4, -2*b*x)*sinh((m + 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 3)*log(x))
)/(b*m + 4*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*cosh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+3} \cosh{(bx+a)}^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*cosh(b*x + a)^2, x)
```

3.89 $\int x^{2+m} \cosh^2(a+bx)\,dx$

Optimal. Leaf size=85

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\text{Gamma}(m+3,-2bx)}{b^3}-\frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\text{Gamma}(m+3,2bx)}{b^3}+\frac{x^{m+3}}{2(m+3)}$$

```
[Out] x^(3 + m)/(2*(3 + m)) + (2^(-5 - m)*E^(2*a)*x^m*Gamma[3 + m, -2*b*x])/(b^3*
(-(b*x))^m) - (2^(-5 - m)*x^m*Gamma[3 + m, 2*b*x])/(b^3*E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.133411, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\text{Gamma}(m+3,-2bx)}{b^3}-\frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\text{Gamma}(m+3,2bx)}{b^3}+\frac{x^{m+3}}{2(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] x^(3 + m)/(2*(3 + m)) + (2^(-5 - m)*E^(2*a)*x^m*Gamma[3 + m, -2*b*x])/(b^3*
(-(b*x))^m) - (2^(-5 - m)*x^m*Gamma[3 + m, 2*b*x])/(b^3*E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{2+m}\cosh^2(a+bx)\,dx &= \int \left(\frac{x^{2+m}}{2}+\frac{1}{2}x^{2+m}\cosh(2a+2bx)\right)dx\\
&= \frac{x^{3+m}}{2(3+m)}+\frac{1}{2}\int x^{2+m}\cosh(2a+2bx)\,dx\\
&= \frac{x^{3+m}}{2(3+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{2+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{2+m}\,dx\\
&= \frac{x^{3+m}}{2(3+m)}+\frac{2^{-5-m}e^{2a}x^m(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3}-\frac{2^{-5-m}e^{-2a}x^m(bx)^{-m}\Gamma(3+m,2bx)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.0951888, size = 78, normalized size = 0.92

$$\frac{1}{32}x^m\left(\frac{e^{2a}2^{-m}(-bx)^{-m}\text{Gamma}(m+3,-2bx)}{b^3}-\frac{e^{-2a}2^{-m}(bx)^{-m}\text{Gamma}(m+3,2bx)}{b^3}+\frac{16x^3}{m+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] (x^m*((16*x^3)/(3 + m) + (E^(2*a)*Gamma[3 + m, -2*b*x])/(2^m*b^3*(-(b*x))^m
) - Gamma[3 + m, 2*b*x]/(2^m*b^3*E^(2*a)*(b*x)^m)))/32
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int x^{2+m}\left(\cosh\left(bx+a\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*cosh(b*x+a)^2,x)
```

```
[Out] int(x^(2+m)*cosh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80286, size = 425, normalized size = 5.

$$\frac{4\,bx\cosh\left((m+2)\log\left(x\right)\right)-(m+3)\cosh\left((m+2)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m+3,2\,bx\right)+(m+3)\cosh\left((m+2)\log\left(-2\,b\right)-2\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh((m + 2)*log(x)) - (m + 3)*cosh((m + 2)*log(2*b) + 2*a)*gamm
a(m + 3, 2*b*x) + (m + 3)*cosh((m + 2)*log(-2*b) - 2*a)*gamma(m + 3, -2*b*x
) + (m + 3)*gamma(m + 3, 2*b*x)*sinh((m + 2)*log(2*b) + 2*a) - (m + 3)*gamm
a(m + 3, -2*b*x)*sinh((m + 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 2)*log(x))
)/(b*m + 3*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*cosh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+2} \cosh (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*cosh(b*x + a)^2, x)
```

3.90 $\int x^{1+m}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=86

$$-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+2,-2bx)}{b^2}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{Gamma}(m+2,2bx)}{b^2}+\frac{x^{m+2}}{2(m+2)}$$

```
[Out] x^(2 + m)/(2*(2 + m)) - (2^(-4 - m)*E^(2*a)*x^m*Gamma[2 + m, -2*b*x])/(b^2*
(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma[2 + m, 2*b*x])/(b^2*E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.133434, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+2,-2bx)}{b^2}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{Gamma}(m+2,2bx)}{b^2}+\frac{x^{m+2}}{2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] x^(2 + m)/(2*(2 + m)) - (2^(-4 - m)*E^(2*a)*x^m*Gamma[2 + m, -2*b*x])/(b^2*
(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma[2 + m, 2*b*x])/(b^2*E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{1+m}\cosh^2(a+bx)\,dx &= \int\left(\frac{x^{1+m}}{2}+\frac{1}{2}x^{1+m}\cosh(2a+2bx)\right)dx\\
&= \frac{x^{2+m}}{2(2+m)}+\frac{1}{2}\int x^{1+m}\cosh(2a+2bx)\,dx\\
&= \frac{x^{2+m}}{2(2+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{1+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{1+m}\,dx\\
&= \frac{x^{2+m}}{2(2+m)}-\frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2}-\frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(2+m,2bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.105677, size = 79, normalized size = 0.92

$$\frac{1}{16}x^m\left(-\frac{e^{2a}2^{-m}(-bx)^{-m}\text{Gamma}(m+2,-2bx)}{b^2}-\frac{e^{-2a}2^{-m}(bx)^{-m}\text{Gamma}(m+2,2bx)}{b^2}+\frac{8x^2}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] (x^m*((8*x^2)/(2 + m) - (E^(2*a)*Gamma[2 + m, -2*b*x])/(2^m*b^2*(-(b*x))^m)
 - Gamma[2 + m, 2*b*x]/(2^m*b^2*E^(2*a)*(b*x)^m)))/16
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int x^{1+m}\left(\cosh\left(bx+a\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*cosh(b*x+a)^2,x)
```

```
[Out] int(x^(1+m)*cosh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74125, size = 425, normalized size = 4.94

$$4\,bx\cosh\left((m+1)\log\left(x\right)\right)-(m+2)\cosh\left((m+1)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m+2,2\,bx\right)+(m+2)\cosh\left((m+1)\log\left(-2\,b\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh((m + 1)*log(x)) - (m + 2)*cosh((m + 1)*log(2*b) + 2*a)*gamm
a(m + 2, 2*b*x) + (m + 2)*cosh((m + 1)*log(-2*b) - 2*a)*gamma(m + 2, -2*b*x
) + (m + 2)*gamma(m + 2, 2*b*x)*sinh((m + 1)*log(2*b) + 2*a) - (m + 2)*gamm
a(m + 2, -2*b*x)*sinh((m + 1)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 1)*log(x))
)/(b*m + 2*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1}\cosh^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*cosh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 1)*cosh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1}\cosh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*cosh(b*x + a)^2, x)
```

3.91 $\int x^m \cosh^2(a+bx)\,dx$

Optimal. Leaf size=85

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\text{Gamma}(m+1,2bx)}{b}+\frac{x^{m+1}}{2(m+1)}$$

```
[Out] x^(1 + m)/(2*(1 + m)) + (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-
(b*x))^m) - (2^(-3 - m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.126934, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3307, 2181}

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\text{Gamma}(m+1,2bx)}{b}+\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) + (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-
(b*x))^m) - (2^(-3 - m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^m \cosh^2(a+bx)\,dx &= \int \left(\frac{x^m}{2}+\frac{1}{2}x^m\cosh(2a+2bx)\right)dx\\
&= \frac{x^{1+m}}{2(1+m)}+\frac{1}{2}\int x^m\cosh(2a+2bx)\,dx\\
&= \frac{x^{1+m}}{2(1+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx\\
&= \frac{x^{1+m}}{2(1+m)}+\frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}-\frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0852304, size = 76, normalized size = 0.89

$$\frac{1}{8}x^m\left(\frac{e^{2a}2^{-m}(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m}(bx)^{-m}\text{Gamma}(m+1,2bx)}{b}+\frac{4x}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^2,x]
```

```
[Out] (x^m*((4*x)/(1 + m) + (E^(2*a)*Gamma[1 + m, -2*b*x])/(2^m*b*(-(b*x))^m) - G
amma[1 + m, 2*b*x]/(2^m*b*E^(2*a)*(b*x)^m)))/8
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int x^m (\cosh (bx + a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86069, size = 374, normalized size = 4.4

$$\frac{4\,bx\cosh\left(m\log\left(x\right)\right)-\left(m+1\right)\cosh\left(m\log\left(2\,b\right)+2\,a\right)\Gamma\left(m+1,2\,bx\right)+\left(m+1\right)\cosh\left(m\log\left(-2\,b\right)-2\,a\right)\Gamma\left(m+1,-2\,bx\right)}{8\,(bm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh(m*log(x)) - (m + 1)*cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2*b
*x) + (m + 1)*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + (m + 1)*gamma(
m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) - (m + 1)*gamma(m + 1, -2*b*x)*sinh(m*
log(-2*b) - 2*a) + 4*b*x*sinh(m*log(x)))/(b*m + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh^2 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*cosh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \cosh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2, x)
```

3.92 $\int x^{-1+m} \cosh^2(a+bx)\,dx$

Optimal. Leaf size=72

$$e^{2a}\left(-2^{-m-2}\right)x^m(-bx)^{-m}\text{Gamma}(m,-2bx)-e^{-2a}2^{-m-2}x^m(bx)^{-m}\text{Gamma}(m,2bx)+\frac{x^m}{2m}$$

```
[Out] x^m/(2*m) - (2^(-2 - m)*E^(2*a)*x^m*Gamma[m, -2*b*x])/(-(b*x))^m - (2^(-2 -
 m)*x^m*Gamma[m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.124953, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$e^{2a}\left(-2^{-m-2}\right)x^m(-bx)^{-m}\text{Gamma}(m,-2bx)-e^{-2a}2^{-m-2}x^m(bx)^{-m}\text{Gamma}(m,2bx)+\frac{x^m}{2m}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] x^m/(2*m) - (2^(-2 - m)*E^(2*a)*x^m*Gamma[m, -2*b*x])/(-(b*x))^m - (2^(-2 -
 m)*x^m*Gamma[m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{-1+m}\cosh^2(a+bx)\,dx &= \int\left(\frac{x^{-1+m}}{2}+\frac{1}{2}x^{-1+m}\cosh(2a+2bx)\right)dx\\
&= \frac{x^m}{2m}+\frac{1}{2}\int x^{-1+m}\cosh(2a+2bx)\,dx\\
&= \frac{x^m}{2m}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{-1+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{-1+m}\,dx\\
&= \frac{x^m}{2m}-2^{-2-m}e^{2a}x^m(-bx)^{-m}\Gamma(m,-2bx)-2^{-2-m}e^{-2a}x^m(bx)^{-m}\Gamma(m,2bx)
\end{aligned}$$

Mathematica [A] time = 0.0560609, size = 64, normalized size = 0.89

$$\frac{1}{4}x^m \left(e^{2a}\left(-2^{-m}\right)(-bx)^{-m}\text{Gamma}(m,-2bx) - e^{-2a}2^{-m}(bx)^{-m}\text{Gamma}(m,2bx) + \frac{2}{m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] (x^m*(2/m - (E^(2*a)*Gamma[m, -2*b*x])/(2^m*(-(b*x))^m) - Gamma[m, 2*b*x]/(
2^m*E^(2*a)*(b*x)^m)))/4
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int x^{-1+m}\left(\cosh\left(bx+a\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*cosh(b*x+a)^2,x)
```

```
[Out] int(x^(-1+m)*cosh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98085, size = 363, normalized size = 5.04

$$\frac{4\,bx\cosh\left((m-1)\log\left(x\right)\right) - m\cosh\left((m-1)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m,2\,bx\right) + m\cosh\left((m-1)\log\left(-2\,b\right)-2\,a\right)\Gamma\left(m,-2\,bx\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh((m - 1)*log(x)) - m*cosh((m - 1)*log(2*b) + 2*a)*gamma(m, 2
*b*x) + m*cosh((m - 1)*log(-2*b) - 2*a)*gamma(m, -2*b*x) + m*gamma(m, 2*b*x
)*sinh((m - 1)*log(2*b) + 2*a) - m*gamma(m, -2*b*x)*sinh((m - 1)*log(-2*b)
- 2*a) + 4*b*x*sinh((m - 1)*log(x)))/(b*m)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1}\cosh^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*cosh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 1)*cosh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1}\cosh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*cosh(b*x + a)^2, x)
```

3.93 $\int x^{-2+m}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=83

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m-1,-2bx)-e^{-2a}b2^{-m-1}x^m(bx)^{-m}\text{Gamma}(m-1,2bx)-\frac{x^{m-1}}{2(1-m)}$$

```
[Out] -x^(-1 + m)/(2*(1 - m)) + (2^(-1 - m)*b*E^(2*a)*x^m*Gamma[-1 + m, -2*b*x])/
(-(b*x))^m - (2^(-1 - m)*b*x^m*Gamma[-1 + m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.12958, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m-1,-2bx)-e^{-2a}b2^{-m-1}x^m(bx)^{-m}\text{Gamma}(m-1,2bx)-\frac{x^{m-1}}{2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] -x^(-1 + m)/(2*(1 - m)) + (2^(-1 - m)*b*E^(2*a)*x^m*Gamma[-1 + m, -2*b*x])/
(-(b*x))^m - (2^(-1 - m)*b*x^m*Gamma[-1 + m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{-2+m}\cosh^2(a+bx)\,dx &= \int\left(\frac{x^{-2+m}}{2}+\frac{1}{2}x^{-2+m}\cosh(2a+2bx)\right)dx\\
&= -\frac{x^{-1+m}}{2(1-m)}+\frac{1}{2}\int x^{-2+m}\cosh(2a+2bx)\,dx\\
&= -\frac{x^{-1+m}}{2(1-m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{-2+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{-2+m}\,dx\\
&= -\frac{x^{-1+m}}{2(1-m)}+2^{-1-m}be^{2a}x^m(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-1-m}be^{-2a}x^m(bx)^{-m}\Gamma(-1+m,2bx)
\end{aligned}$$

Mathematica [A] time = 0.0975381, size = 73, normalized size = 0.88

$$\frac{1}{2}x^m\left(e^{2a}b2^{-m}(-bx)^{-m}\text{Gamma}(m-1,-2bx)-e^{-2a}b2^{-m}(bx)^{-m}\text{Gamma}(m-1,2bx)+\frac{1}{(m-1)x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] (x^m*(1/((-1 + m)*x) + (b*E^(2*a)*Gamma[-1 + m, -2*b*x])/(2^m*(-(b*x))^m) -
 (b*Gamma[-1 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)))/2
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int x^{-2+m}\left(\cosh\left(bx+a\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)*cosh(b*x+a)^2,x)
```

```
[Out] int(x^(-2+m)*cosh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09172, size = 423, normalized size = 5.1

$$\frac{4\,bx\cosh\left((m-2)\log\left(x\right)\right)-(m-1)\cosh\left((m-2)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m-1,2\,bx\right)+(m-1)\cosh\left((m-2)\log\left(-2\,b\right)-2\,a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh((m - 2)*log(x)) - (m - 1)*cosh((m - 2)*log(2*b) + 2*a)*gamm
a(m - 1, 2*b*x) + (m - 1)*cosh((m - 2)*log(-2*b) - 2*a)*gamma(m - 1, -2*b*x
) + (m - 1)*gamma(m - 1, 2*b*x)*sinh((m - 2)*log(2*b) + 2*a) - (m - 1)*gamm
a(m - 1, -2*b*x)*sinh((m - 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 2)*log(x))
)/(b*m - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*cosh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-2} \cosh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*cosh(b*x + a)^2, x)
```

3.94 $\int x^{-3+m}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=84

$$-e^{2a}b^2 2^{-m}x^m(-bx)^{-m}\mathrm{Gamma}(m-2,-2bx)-e^{-2a}b^2 2^{-m}x^m(bx)^{-m}\mathrm{Gamma}(m-2,2bx)-\frac{x^{m-2}}{2(2-m)}$$

```
[Out] -x^(-2 + m)/(2*(2 - m)) - (b^2*E^(2*a)*x^m*Gamma[-2 + m, -2*b*x])/(2^m*(-(b
*x))^m) - (b^2*x^m*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.139859, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$-e^{2a}b^2 2^{-m}x^m(-bx)^{-m}\mathrm{Gamma}(m-2,-2bx)-e^{-2a}b^2 2^{-m}x^m(bx)^{-m}\mathrm{Gamma}(m-2,2bx)-\frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] -x^(-2 + m)/(2*(2 - m)) - (b^2*E^(2*a)*x^m*Gamma[-2 + m, -2*b*x])/(2^m*(-(b
*x))^m) - (b^2*x^m*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{-3+m}\cosh^2(a+bx)\,dx &= \int\left(\frac{x^{-3+m}}{2}+\frac{1}{2}x^{-3+m}\cosh(2a+2bx)\right)dx\\
&= -\frac{x^{-2+m}}{2(2-m)}+\frac{1}{2}\int x^{-3+m}\cosh(2a+2bx)\,dx\\
&= -\frac{x^{-2+m}}{2(2-m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{-3+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{-3+m}\,dx\\
&= -\frac{x^{-2+m}}{2(2-m)}-2^{-m}b^2e^{2a}x^m(-bx)^{-m}\Gamma(-2+m,-2bx)-2^{-m}b^2e^{-2a}x^m(bx)^{-m}\Gamma(-2+m,2bx)
\end{aligned}$$

Mathematica [A] time = 0.101745, size = 84, normalized size = 1.

$$-e^{2a}b^2 2^{-m}x^m(-bx)^{-m}\mathrm{Gamma}(m-2,-2bx)-e^{-2a}b^2 2^{-m}x^m(bx)^{-m}\mathrm{Gamma}(m-2,2bx)-\frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] -x^(-2 + m)/(2*(2 - m)) - (b^2*E^(2*a)*x^m*Gamma[-2 + m, -2*b*x])/(2^m*(-(b
*x))^m) - (b^2*x^m*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int x^{-3+m}\left(\cosh\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)*cosh(b*x+a)^2,x)
```

```
[Out] int(x^(-3+m)*cosh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.19837, size = 425, normalized size = 5.06

$$\frac{4\,bx\cosh\left((m-3)\log\left(x\right)\right)-(m-2)\cosh\left((m-3)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m-2,2\,bx\right)+(m-2)\cosh\left((m-3)\log\left(-2\,b\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh((m - 3)*log(x)) - (m - 2)*cosh((m - 3)*log(2*b) + 2*a)*gamm
a(m - 2, 2*b*x) + (m - 2)*cosh((m - 3)*log(-2*b) - 2*a)*gamma(m - 2, -2*b*x
) + (m - 2)*gamma(m - 2, 2*b*x)*sinh((m - 3)*log(2*b) + 2*a) - (m - 2)*gamm
a(m - 2, -2*b*x)*sinh((m - 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 3)*log(x))
)/(b*m - 2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*cosh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-3}\cosh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)*cosh(b*x + a)^2, x)
```

3.95 $\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\operatorname{sech}(x)}\right)dx$

Optimal. Leaf size=24

$$\frac{2x\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}-\frac{4}{9\operatorname{sech}^{\frac{3}{2}}(x)}$$

```
[Out] -4/(9*Sech[x]^(3/2)) + (2*x*Sinh[x])/(3*Sqrt[Sech[x]])
```

Rubi [A] time = 0.0906282, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4187, 4189}

$$\frac{2x\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}-\frac{4}{9\operatorname{sech}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sech[x]^(3/2) - (x*Sqrt[Sech[x]])/3,x]
```

```
[Out] -4/(9*Sech[x]^(3/2)) + (2*x*Sinh[x])/(3*Sqrt[Sech[x]])
```

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\operatorname{sech}(x)}\right)dx &= -\left(\frac{1}{3}\int x\sqrt{\operatorname{sech}(x)}\,dx\right)+\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}\,dx\\ &= -\frac{4}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}+\frac{1}{3}\int x\sqrt{\operatorname{sech}(x)}\,dx-\frac{1}{3}\left(\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\right)\int \frac{}{\sqrt{c}}\\ &= -\frac{4}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}\end{aligned}$$

Mathematica [A] time = 0.0874916, size = 17, normalized size = 0.71

$$\frac{2(3x\tanh(x)-2)}{9\operatorname{sech}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sech[x]^(3/2) - (x*Sqrt[Sech[x]])/3,x]
```

```
[Out] (2*(-2 + 3*x*Tanh[x]))/(9*Sech[x]^(3/2))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x\left(\operatorname{sech}(x)\right)^{-\frac{3}{2}}-\frac{x}{3}\sqrt{\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x)
```

```
[Out] int(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{3}\,x\sqrt{\operatorname{sech}(x)}+\frac{x}{\operatorname{sech}(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x*sqrt(sech(x)) + x/sech(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{3x}{\operatorname{sech}^{\frac{3}{2}}(x)}\,dx+\int x\sqrt{\operatorname{sech}(x)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)**(3/2)-1/3*x*sech(x)**(1/2),x)
```

```
[Out] -(Integral(-3*x/sech(x)**(3/2), x) + Integral(x*sqrt(sech(x)), x))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{3}\,x\sqrt{\operatorname{sech}(x)} + \frac{x}{\operatorname{sech}(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x*sqrt(sech(x)) + x/sech(x)^(3/2), x)
```

3.96 $\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right) dx$

Optimal. Leaf size=24

$$\frac{2x\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)}$$

```
[Out] -4/(25*Sech[x]^(5/2)) + (2*x*Sinh[x])/(5*Sech[x]^(3/2))
```

Rubi [A] time = 0.0877239, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4187, 4189}

$$\frac{2x\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sech[x]^(5/2) - (3*x)/(5*Sqrt[Sech[x]]),x]
```

```
[Out] -4/(25*Sech[x]^(5/2)) + (2*x*Sinh[x])/(5*Sech[x]^(3/2))
```

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right) dx &= -\left(\frac{3}{5}\int \frac{x}{\sqrt{\operatorname{sech}(x)}}\,dx\right) + \int \frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}\,dx \\
&= -\frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)} + \frac{2x\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)} + \frac{3}{5}\int \frac{x}{\sqrt{\operatorname{sech}(x)}}\,dx - \frac{1}{5}\left(3\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\right)\int x\sqrt{\operatorname{co}
\end{aligned}$$

$$= -\frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)} + \frac{2x\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}$$

Mathematica [A] time = 0.139346, size = 17, normalized size = 0.71

$$\frac{2(5x\tanh(x) - 2)}{25\operatorname{sech}^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sech[x]^(5/2) - (3*x)/(5*Sqrt[Sech[x]]),x]
```

```
[Out] (2*(-2 + 5*x*Tanh[x]))/(25*Sech[x]^(5/2))
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int x(\operatorname{sech}(x))^{-\frac{5}{2}}-\frac{3x}{5}\frac{1}{\sqrt{\operatorname{sech}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x)
```

```
[Out] int(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{3x}{5\sqrt{\operatorname{sech}(x)}}+\frac{x}{\operatorname{sech}(x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-3/5*x/sqrt(sech(x)) + x/sech(x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{5x}{\operatorname{sech}^{\frac{5}{2}}(x)}\,dx+\int \frac{3x}{\sqrt{\operatorname{sech}(x)}}\,dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)**(5/2)-3/5*x/sech(x)**(1/2),x)
```

```
[Out] -(Integral(-5*x/sech(x)**(5/2), x) + Integral(3*x/sqrt(sech(x)), x))/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{3x}{5\sqrt{\operatorname{sech}(x)}} + \frac{x}{\operatorname{sech}(x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-3/5*x/sqrt(sech(x)) + x/sech(x)^(5/2), x)
```

3.97 $\int \left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{sech}(x)}\right) dx$

Optimal. Leaf size=47

$$-\frac{20}{63\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{4}{49\operatorname{sech}^{\frac{7}{2}}(x)}+\frac{2x\sinh(x)}{7\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{10x\sinh(x)}{21\sqrt{\operatorname{sech}(x)}}$$

```
[Out] -4/(49*Sech[x]^(7/2)) - 20/(63*Sech[x]^(3/2)) + (2*x*Sinh[x])/(7*Sech[x]^(5
/2)) + (10*x*Sinh[x])/(21*Sqrt[Sech[x]])
```

Rubi [A] time = 0.104468, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4187, 4189}

$$-\frac{20}{63\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{4}{49\operatorname{sech}^{\frac{7}{2}}(x)}+\frac{2x\sinh(x)}{7\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{10x\sinh(x)}{21\sqrt{\operatorname{sech}(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sech[x]^(7/2) - (5*x*Sqrt[Sech[x]])/21,x]
```

```
[Out] -4/(49*Sech[x]^(7/2)) - 20/(63*Sech[x]^(3/2)) + (2*x*Sinh[x])/(7*Sech[x]^(5
/2)) + (10*x*Sinh[x])/(21*Sqrt[Sech[x]])
```

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{sech}(x)}\right) dx &= -\left(\frac{5}{21}\int x\sqrt{\operatorname{sech}(x)}\,dx\right)+\int \frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}\,dx\\
&= -\frac{4}{49\operatorname{sech}^{\frac{7}{2}}(x)}+\frac{2x\sinh(x)}{7\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{5}{7}\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}\,dx-\frac{1}{21}\left(5\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\right)\int\\
&= -\frac{4}{49\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{20}{63\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{7\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{10x\sinh(x)}{21\sqrt{\operatorname{sech}(x)}}+\frac{5}{21}\int x\sqrt{\operatorname{sech}(x)}\,d\\
&= -\frac{4}{49\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{20}{63\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{7\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{10x\sinh(x)}{21\sqrt{\operatorname{sech}(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.103575, size = 45, normalized size = 0.96

$$\sqrt{\operatorname{sech}(x)}\left(\frac{13}{42}x\sinh(2x)+\frac{1}{28}x\sinh(4x)-\frac{88}{441}\cosh(2x)-\frac{1}{98}\cosh(4x)-\frac{167}{882}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sech[x]^(7/2) - (5*x*Sqrt[Sech[x]])/21,x]
```

```
[Out] Sqrt[Sech[x]]*(-167/882 - (88*Cosh[2*x])/441 - Cosh[4*x]/98 + (13*x*Sinh[2*
x])/42 + (x*Sinh[4*x])/28)
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int x\left(\operatorname{sech}(x)\right)^{-\frac{7}{2}}-\frac{5x}{21}\sqrt{\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x)
```

```
[Out] int(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{5}{21}\,x\sqrt{\operatorname{sech}(x)}+\frac{x}{\operatorname{sech}(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-5/21*x*sqrt(sech(x)) + x/sech(x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)**(7/2)-5/21*x*sech(x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{5}{21}\,x\sqrt{\operatorname{sech}\left(x\right)} + \frac{x}{\operatorname{sech}\left(x\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-5/21*x*sqrt(sech(x)) + x/sech(x)^(7/2), x)
```

3.98 $\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3}x^2\sqrt{\operatorname{sech}(x)} \right) dx$

Optimal. Leaf size=66

$$-\frac{16}{27}i\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\text{EllipticF}\left(\frac{ix}{2},2\right)+\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}-\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}}$$

```
[Out] (-8*x)/(9*Sech[x]^(3/2)) - ((16*I)/27)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2]*
Sqrt[Sech[x]] + (16*Sinh[x])/(27*Sqrt[Sech[x]]) + (2*x^2*Sinh[x])/(3*Sqrt[S
ech[x]])
```

Rubi [A] time = 0.169032, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4188, 4189, 3769, 3771, 2641}

$$\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}-\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}}-\frac{16}{27}i\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}F\left(\frac{ix}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sech[x]^(3/2) - (x^2*Sqrt[Sech[x]])/3,x]
```

```
[Out] (-8*x)/(9*Sech[x]^(3/2)) - ((16*I)/27)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2]*
Sqrt[Sech[x]] + (16*Sinh[x])/(27*Sqrt[Sech[x]]) + (2*x^2*Sinh[x])/(3*Sqrt[S
ech[x]])
```

Rule 4188

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(n + 1)/(b^2*n), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n + 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^n, x], x]
+ Simp[((c + d*x)^m*Cos[e + f*x]*(b*Csc[e + f*x])^(n + 1))/(b*f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right)dx=-\left(\frac{1}{3}\int x^2\sqrt{\operatorname{sech}(x)}\,dx\right)+\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}\,dx$$
$$=-\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}+\frac{1}{3}\int x^2\sqrt{\operatorname{sech}(x)}\,dx+\frac{8}{9}\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(x)}\,dx-\frac{1}{3}\left(\sqrt{\mathrm{c}\right.$$
$$=-\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}}+\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}+\frac{8}{27}\int \sqrt{\operatorname{sech}(x)}\,dx$$
$$=-\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}}+\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}+\frac{1}{27}\left(8\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\right)\int \frac{}{\sqrt{\mathrm{co}}}$$
$$=-\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{16}{27}i\sqrt{\cosh(x)}F\left(\frac{ix}{2}\middle|2\right)\sqrt{\operatorname{sech}(x)}+\frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}}+\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}$$

Mathematica [A] time = 0.100945, size = 55, normalized size = 0.83

$$\frac{1}{27}\sqrt{\operatorname{sech}(x)}\left(-16i\sqrt{\cosh(x)}\text{EllipticF}\left(\frac{ix}{2},2\right)+9x^2\sinh(2x)-12x+8\sinh(2x)-12x\cosh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sech[x]^(3/2) - (x^2*Sqrt[Sech[x]])/3,x]
```

```
[Out] (Sqrt[Sech[x]]*(-12*x - 12*x*Cosh[2*x] - (16*I)*Sqrt[Cosh[x]]*EllipticF[(I/
2)*x, 2] + 8*Sinh[2*x] + 9*x^2*Sinh[2*x]))/27
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int x^2(\operatorname{sech}(x))^{-\frac{3}{2}}-\frac{x^2}{3}\sqrt{\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x)
```

```
[Out] int(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}+\frac{x^2}{\operatorname{sech}(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x^2*sqrt(sech(x)) + x^2/sech(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{3x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}\,dx + \int x^2\sqrt{\operatorname{sech}(x)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sech(x)**(3/2)-1/3*x**2*sech(x)**(1/2),x)
```

```
[Out] -(Integral(-3*x**2/sech(x)**(3/2), x) + Integral(x**2*sqrt(sech(x)), x))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{3}\,x^2\sqrt{\operatorname{sech}(x)} + \frac{x^2}{\operatorname{sech}(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x^2*sqrt(sech(x)) + x^2/sech(x)^(3/2), x)
```

3.99 $\int (c+dx)^3 (a+a\cosh(e+fx))\,dx$

Optimal. Leaf size=89

$$\frac{6ad^2(c+dx)\sinh(e+fx)}{f^3}-\frac{3ad(c+dx)^2\cosh(e+fx)}{f^2}+\frac{a(c+dx)^3\sinh(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6ad^3\cosh(e+fx)}{f^4}$$

```
[Out] (a*(c + d*x)^4)/(4*d) - (6*a*d^3*Cosh[e + f*x])/f^4 - (3*a*d*(c + d*x)^2*Co
sh[e + f*x])/f^2 + (6*a*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (a*(c + d*x)^3*S
inh[e + f*x])/f
```

Rubi [A] time = 0.13154, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3296, 2638}

$$\frac{6ad^2(c+dx)\sinh(e+fx)}{f^3}-\frac{3ad(c+dx)^2\cosh(e+fx)}{f^2}+\frac{a(c+dx)^3\sinh(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6ad^3\cosh(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + a*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) - (6*a*d^3*Cosh[e + f*x])/f^4 - (3*a*d*(c + d*x)^2*Co
sh[e + f*x])/f^2 + (6*a*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (a*(c + d*x)^3*S
inh[e + f*x])/f
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^3(a+a\cosh(e+fx))\,dx = \int \left(a(c+dx)^3 + a(c+dx)^3\cosh(e+fx)\right)dx$$
$$= \frac{a(c+dx)^4}{4d} + a\int (c+dx)^3\cosh(e+fx)\,dx$$
$$= \frac{a(c+dx)^4}{4d} + \frac{a(c+dx)^3\sinh(e+fx)}{f} - \frac{(3ad)\int (c+dx)^2\sinh(e+fx)\,dx}{f}$$
$$= \frac{a(c+dx)^4}{4d} - \frac{3ad(c+dx)^2\cosh(e+fx)}{f^2} + \frac{a(c+dx)^3\sinh(e+fx)}{f} + \frac{(6ad^2)\int (c+$$
$$= \frac{a(c+dx)^4}{4d} - \frac{3ad(c+dx)^2\cosh(e+fx)}{f^2} + \frac{6ad^2(c+dx)\sinh(e+fx)}{f^3} + \frac{a(c+dx)^3 s}{}$$
$$= \frac{a(c+dx)^4}{4d} - \frac{6ad^3\cosh(e+fx)}{f^4} - \frac{3ad(c+dx)^2\cosh(e+fx)}{f^2} + \frac{6ad^2(c+dx)\sinh(}{f^3}$$

Mathematica [A] time = 0.541269, size = 122, normalized size = 1.37

$$a\left(\frac{(c+dx)\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+6\right)\right)\sinh(e+fx)}{f^3} - \frac{3d\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\cosh(e+fx)}{f^4} + \frac{1}{4}x\left(6c^2d\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + a*Cosh[e + f*x]),x]
```

```
[Out] a*((x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (3*d*(c^2*f^2 + 2*c*
d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x])/f^4 + ((c + d*x)*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(6 + f^2*x^2))*Sinh[e + f*x])/f^3)
```

Maple [B] time = 0.016, size = 482, normalized size = 5.4

$$\frac{1}{f}\left(\frac{d^3a\left(fx+e\right)^4}{4f^3} + \frac{d^3a\left(\left(fx+e\right)^3\sinh\left(fx+e\right)-3\left(fx+e\right)^2\cosh\left(fx+e\right)+6\left(fx+e\right)\sinh\left(fx+e\right)-6\cosh\left(fx+\right.\right.}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+a*cosh(f*x+e)),x)
```

```
[Out] 1/f*(1/4/f^3*d^3*a*(f*x+e)^4+1/f^3*d^3*a*((f*x+e)^3*sinh(f*x+e)-3*(f*x+e)^2
*cosh(f*x+e)+6*(f*x+e)*sinh(f*x+e)-6*cosh(f*x+e))-1/f^3*d^3*e*a*(f*x+e)^3-3
/f^3*d^3*e*a*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f*x+e)+2*sinh(f*x+e))+1/
f^2*d^2*c*a*(f*x+e)^3+3/f^2*d^2*c*a*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f
*x+e)+2*sinh(f*x+e))+3/2/f^3*d^3*e^2*a*(f*x+e)^2+3/f^3*d^3*e^2*a*((f*x+e)*s
inh(f*x+e)-cosh(f*x+e))-3/f^2*d^2*e*c*a*(f*x+e)^2-6/f^2*d^2*e*c*a*((f*x+e)*
sinh(f*x+e)-cosh(f*x+e))+3/2/f*d*c^2*a*(f*x+e)^2+3/f*d*c^2*a*((f*x+e)*sinh(
f*x+e)-cosh(f*x+e))-d^3*e^3/f^3*a*(f*x+e)-d^3*e^3/f^3*a*sinh(f*x+e)+3*d^2*e
^2/f^2*c*a*(f*x+e)+3*d^2*e^2/f^2*c*a*sinh(f*x+e)-3*d*e/f*c^2*a*(f*x+e)-3*d*
e/f*c^2*a*sinh(f*x+e)+c^3*a*(f*x+e)+a*c^3*sinh(f*x+e))
```

Maxima [B] time = 1.12106, size = 320, normalized size = 3.6

$$\frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x + \frac{3}{2}ac^2d\left(\frac{\left(fxe^e-e^e\right)e^{(fx)}}{f^2} - \frac{\left(fx+1\right)e^{(-fx-e)}}{f^2}\right) + \frac{3}{2}acd^2\left(\frac{\left(f^2x^2e^e-2fxe^e+2e^e\right)e^{(fx}}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*a*c^2*d*((f*x
*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*a*c*d^2*((f^2*x
^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e
)/f^3) + 1/2*a*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*
x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + a*c^3*sinh(f
*x + e)/f
```

Fricas [A] time = 2.01208, size = 365, normalized size = 4.1

$$\frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x - 12\left(ad^3f^2x^2 + 2\,acd^2f^2x + ac^2df^2 + 2\,ad^3\right)\cosh\left(fx+e\right) + 4\left(ad^3f^3\right.}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
- 12*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*x + a*c^2*d*f^2 + 2*a*d^3)*cosh(f*x + e
) + 4*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + a*c^3*f^3 + 6*a*c*d^2*f + 3*(a*c
^2*d*f^3 + 2*a*d^3*f)*x)*sinh(f*x + e))/f^4
```

Sympy [A] time = 2.41751, size = 264, normalized size = 2.97

$$\begin{cases} ac^3x + \frac{ac^3\sinh\left(e+fx\right)}{f} + \frac{3ac^2dx^2}{2} + \frac{3ac^2dx\sinh\left(e+fx\right)}{f} - \frac{3ac^2d\cosh\left(e+fx\right)}{f^2} + acd^2x^3 + \frac{3acd^2x^2\sinh\left(e+fx\right)}{f} - \frac{6acd^2x\cosh\left(e+fx\right)}{f^2} + \frac{6ac\cdots}{} \\ \left(a\cosh\left(e\right) + a\right)\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+a*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x + a*c**3*sinh(e + f*x)/f + 3*a*c**2*d*x**2/2 + 3*a*c**2
*d*x**sinh(e + f*x)/f - 3*a*c**2*d*cosh(e + f*x)/f**2 + a*c*d**2*x**3 + 3*a*
c*d**2*x**2*sinh(e + f*x)/f - 6*a*c*d**2*x*cosh(e + f*x)/f**2 + 6*a*c*d**2*
sinh(e + f*x)/f**3 + a*d**3*x**4/4 + a*d**3*x**3*sinh(e + f*x)/f - 3*a*d**3
*x**2*cosh(e + f*x)/f**2 + 6*a*d**3*x*sinh(e + f*x)/f**3 - 6*a*d**3*cosh(e
+ f*x)/f**4, Ne(f, 0)), ((a*cosh(e) + a)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2
*x**3 + d**3*x**4/4), True))
```

Giac [B] time = 1.25464, size = 351, normalized size = 3.94

$$\frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{\left(ad^3f^3x^3 + 3\,acd^2f^3x^2 + 3\,ac^2df^3x - 3\,ad^3f^2x^2 + ac^3f^3 - 6\,acd^2f^2x - 3\,ac^2d\cdots\right.}{2\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 1/2*(a*d^3*f^3*x^
3 + 3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x - 3*a*d^3*f^2*x^2 + a*c^3*f^3 - 6*a
*c*d^2*f^2*x - 3*a*c^2*d*f^2 + 6*a*d^3*f*x + 6*a*c*d^2*f - 6*a*d^3)*e^(f*x
+ e)/f^4 - 1/2*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x + 3*a*d
^3*f^2*x^2 + a*c^3*f^3 + 6*a*c*d^2*f^2*x + 3*a*c^2*d*f^2 + 6*a*d^3*f*x + 6*
a*c*d^2*f + 6*a*d^3)*e^(-f*x - e)/f^4
```

3.100 $\int (c+dx)^2(a+a\cosh(e+fx))\,dx$

Optimal. Leaf size=67

$$-\frac{2ad(c+dx)\cosh(e+fx)}{f^2}+\frac{a(c+dx)^2\sinh(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2ad^2\sinh(e+fx)}{f^3}$$

```
[Out] (a*(c + d*x)^3)/(3*d) - (2*a*d*(c + d*x)*Cosh[e + f*x])/f^2 + (2*a*d^2*Sinh
[e + f*x])/f^3 + (a*(c + d*x)^2*Sinh[e + f*x])/f
```

Rubi [A] time = 0.0906686, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3296, 2637}

$$-\frac{2ad(c+dx)\cosh(e+fx)}{f^2}+\frac{a(c+dx)^2\sinh(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2ad^2\sinh(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + a*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) - (2*a*d*(c + d*x)*Cosh[e + f*x])/f^2 + (2*a*d^2*Sinh
[e + f*x])/f^3 + (a*(c + d*x)^2*Sinh[e + f*x])/f
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2(a+a\cosh(e+fx))\,dx &= \int \left(a(c+dx)^2+a(c+dx)^2\cosh(e+fx)\right)dx\\
&= \frac{a(c+dx)^3}{3d}+a\int (c+dx)^2\cosh(e+fx)\,dx\\
&= \frac{a(c+dx)^3}{3d}+\frac{a(c+dx)^2\sinh(e+fx)}{f}-\frac{(2ad)\int (c+dx)\sinh(e+fx)\,dx}{f}\\
&= \frac{a(c+dx)^3}{3d}-\frac{2ad(c+dx)\cosh(e+fx)}{f^2}+\frac{a(c+dx)^2\sinh(e+fx)}{f}+\frac{\left(2ad^2\right)\int \text{co}}{}\\
&= \frac{a(c+dx)^3}{3d}-\frac{2ad(c+dx)\cosh(e+fx)}{f^2}+\frac{2ad^2\sinh(e+fx)}{f^3}+\frac{a(c+dx)^2\sinh(e}{f}
\end{aligned}
$$

Mathematica [A] time = 0.314141, size = 80, normalized size = 1.19

$$a\left(\frac{\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\sinh(e+fx)}{f^3}+c^2x-\frac{2d(c+dx)\cosh(e+fx)}{f^2}+cdx^2+\frac{d^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + a*Cosh[e + f*x]),x]
```

```
[Out] a*(c^2*x + c*d*x^2 + (d^2*x^3)/3 - (2*d*(c + d*x)*Cosh[e + f*x])/f^2 + ((c^
2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x])/f^3)
```

Maple [B] time = 0.012, size = 240, normalized size = 3.6

$$\frac{1}{f}\left(\frac{ad^2\left(fx+e\right)^3}{3f^2}+\frac{ad^2\left(\left(fx+e\right)^2\sinh\left(fx+e\right)-2\left(fx+e\right)\cosh\left(fx+e\right)+2\sinh\left(fx+e\right)\right)}{f^2}-\frac{d^2ea\left(fx+e\right)^2}{f^2}-2\frac{d^2e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+a*cosh(f*x+e)),x)
```

```
[Out] 1/f*(1/3/f^2*d^2*a*(f*x+e)^3+1/f^2*d^2*a*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*c
osh(f*x+e)+2*sinh(f*x+e))-1/f^2*d^2*e*a*(f*x+e)^2-2/f^2*d^2*e*a*((f*x+e)*si
nh(f*x+e)-cosh(f*x+e))+1/f*d*c*a*(f*x+e)^2+2/f*d*c*a*((f*x+e)*sinh(f*x+e)-c
osh(f*x+e))+d^2*e^2/f^2*a*(f*x+e)+d^2*e^2/f^2*a*sinh(f*x+e)-2*d*e/f*c*a*(f*
x+e)-2*d*e/f*c*a*sinh(f*x+e)+c^2*a*(f*x+e)+c^2*a*sinh(f*x+e))
```

Maxima [B] time = 1.09106, size = 190, normalized size = 2.84

$$\frac{1}{3}ad^2x^3+acdx^2+ac^2x+acd\left(\frac{\left(fxe^e-e^e\right)e^{(fx)}}{f^2}-\frac{\left(fx+1\right)e^{(-fx-e)}}{f^2}\right)+\frac{1}{2}ad^2\left(\frac{\left(f^2x^2e^e-2fxe^e+2e^e\right)e^{(fx)}}{f^3}-\frac{\left(f^2x^2+2fx\right.}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + a*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2 -
(f*x + 1)*e^(-f*x - e)/f^2) + 1/2*a*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*
e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a*c^2*sinh(f*x + e)
/f
```

Fricas [A] time = 1.9719, size = 231, normalized size = 3.45

$$\frac{ad^2f^3x^3+3acdf^3x^2+3ac^2f^3x-6\left(ad^2fx+acdf\right)\cosh\left(fx+e\right)+3\left(ad^2f^2x^2+2acdf^2x+ac^2f^2+2ad^2\right)\sinh\left(fx+\right.}{3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x - 6*(a*d^2*f*x + a*c*d
*f)*cosh(f*x + e) + 3*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 + 2*a*d^2)
*sinh(f*x + e))/f^3
```

Sympy [A] time = 0.945921, size = 151, normalized size = 2.25

$$\begin{cases} ac^2x + \frac{ac^2\sinh(e+fx)}{f} + acdx^2 + \frac{2acdx\sinh(e+fx)}{f} - \frac{2acd\cosh(e+fx)}{f^2} + \frac{ad^2x^3}{3} + \frac{ad^2x^2\sinh(e+fx)}{f} - \frac{2ad^2x\cosh(e+fx)}{f^2} + \frac{2ad^2\sinh(e\ldots}{f^3} \\ (a\cosh(e) + a)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x + a*c**2*sinh(e + f*x)/f + a*c*d*x**2 + 2*a*c*d*x*sinh(
e + f*x)/f - 2*a*c*d*cosh(e + f*x)/f**2 + a*d**2*x**3/3 + a*d**2*x**2*sinh(
e + f*x)/f - 2*a*d**2*x*cosh(e + f*x)/f**2 + 2*a*d**2*sinh(e + f*x)/f**3, N
e(f, 0)), ((a*cosh(e) + a)*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [B] time = 1.29276, size = 200, normalized size = 2.99

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x + \frac{\left(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 - 2\,ad^2fx - 2\,acdf + 2\,ad^2\right)e^{(fx+e)}}{2\,f^3} - \frac{\left(ad^2f^2x^2 + 2\,acdf^2x + \ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 1/2*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x +
a*c^2*f^2 - 2*a*d^2*f*x - 2*a*c*d*f + 2*a*d^2)*e^(f*x + e)/f^3 - 1/2*(a*d^2
*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 + 2*a*d^2*f*x + 2*a*c*d*f + 2*a*d^2)*e
^(-f*x - e)/f^3
```

3.101 $\int (c+dx)(a+a\cosh(e+fx))\,dx$

Optimal. Leaf size=45

$$\frac{a(c+dx)\sinh(e+fx)}{f}+\frac{a(c+dx)^2}{2d}-\frac{ad\cosh(e+fx)}{f^2}$$

```
[Out] (a*(c + d*x)^2)/(2*d) - (a*d*Cosh[e + f*x])/f^2 + (a*(c + d*x)*Sinh[e + f*x
])/f
```

Rubi [A] time = 0.044669, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3317, 3296, 2638}

$$\frac{a(c+dx)\sinh(e+fx)}{f}+\frac{a(c+dx)^2}{2d}-\frac{ad\cosh(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + a*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) - (a*d*Cosh[e + f*x])/f^2 + (a*(c + d*x)*Sinh[e + f*x
])/f
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)(a+a\cosh(e+fx))\,dx &= \int (a(c+dx)+a(c+dx)\cosh(e+fx))\,dx\\
&= \frac{a(c+dx)^2}{2d}+a\int (c+dx)\cosh(e+fx)\,dx\\
&= \frac{a(c+dx)^2}{2d}+\frac{a(c+dx)\sinh(e+fx)}{f}-\frac{(ad)\int \sinh(e+fx)\,dx}{f}\\
&= \frac{a(c+dx)^2}{2d}-\frac{ad\cosh(e+fx)}{f^2}+\frac{a(c+dx)\sinh(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.217551, size = 52, normalized size = 1.16

$$\frac{a(-2(e+fx)(-2cf+de-dfx)+4f(c+dx)\sinh(e+fx)-4d\cosh(e+fx))}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Cosh[e + f*x]),x]
```

```
[Out] (a*(-2*(e + f*x)*(d*e - 2*c*f - d*f*x) - 4*d*Cosh[e + f*x] + 4*f*(c + d*x)*
Sinh[e + f*x]))/(4*f^2)
```

Maple [B] time = 0.012, size = 91, normalized size = 2.

$$\frac{1}{f}\left(\frac{da\left(fx+e\right)^{2}}{2f}+\frac{da\left(\left(fx+e\right)\sinh\left(fx+e\right)-\cosh\left(fx+e\right)\right)}{f}-\frac{dea\left(fx+e\right)}{f}-\frac{dea\sinh\left(fx+e\right)}{f}+ac\left(fx+e\right)+ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+a*cosh(f*x+e)),x)
```

```
[Out] 1/f*(1/2/f*d*a*(f*x+e)^2+1/f*d*a*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))-d*e/f*a*
(f*x+e)-d*e/f*a*sinh(f*x+e)+a*c*(f*x+e)+c*a*sinh(f*x+e))
```

Maxima [A] time = 1.05972, size = 89, normalized size = 1.98

$$\frac{1}{2}\,adx^2+acx+\frac{1}{2}\,ad\left(\frac{\left(fxe^e-e^e\right)e^{(fx)}}{f^2}-\frac{\left(fx+1\right)e^{(-fx-e)}}{f^2}\right)+\frac{ac\sinh\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*a*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-
f*x - e)/f^2) + a*c*sinh(f*x + e)/f
```

Fricas [A] time = 1.9827, size = 128, normalized size = 2.84

$$\frac{adf^2x^2+2\,acf^2x-2\,ad\cosh\left(fx+e\right)+2\left(adfx+acf\right)\sinh\left(fx+e\right)}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x - 2*a*d*cosh(f*x + e) + 2*(a*d*f*x + a*c*f)*
sinh(f*x + e))/f^2
```

Sympy [A] time = 0.401968, size = 68, normalized size = 1.51

$$\begin{cases} acx + \frac{ac\sinh\left(e+fx\right)}{f} + \frac{adx^2}{2} + \frac{adx\sinh\left(e+fx\right)}{f} - \frac{ad\cosh\left(e+fx\right)}{f^2} & \text{for } f \neq 0 \\ \left(a\cosh\left(e\right) + a\right)\left(cx + \frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*c*sinh(e + f*x)/f + a*d*x**2/2 + a*d*x*sinh(e + f*x)/f
 - a*d*cosh(e + f*x)/f**2, Ne(f, 0)), ((a*cosh(e) + a)*(c*x + d*x**2/2), Tr
ue))
```

Giac [A] time = 1.25261, size = 89, normalized size = 1.98

$$\frac{1}{2}\,adx^2 + acx + \frac{\left(adfx + acf - ad\right)e^{\left(fx+e\right)}}{2\,f^2} - \frac{\left(adfx + acf + ad\right)e^{\left(-fx-e\right)}}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*(a*d*f*x + a*c*f - a*d)*e^(f*x + e)/f^2 - 1/2*(a*
d*f*x + a*c*f + a*d)*e^(-f*x - e)/f^2
```

3.102 $\int \frac{a+a\cosh(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=64

$$\frac{a\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{a\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

```
[Out] (a*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d +
(a*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Rubi [A] time = 0.13797, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3317, 3303, 3298, 3301}

$$\frac{a\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{a\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[e + f*x])/(c + d*x),x]
```


```
[Out] (a*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d +
(a*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{a+a\cosh(e+fx)}{c+dx}\,dx = \int \left(\frac{a}{c+dx}+\frac{a\cosh(e+fx)}{c+dx}\right)dx$$
$$= \frac{a\log(c+dx)}{d} + a\int \frac{\cosh(e+fx)}{c+dx}\,dx$$
$$= \frac{a\log(c+dx)}{d} + \left(a\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx + \left(a\sinh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{cf}{d}+f\right.}{c+dx}$$
$$= \frac{a\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d} + \frac{a\log(c+dx)}{d} + \frac{a\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

Mathematica [A] time = 0.107856, size = 54, normalized size = 0.84

$$\frac{a\left(\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\cosh\left(e-\frac{cf}{d}\right)+\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+\log(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])/(c + d*x),x]
```

```
[Out] (a*(Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + Log[c + d*x] + Sinh[e - (
c*f)/d]*SinhIntegral[f*(c/d + x)]))/d
```

Maple [A] time = 0.042, size = 94, normalized size = 1.5

$$\frac{a\ln(dx+c)}{d} - \frac{a}{2d}\mathrm{e}^{\frac{cf-de}{d}}\mathrm{Ei}\left(1, fx+e+\frac{cf-de}{d}\right) - \frac{a}{2d}\mathrm{e}^{-\frac{cf-de}{d}}\mathrm{Ei}\left(1, -fx-e-\frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(f*x+e))/(d*x+c),x)
```

```
[Out] a*ln(d*x+c)/d-1/2*a/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*a/d*exp(
-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A] time = 1.26153, size = 95, normalized size = 1.48

$$-\frac{1}{2}a\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right) + \frac{a\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d + e^(e - c*f/d)*e
xp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

Fricas [A] time = 2.04592, size = 230, normalized size = 3.59

$$\frac{\left(a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+2\,a\log\left(dx+c\right)-\left(a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{de-cf}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((a*Ei((d*f*x + c*f)/d) + a*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d)
+ 2*a*log(d*x + c) - (a*Ei((d*f*x + c*f)/d) - a*Ei(-(d*f*x + c*f)/d))*sinh(
-(d*e - c*f)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{\cosh\left(e+fx\right)}{c+dx}\,dx+\int\frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c),x)
```

```
[Out] a*(Integral(cosh(e + f*x)/(c + d*x), x) + Integral(1/(c + d*x), x))
```

Giac [A] time = 1.29379, size = 93, normalized size = 1.45

$$\frac{a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+2\,a\log\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + a*Ei((d*f*x + c*f)/d)*e^(-c*f/d
 + e) + 2*a*log(d*x + c))/d
```

3.103 $\int \frac{a+a\cosh(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=87

$$\frac{af\,\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{af\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a\cosh(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

```
[Out] -(a/(d*(c + d*x))) - (a*Cosh[e + f*x])/(d*(c + d*x)) + (a*f*CoshIntegral[(c
*f)/d + f*x]*Sinh[e - (c*f)/d])/d^2 + (a*f*Cosh[e - (c*f)/d]*SinhIntegral[(
c*f)/d + f*x])/d^2
```

Rubi [A] time = 0.172057, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3317, 3297, 3303, 3298, 3301}

$$\frac{af\,\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{af\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a\cosh(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) - (a*Cosh[e + f*x])/(d*(c + d*x)) + (a*f*CoshIntegral[(c
*f)/d + f*x]*Sinh[e - (c*f)/d])/d^2 + (a*f*Cosh[e - (c*f)/d]*SinhIntegral[(
c*f)/d + f*x])/d^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\cosh(e+fx)}{(c+dx)^2}\,dx &= \int \left(\frac{a}{(c+dx)^2}+\frac{a\cosh(e+fx)}{(c+dx)^2}\right)dx\\ &= -\frac{a}{d(c+dx)}+a\int \frac{\cosh(e+fx)}{(c+dx)^2}\,dx\\ &= -\frac{a}{d(c+dx)}-\frac{a\cosh(e+fx)}{d(c+dx)}+\frac{(af)\int \frac{\sinh(e+fx)}{c+dx}\,dx}{d}\\ &= -\frac{a}{d(c+dx)}-\frac{a\cosh(e+fx)}{d(c+dx)}+\frac{\left(af\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}+\frac{\left(af\sinh\left(e-\frac{cf}{d}\right)\right)\int}{d}\\ &= -\frac{a}{d(c+dx)}-\frac{a\cosh(e+fx)}{d(c+dx)}+\frac{af\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{af\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}\right.}{d^2}\end{aligned}$$

Mathematica [A] time = 0.297618, size = 68, normalized size = 0.78

$$\frac{a\left(f\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+f\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)-\frac{d(\cosh(e+fx)+1)}{c+dx}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] (a*(-((d*(1 + Cosh[e + f*x]))/(c + d*x)) + f*CoshIntegral[f*(c/d + x)]*Sinh
[e - (c*f)/d] + f*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/d^2
```

Maple [A] time = 0.052, size = 149, normalized size = 1.7

$$-\frac{a}{d\,(dx+c)}-\frac{af\mathrm{e}^{-fx-e}}{2\,d\left(dfx+cf\right)}+\frac{af}{2\,d^2}\mathrm{e}^{\frac{cf-de}{d}}\mathrm{Ei}\left(1,fx+e+\frac{cf-de}{d}\right)-\frac{af\mathrm{e}^{fx+e}}{2\,d^2}\left(\frac{cf}{d}+fx\right)^{-1}-\frac{af}{2\,d^2}\mathrm{e}^{-\frac{cf-de}{d}}\mathrm{Ei}\left(1,-fx-e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(f*x+e))/(d*x+c)^2,x)
```

```
[Out] -a/d/(d*x+c)-1/2*a*f*exp(-f*x-e)/d/(d*f*x+c*f)+1/2*a*f/d^2*exp((c*f-d*e)/d)
*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*a*f/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*a*f/d^2*exp
(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A] time = 1.23123, size = 117, normalized size = 1.34

$$-\frac{1}{2}a\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) + e^(
e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x + c*
d)
```

Fricas [A] time = 2.00517, size = 351, normalized size = 4.03

$$-\frac{2\,ad\cosh\left(fx+e\right)+2\,ad-\left(\left(adfx+acf\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-\left(adfx+acf\right)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left(\left(adfx+acf\right)\mathrm{Ei}\left(\cdots\right.\right.}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*cosh(f*x + e) + 2*a*d - ((a*d*f*x + a*c*f)*Ei((d*f*x + c*f)/d)
- (a*d*f*x + a*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + ((a*d*f*x
+ a*c*f)*Ei((d*f*x + c*f)/d) + (a*d*f*x + a*c*f)*Ei(-(d*f*x + c*f)/d))*sinh
(-(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27104, size = 227, normalized size = 2.61

$$-\frac{\left(dfx\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}-dfx\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+cf\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}-cf\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+de^{\left(fx+e\right)}+de^{\left(-fx\cdots\right.}\right.}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(d*f*x*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) - d*f*x*Ei((d*f*x + c*f)/d)*
e^(-c*f/d + e) + c*f*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) - c*f*Ei((d*f*x + c
*f)/d)*e^(-c*f/d + e) + d*e^(f*x + e) + d*e^(-f*x - e))*a/(d^3*x + c*d^2) -
 a/((d*x + c)*d)
```

3.104 $\int \frac{a+a\cosh(e+fx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=123

$$\frac{af^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{af^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{af\sinh(e+fx)}{2d^2(c+dx)}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

```
[Out] -a/(2*d*(c + d*x)^2) - (a*Cosh[e + f*x])/(2*d*(c + d*x)^2) + (a*f^2*Cosh[e
- (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/(2*d^3) - (a*f*Sinh[e + f*x])/(2*d^
2*(c + d*x)) + (a*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(2*d^3
)
```

Rubi [A] time = 0.215713, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3317, 3297, 3303, 3298, 3301}

$$\frac{af^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{af^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{af\sinh(e+fx)}{2d^2(c+dx)}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] -a/(2*d*(c + d*x)^2) - (a*Cosh[e + f*x])/(2*d*(c + d*x)^2) + (a*f^2*Cosh[e
- (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/(2*d^3) - (a*f*Sinh[e + f*x])/(2*d^
2*(c + d*x)) + (a*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(2*d^3
)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\cosh(e+fx)}{(c+dx)^3}\,dx &= \int \left(\frac{a}{(c+dx)^3}+\frac{a\cosh(e+fx)}{(c+dx)^3}\right)dx\\
&= -\frac{a}{2d(c+dx)^2}+a\int \frac{\cosh(e+fx)}{(c+dx)^3}\,dx\\
&= -\frac{a}{2d(c+dx)^2}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}+\frac{(af)\int \frac{\sinh(e+fx)}{(c+dx)^2}\,dx}{2d}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}-\frac{af\sinh(e+fx)}{2d^2(c+dx)}+\frac{\left(af^2\right)\int \frac{\cosh(e+fx)}{c+dx}\,dx}{2d^2}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}-\frac{af\sinh(e+fx)}{2d^2(c+dx)}+\frac{\left(af^2\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{2d^2}+\cdots\\
&= -\frac{a}{2d(c+dx)^2}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}+\frac{af^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{2d^3}-\frac{af\sinh(e+fx)}{2d^2(c+dx)}+\frac{af^2\cdots}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.455808, size = 90, normalized size = 0.73

$$\frac{a\left(f^2\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\cosh\left(e-\frac{cf}{d}\right)+f^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)-\frac{d(f(c+dx)\sinh(e+fx)+d\cosh(e+fx)+d)}{(c+dx)^2}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] (a*(f^2*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - (d*(d + d*Cosh[e + f*
x] + f*(c + d*x)*Sinh[e + f*x]))/(c + d*x)^2 + f^2*Sinh[e - (c*f)/d]*SinhIn
tegral[f*(c/d + x)]))/(2*d^3)
```

Maple [B] time = 0.053, size = 296, normalized size = 2.4

$$-\frac{a}{2\,d\,(dx+c)^2}+\frac{f^3a\mathrm{e}^{-fx-e}x}{4\,d\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)}+\frac{f^3a\mathrm{e}^{-fx-e}c}{4\,d^2\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)}-\frac{f^2a\mathrm{e}^{-fx-e}}{4\,d\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)}-\frac{f^2\cdots}{4\,d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(f*x+e))/(d*x+c)^3,x)
```

```
[Out] -1/2*a/d/(d*x+c)^2+1/4*a*f^3*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2
)*x+1/4*a*f^3*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/4*a*f^2
*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/4*a*f^2/d^3*exp((c*f-d*e
)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*f^2*a/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/4*f^2
*a/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*f^2*a/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(
c*f-d*e)/d)
```

Maxima [A] time = 1.22622, size = 132, normalized size = 1.07

$$-\frac{1}{2}a\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-\frac{a}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a*(e^(-e + c*f/d)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) + e
^(e - c*f/d)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/(d^
3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] time = 2.07163, size = 572, normalized size = 4.65

$$-\frac{2\,ad^2\cosh\left(fx+e\right)+2\,ad^2-\left(\left(ad^2f^2x^2+2\,acdf^2x+ac^2f^2\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+\left(ad^2f^2x^2+2\,acdf^2x+ac^2f^2\right)\mathrm{Ei}\left(-\frac{dfx+c}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d^2*cosh(f*x + e) + 2*a*d^2 - ((a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a
*c^2*f^2)*Ei((d*f*x + c*f)/d) + (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2)
*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + 2*(a*d^2*f*x + a*c*d*f)*sinh(
f*x + e) + ((a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2)*Ei((d*f*x + c*f)/d)
 - (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*sinh(-
(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19746, size = 443, normalized size = 3.6

$$\frac{ad^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+ad^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+2\,acdf^2x\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+2\,acdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + a*d^2*f^2*x^2*Ei((d
*f*x + c*f)/d)*e^(-c*f/d + e) + 2*a*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(c*f/d
 - e) + 2*a*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) + a*c^2*f^2*Ei(-(d
*f*x + c*f)/d)*e^(c*f/d - e) + a*c^2*f^2*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e)
 - a*d^2*f*x*e^(f*x + e) + a*d^2*f*x*e^(-f*x - e) - a*c*d*f*e^(f*x + e) + a
*c*d*f*e^(-f*x - e) - a*d^2*e^(f*x + e) - a*d^2*e^(-f*x - e) - 2*a*d^2)/(d^
5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.105 $\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx$

Optimal. Leaf size=237

$$\frac{12a^2d^2(c+dx)\sinh(e+fx)}{f^3}+\frac{3a^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{3a^2cd^2x}{4f^2}-\frac{3a^2d(c+dx)^2\cosh^2(e+fx)}{4f^2}-$$

```
[Out] (3*a^2*c*d^2*x)/(4*f^2) + (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8*
d) - (12*a^2*d^3*Cosh[e + f*x])/f^4 - (6*a^2*d*(c + d*x)^2*Cosh[e + f*x])/f
^2 - (3*a^2*d^3*Cosh[e + f*x]^2)/(8*f^4) - (3*a^2*d*(c + d*x)^2*Cosh[e + f*
x]^2)/(4*f^2) + (12*a^2*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (2*a^2*(c + d*x)
^3*Sinh[e + f*x])/f + (3*a^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(4*
f^3) + (a^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)
```

Rubi [A] time = 0.265398, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3317, 3296, 2638, 3311, 32, 3310}

$$\frac{12a^2d^2(c+dx)\sinh(e+fx)}{f^3}+\frac{3a^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{3a^2cd^2x}{4f^2}-\frac{3a^2d(c+dx)^2\cosh^2(e+fx)}{4f^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + a*Cosh[e + f*x])^2,x]
```


```
[Out] (3*a^2*c*d^2*x)/(4*f^2) + (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8*
d) - (12*a^2*d^3*Cosh[e + f*x])/f^4 - (6*a^2*d*(c + d*x)^2*Cosh[e + f*x])/f
^2 - (3*a^2*d^3*Cosh[e + f*x]^2)/(8*f^4) - (3*a^2*d*(c + d*x)^2*Cosh[e + f*
x]^2)/(4*f^2) + (12*a^2*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (2*a^2*(c + d*x)
^3*Sinh[e + f*x])/f + (3*a^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(4*
f^3) + (a^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx = \int \left(a^2(c+dx)^3+2a^2(c+dx)^3\cosh(e+fx)+a^2(c+dx)^3\cosh^2(e+fx)\right)dx$$

$$= \frac{a^2(c+dx)^4}{4d}+a^2\int (c+dx)^3\cosh^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)^3\cosh(e+fx)\,dx$$

$$= \frac{a^2(c+dx)^4}{4d}-\frac{3a^2d(c+dx)^2\cosh^2(e+fx)}{4f^2}+\frac{2a^2(c+dx)^3\sinh(e+fx)}{f}+\frac{a^2(c+$$

$$= \frac{3a^2(c+dx)^4}{8d}-\frac{6a^2d(c+dx)^2\cosh(e+fx)}{f^2}-\frac{3a^2d^3\cosh^2(e+fx)}{8f^4}-\frac{3a^2d(c+dx)}{4}$$

$$= \frac{3a^2cd^2x}{4f^2}+\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}-\frac{6a^2d(c+dx)^2\cosh(e+fx)}{f^2}-\frac{3a^2d^3\cosh^2(e}{8f^4}$$

$$= \frac{3a^2cd^2x}{4f^2}+\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}-\frac{12a^2d^3\cosh(e+fx)}{f^4}-\frac{6a^2d(c+dx)^2\cosh(e}{f^2}$$

Mathematica [A] time = 1.40764, size = 217, normalized size = 0.92

$$a^2\left(2f\left(16(c+dx)\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+6\right)\right)\sinh(e+fx)+(c+dx)\left(2c^2f^2+4cdf^2x+d^2\left(2f^2x^2+3\right)\right)\sinh(2(e\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (a^2*(-96*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x] - 3*d
*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e + f*x)] + 2*f*(3
*f^3*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 16*(c + d*x)*(c^2*f^2
+ 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Sinh[e + f*x] + (c + d*x)*(2*c^2*f^2 + 4
*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e + f*x)])))/(16*f^4)
```

Maple [B] time = 0.016, size = 1071, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+a*cosh(f*x+e))^2,x)
```

```
[Out] 1/f*(6/f^2*c*d^2*a^2*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f*x+e)+2*sinh(f*
x+e))+3/f^2*c*d^2*a^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/6*(f*x+e)^3-
1/2*(f*x+e)*cosh(f*x+e)^2+1/4*sinh(f*x+e)*cosh(f*x+e)+1/4*f*x+1/4*e)+3/f*c^
2*d*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^
2)+6/f*c^2*d*a^2*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))-1/f^3*d^3*e*a^2*(f*x+e)^
3-1/f^3*d^3*e^3*a^2*(f*x+e)+6/f^3*d^3*e^2*a^2*((f*x+e)*sinh(f*x+e)-cosh(f*x
+e))+3/f^3*d^3*e^2*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1
/4*cosh(f*x+e)^2)-1/f^3*d^3*e^3*a^2*(1/2*sinh(f*x+e)*cosh(f*x+e)+1/2*f*x+1/
2*e)-3/f^3*d^3*e*a^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/6*(f*x+e)^3-1
/2*(f*x+e)*cosh(f*x+e)^2+1/4*sinh(f*x+e)*cosh(f*x+e)+1/4*f*x+1/4*e)+3/2/f*c
^2*d*a^2*(f*x+e)^2+3/2/f^3*d^3*e^2*a^2*(f*x+e)^2+1/f^2*c*d^2*a^2*(f*x+e)^3-
6/f^3*d^3*e*a^2*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f*x+e)+2*sinh(f*x+e))
-2/f^3*d^3*e^3*a^2*sinh(f*x+e)-3/f^2*c*d^2*e*a^2*(f*x+e)^2+3/f^2*c*d^2*e^2*
a^2*(f*x+e)-3/f*c^2*d*e*a^2*(f*x+e)-6/f^2*c*d^2*e*a^2*(1/2*(f*x+e)*cosh(f*x
+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-12/f^2*c*d^2*e*a^2*((f*x+e
)*sinh(f*x+e)-cosh(f*x+e))+3/f^2*c*d^2*e^2*a^2*(1/2*sinh(f*x+e)*cosh(f*x+e)
+1/2*f*x+1/2*e)-3/f*c^2*d*e*a^2*(1/2*sinh(f*x+e)*cosh(f*x+e)+1/2*f*x+1/2*e)
-6/f*c^2*d*e*a^2*sinh(f*x+e)+6/f^2*c*d^2*e^2*a^2*sinh(f*x+e)+1/f^3*d^3*a^2*
(1/2*(f*x+e)^3*cosh(f*x+e)*sinh(f*x+e)+1/8*(f*x+e)^4-3/4*(f*x+e)^2*cosh(f*x
+e)^2+3/4*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/8*(f*x+e)^2-3/8*cosh(f*x+e)^2)+
2/f^3*d^3*a^2*((f*x+e)^3*sinh(f*x+e)-3*(f*x+e)^2*cosh(f*x+e)+6*(f*x+e)*sinh
(f*x+e)-6*cosh(f*x+e))+1/4/f^3*d^3*a^2*(f*x+e)^4+c^3*a^2*(1/2*sinh(f*x+e)*c
osh(f*x+e)+1/2*f*x+1/2*e)+2*c^3*a^2*sinh(f*x+e)+c^3*a^2*(f*x+e))
```

Maxima [B] time = 1.28064, size = 711, normalized size = 3.

$$\frac{1}{4}a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + \frac{3}{16}\left(4x^2 + \frac{\left(2fxe^{(2e)} - e^{(2e)}\right)e^{(2fx)}}{f^2} - \frac{\left(2fx+1\right)e^{(-2fx-2e)}}{f^2}\right)a^2c^2d + \frac{1}{16}\left(8x^3 + \frac{3(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + 3/16*(4*x^2 + (2*f*x*
e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 - (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*c^
2*d + 1/16*(8*x^3 + 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*
x)/f^3 - 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*c*d^2 + 1/32*(
4*x^4 + (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(2*e))
*e^(2*f*x)/f^4 - (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)/f^4)*
a^2*d^3 + 1/8*a^2*c^3*(4*x + e^(2*f*x + 2*e)/f - e^(-2*f*x - 2*e)/f) + a^2*
c^3*x + 3*a^2*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f
^2) + 3*a^2*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2
 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a^2*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6
*f*x*e^e - 6*e^e)*e^(f*x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x -
 e)/f^4) + 2*a^2*c^3*sinh(f*x + e)/f
```

Fricas [A] time = 2.15764, size = 828, normalized size = 3.49

$$\frac{6a^2d^3f^4x^4 + 24a^2cd^2f^4x^3 + 36a^2c^2df^4x^2 + 24a^2c^3f^4x - 3\left(2a^2d^3f^2x^2 + 4a^2cd^2f^2x + 2a^2c^2df^2 + a^2d^3\right)\cosh\left(fx +\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(6*a^2*d^3*f^4*x^4 + 24*a^2*c*d^2*f^4*x^3 + 36*a^2*c^2*d*f^4*x^2 + 24*
a^2*c^3*f^4*x - 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2
+ a^2*d^3)*cosh(f*x + e)^2 - 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a
^2*c^2*d*f^2 + a^2*d^3)*sinh(f*x + e)^2 - 96*(a^2*d^3*f^2*x^2 + 2*a^2*c*d^2
*f^2*x + a^2*c^2*d*f^2 + 2*a^2*d^3)*cosh(f*x + e) + 4*(8*a^2*d^3*f^3*x^3 +
24*a^2*c*d^2*f^3*x^2 + 8*a^2*c^3*f^3 + 48*a^2*c*d^2*f + 24*(a^2*c^2*d*f^3 +
 2*a^2*d^3*f)*x + (2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3*x^2 + 2*a^2*c^3*f^3
+ 3*a^2*c*d^2*f + 3*(2*a^2*c^2*d*f^3 + a^2*d^3*f)*x)*cosh(f*x + e))*sinh(f*
x + e))/f^4
```

Sympy [A] time = 6.98515, size = 779, normalized size = 3.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((-a**2*c**3*x*sinh(e + f*x)**2/2 + a**2*c**3*x*cosh(e + f*x)**2/2
 + a**2*c**3*x + a**2*c**3*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*c**3*
sinh(e + f*x)/f - 3*a**2*c**2*d*x**2*sinh(e + f*x)**2/4 + 3*a**2*c**2*d*x**
2*cosh(e + f*x)**2/4 + 3*a**2*c**2*d*x**2/2 + 3*a**2*c**2*d*x*sinh(e + f*x)
*cosh(e + f*x)/(2*f) + 6*a**2*c**2*d*x*sinh(e + f*x)/f - 3*a**2*c**2*d*sinh
(e + f*x)**2/(4*f**2) - 6*a**2*c**2*d*cosh(e + f*x)/f**2 - a**2*c*d**2*x**3
*sinh(e + f*x)**2/2 + a**2*c*d**2*x**3*cosh(e + f*x)**2/2 + a**2*c*d**2*x**
3 + 3*a**2*c*d**2*x**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 6*a**2*c*d**2*x*
*2*sinh(e + f*x)/f - 3*a**2*c*d**2*x*sinh(e + f*x)**2/(4*f**2) - 3*a**2*c*d
**2*x*cosh(e + f*x)**2/(4*f**2) - 12*a**2*c*d**2*x*cosh(e + f*x)/f**2 + 3*a
**2*c*d**2*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) + 12*a**2*c*d**2*sinh(e + f
*x)/f**3 - a**2*d**3*x**4*sinh(e + f*x)**2/8 + a**2*d**3*x**4*cosh(e + f*x)
**2/8 + a**2*d**3*x**4/4 + a**2*d**3*x**3*sinh(e + f*x)*cosh(e + f*x)/(2*f)
 + 2*a**2*d**3*x**3*sinh(e + f*x)/f - 3*a**2*d**3*x**2*sinh(e + f*x)**2/(8*
f**2) - 3*a**2*d**3*x**2*cosh(e + f*x)**2/(8*f**2) - 6*a**2*d**3*x**2*cosh(
e + f*x)/f**2 + 3*a**2*d**3*x*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) + 12*a**
2*d**3*x*sinh(e + f*x)/f**3 - 3*a**2*d**3*sinh(e + f*x)**2/(8*f**4) - 12*a*
*2*d**3*cosh(e + f*x)/f**4, Ne(f, 0)), ((a*cosh(e) + a)**2*(c**3*x + 3*c**2
*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), True))
```

Giac [B] time = 1.25604, size = 784, normalized size = 3.31

$$\frac{3}{8}\,a^2d^3x^4+\frac{3}{2}\,a^2cd^2x^3+\frac{9}{4}\,a^2c^2dx^2+\frac{3}{2}\,a^2c^3x+\frac{\left(4\,a^2d^3f^3x^3+12\,a^2cd^2f^3x^2+12\,a^2c^2df^3x-6\,a^2d^3f^2x^2+4\,a^2c^3f^3-12\,a\right.}{32\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x + 1
/32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x - 6*a^2*
d^3*f^2*x^2 + 4*a^2*c^3*f^3 - 12*a^2*c*d^2*f^2*x - 6*a^2*c^2*d*f^2 + 6*a^2*
d^3*f*x + 6*a^2*c*d^2*f - 3*a^2*d^3)*e^(2*f*x + 2*e)/f^4 + (a^2*d^3*f^3*x^3
 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x - 3*a^2*d^3*f^2*x^2 + a^2*c^3*f^
3 - 6*a^2*c*d^2*f^2*x - 3*a^2*c^2*d*f^2 + 6*a^2*d^3*f*x + 6*a^2*c*d^2*f - 6
*a^2*d^3)*e^(f*x + e)/f^4 - (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*
```

```
c^2*d*f^3*x + 3*a^2*d^3*f^2*x^2 + a^2*c^3*f^3 + 6*a^2*c*d^2*f^2*x + 3*a^2*c
^2*d*f^2 + 6*a^2*d^3*f*x + 6*a^2*c*d^2*f + 6*a^2*d^3)*e^(-f*x - e)/f^4 - 1/
32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x + 6*a^2*d
^3*f^2*x^2 + 4*a^2*c^3*f^3 + 12*a^2*c*d^2*f^2*x + 6*a^2*c^2*d*f^2 + 6*a^2*d
^3*f*x + 6*a^2*c*d^2*f + 3*a^2*d^3)*e^(-2*f*x - 2*e)/f^4
```

3.106 $\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx$

Optimal. Leaf size=168

$$-\frac{a^2d(c+dx)\cosh^2(e+fx)}{2f^2}-\frac{4a^2d(c+dx)\cosh(e+fx)}{f^2}+\frac{2a^2(c+dx)^2\sinh(e+fx)}{f}+\frac{a^2(c+dx)^2\sinh(e+fx)\cosh(e}{2f}$$

```
[Out] (a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) - (4*a^2*d*(c + d*x)*Cosh[e +
 f*x])/f^2 - (a^2*d*(c + d*x)*Cosh[e + f*x]^2)/(2*f^2) + (4*a^2*d^2*Sinh[e
+ f*x])/f^3 + (2*a^2*(c + d*x)^2*Sinh[e + f*x])/f + (a^2*d^2*Cosh[e + f*x]*
Sinh[e + f*x])/(4*f^3) + (a^2*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f
)
```

Rubi [A] time = 0.187681, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3317, 3296, 2637, 3311, 32, 2635, 8}

$$-\frac{a^2d(c+dx)\cosh^2(e+fx)}{2f^2}-\frac{4a^2d(c+dx)\cosh(e+fx)}{f^2}+\frac{2a^2(c+dx)^2\sinh(e+fx)}{f}+\frac{a^2(c+dx)^2\sinh(e+fx)\cosh(e}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + a*Cosh[e + f*x])^2,x]
```


```
[Out] (a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) - (4*a^2*d*(c + d*x)*Cosh[e +
 f*x])/f^2 - (a^2*d*(c + d*x)*Cosh[e + f*x]^2)/(2*f^2) + (4*a^2*d^2*Sinh[e
+ f*x])/f^3 + (2*a^2*(c + d*x)^2*Sinh[e + f*x])/f + (a^2*d^2*Cosh[e + f*x]*
Sinh[e + f*x])/(4*f^3) + (a^2*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f
)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx = \int \left(a^2(c+dx)^2+2a^2(c+dx)^2\cosh(e+fx)+a^2(c+dx)^2\cosh^2(e+fx)\right)dx$$
$$= \frac{a^2(c+dx)^3}{3d}+a^2\int (c+dx)^2\cosh^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)^2\cosh(e+fx$$
$$= \frac{a^2(c+dx)^3}{3d}-\frac{a^2d(c+dx)\cosh^2(e+fx)}{2f^2}+\frac{2a^2(c+dx)^2\sinh(e+fx)}{f}+\frac{a^2(c+$$
$$= \frac{a^2(c+dx)^3}{2d}-\frac{4a^2d(c+dx)\cosh(e+fx)}{f^2}-\frac{a^2d(c+dx)\cosh^2(e+fx)}{2f^2}+\frac{2a^2(c$$
$$= \frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}-\frac{4a^2d(c+dx)\cosh(e+fx)}{f^2}-\frac{a^2d(c+dx)\cosh^2(e+fx)}{2f^2}$$

Mathematica [A] time = 0.521083, size = 192, normalized size = 1.14

$$a^2\left(16c^2f^2\sinh(e+fx)+2c^2f^2\sinh(2(e+fx))+12c^2f^3x+32cdf^2x\sinh(e+fx)+4cdf^2x\sinh(2(e+fx))-32df$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 - 32*d*f*(c + d*x)*Cosh
[e + f*x] - 2*d*f*(c + d*x)*Cosh[2*(e + f*x)] + 32*d^2*Sinh[e + f*x] + 16*c
^2*f^2*Sinh[e + f*x] + 32*c*d*f^2*x*Sinh[e + f*x] + 16*d^2*f^2*x^2*Sinh[e +
 f*x] + d^2*Sinh[2*(e + f*x)] + 2*c^2*f^2*Sinh[2*(e + f*x)] + 4*c*d*f^2*x*S
inh[2*(e + f*x)] + 2*d^2*f^2*x^2*Sinh[2*(e + f*x)]))/(8*f^3)
```

Maple [B] time = 0.014, size = 541, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+a*cosh(f*x+e))^2,x)
```

```
[Out] 1/f*(1/3/f^2*d^2*a^2*(f*x+e)^3+2/f^2*d^2*a^2*((f*x+e)^2*sinh(f*x+e)-2*(f*x+
e)*cosh(f*x+e)+2*sinh(f*x+e))+1/f^2*d^2*a^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh
(f*x+e)+1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*sinh(f*x+e)*cosh(f*x+e)
+1/4*f*x+1/4*e)-1/f^2*d^2*e*a^2*(f*x+e)^2-4/f^2*d^2*e*a^2*((f*x+e)*sinh(f*x
+e)-cosh(f*x+e))-2/f^2*d^2*e*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(
f*x+e)^2-1/4*cosh(f*x+e)^2)+1/f^2*d^2*e^2*a^2*(f*x+e)+2/f^2*d^2*e^2*a^2*sin
h(f*x+e)+1/f^2*d^2*e^2*a^2*(1/2*sinh(f*x+e)*cosh(f*x+e)+1/2*f*x+1/2*e)+1/f*
c*d*a^2*(f*x+e)^2+4/f*c*d*a^2*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))+2/f*c*d*a^2
*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-2/f*
c*d*e*a^2*(f*x+e)-4/f*c*d*e*a^2*sinh(f*x+e)-2/f*c*d*e*a^2*(1/2sinh(f*x+e)*
cosh(f*x+e)+1/2*f*x+1/2*e)+a^2*c^2*(f*x+e)+2*a^2*c^2*sinh(f*x+e)+a^2*c^2*(1
/2*sinh(f*x+e)*cosh(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [B] time = 1.24887, size = 441, normalized size = 2.62

$$\frac{1}{3}a^2d^2x^3 + a^2cdx^2 + \frac{1}{8}\left(4x^2 + \frac{\left(2fxe^{(2e)} - e^{(2e)}\right)e^{(2fx)}}{f^2} - \frac{\left(2fx+1\right)e^{(-2fx-2e)}}{f^2}\right)a^2cd + \frac{1}{48}\left(8x^3 + \frac{3\left(2f^2x^2e^{(2e)} - 2fxe^{(2e}}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + 1/8*(4*x^2 + (2*f*x*e^(2*e) - e^(2*e))*e^(2
*f*x)/f^2 - (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*c*d + 1/48*(8*x^3 + 3*(2*
f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 - 3*(2*f^2*x^2 + 2
*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*d^2 + 1/8*a^2*c^2*(4*x + e^(2*f*x + 2*e
)/f - e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*a^2*c*d*((f*x*e^e - e^e)*e^(f*x)/
f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + a^2*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e
^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*a^2*c^2*sinh(
f*x + e)/f
```

Fricas [A] time = 2.11826, size = 491, normalized size = 2.92

$$\frac{2a^2d^2f^3x^3 + 6a^2cdf^3x^2 + 6a^2c^2f^3x - \left(a^2d^2fx + a^2cdf\right)\cosh\left(fx+e\right)^2 - \left(a^2d^2fx + a^2cdf\right)\sinh\left(fx+e\right)^2 - 16\left(a^2d^2f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*d^2*f^3*x^3 + 6*a^2*c*d*f^3*x^2 + 6*a^2*c^2*f^3*x - (a^2*d^2*f*x
 + a^2*c*d*f)*cosh(f*x + e)^2 - (a^2*d^2*f*x + a^2*c*d*f)*sinh(f*x + e)^2 -
 16*(a^2*d^2*f*x + a^2*c*d*f)*cosh(f*x + e) + (8*a^2*d^2*f^2*x^2 + 16*a^2*c
*d*f^2*x + 8*a^2*c^2*f^2 + 16*a^2*d^2 + (2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*
x + 2*a^2*c^2*f^2 + a^2*d^2)*cosh(f*x + e))*sinh(f*x + e))/f^3
```

Sympy [A] time = 3.24659, size = 456, normalized size = 2.71

$$\begin{cases} -\frac{a^2c^2x\sinh^2\left(e+fx\right)}{2} + \frac{a^2c^2x\cosh^2\left(e+fx\right)}{2} + a^2c^2x + \frac{a^2c^2\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{2f} + \frac{2a^2c^2\sinh\left(e+fx\right)}{f} - \frac{a^2cdx^2\sinh^2\left(e+fx\right)}{2} + \frac{a^2cdx^2\cosh^2\left(e\right.}{2} \\ \left(a\cosh\left(e\right)+a\right)^2\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((-a**2*c**2*x*sinh(e + f*x)**2/2 + a**2*c**2*x*cosh(e + f*x)**2/2
 + a**2*c**2*x + a**2*c**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*c**2*
sinh(e + f*x)/f - a**2*c*d*x**2*sinh(e + f*x)**2/2 + a**2*c*d*x**2*cosh(e +
 f*x)**2/2 + a**2*c*d*x**2 + a**2*c*d*x*sinh(e + f*x)*cosh(e + f*x)/f + 4*a
**2*c*d*x*sinh(e + f*x)/f - a**2*c*d*sinh(e + f*x)**2/(2*f**2) - 4*a**2*c*d
*cosh(e + f*x)/f**2 - a**2*d**2*x**3*sinh(e + f*x)**2/6 + a**2*d**2*x**3*co
sh(e + f*x)**2/6 + a**2*d**2*x**3/3 + a**2*d**2*x**2*sinh(e + f*x)*cosh(e +
 f*x)/(2*f) + 2*a**2*d**2*x**2*sinh(e + f*x)/f - a**2*d**2*x*sinh(e + f*x)*
*2/(4*f**2) - a**2*d**2*x*cosh(e + f*x)**2/(4*f**2) - 4*a**2*d**2*x*cosh(e
+ f*x)/f**2 + a**2*d**2*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) + 4*a**2*d**2*
sinh(e + f*x)/f**3, Ne(f, 0)), ((a*cosh(e) + a)**2*(c**2*x + c*d*x**2 + d**
2*x**3/3), True))
```

Giac [B] time = 1.2366, size = 450, normalized size = 2.68

$$\frac{1}{2}a^2d^2x^3 + \frac{3}{2}a^2cdx^2 + \frac{3}{2}a^2c^2x + \frac{\left(2\,a^2d^2f^2x^2 + 4\,a^2cdf^2x + 2\,a^2c^2f^2 - 2\,a^2d^2fx - 2\,a^2cdf + a^2d^2\right)e^{(2\,fx+2\,e)}}{16\,f^3} + \frac{\left(a^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x + 1/16*(2*a^2*d^2*f^2*x^2
 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - 2*a^2*d^2*f*x - 2*a^2*c*d*f + a^2*d^2)
*e^(2*f*x + 2*e)/f^3 + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2 - 2
*a^2*d^2*f*x - 2*a^2*c*d*f + 2*a^2*d^2)*e^(f*x + e)/f^3 - (a^2*d^2*f^2*x^2
+ 2*a^2*c*d*f^2*x + a^2*c^2*f^2 + 2*a^2*d^2*f*x + 2*a^2*c*d*f + 2*a^2*d^2)*
e^(-f*x - e)/f^3 - 1/16*(2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^
2 + 2*a^2*d^2*f*x + 2*a^2*c*d*f + a^2*d^2)*e^(-2*f*x - 2*e)/f^3
```

3.107 $\int (c+dx)(a+a\cosh(e+fx))^2\,dx$

Optimal. Leaf size=118

$$\frac{2a^2(c+dx)\sinh(e+fx)}{f}+\frac{a^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{a^2(c+dx)^2}{2d}+\frac{1}{2}a^2cx-\frac{a^2d\cosh^2(e+fx)}{4f^2}-\frac{2a^2d\cos}{}$$

```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a^2*d*Cosh[e + f
*x])/f^2 - (a^2*d*Cosh[e + f*x]^2)/(4*f^2) + (2*a^2*(c + d*x)*Sinh[e + f*x]
)/f + (a^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)
```

Rubi [A] time = 0.099404, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3317, 3296, 2638, 3310}

$$\frac{2a^2(c+dx)\sinh(e+fx)}{f}+\frac{a^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{a^2(c+dx)^2}{2d}+\frac{1}{2}a^2cx-\frac{a^2d\cosh^2(e+fx)}{4f^2}-\frac{2a^2d\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a^2*d*Cosh[e + f
*x])/f^2 - (a^2*d*Cosh[e + f*x]^2)/(4*f^2) + (2*a^2*(c + d*x)*Sinh[e + f*x]
)/f + (a^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)(a+a\cosh(e+fx))^2\,dx = \int \left(a^2(c+dx) + 2a^2(c+dx)\cosh(e+fx) + a^2(c+dx)\cosh^2(e+fx)\right) dx$$
$$= \frac{a^2(c+dx)^2}{2d} + a^2 \int (c+dx)\cosh^2(e+fx)\,dx + \left(2a^2\right)\int (c+dx)\cosh(e+fx)\,dx$$
$$= \frac{a^2(c+dx)^2}{2d} - \frac{a^2 d\cosh^2(e+fx)}{4f^2} + \frac{2a^2(c+dx)\sinh(e+fx)}{f} + \frac{a^2(c+dx)\cosh(}{}$$
$$= \frac{1}{2}a^2cx + \frac{1}{4}a^2dx^2 + \frac{a^2(c+dx)^2}{2d} - \frac{2a^2d\cosh(e+fx)}{f^2} - \frac{a^2d\cosh^2(e+fx)}{4f^2} + \frac{2a^2}{}$$

Mathematica [A] time = 0.478236, size = 81, normalized size = 0.69

$$\frac{a^2(-6(e+fx)(d(e-fx)-2cf) + 16f(c+dx)\sinh(e+fx) + 2f(c+dx)\sinh(2(e+fx)) - 16d\cosh(e+fx) - d\cosh(}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (a^2*(-6*(e + f*x)*(-2*c*f + d*(e - f*x)) - 16*d*Cosh[e + f*x] - d*Cosh[2*(
e + f*x)] + 16*f*(c + d*x)*Sinh[e + f*x] + 2*f*(c + d*x)*Sinh[2*(e + f*x)])
)/(8*f^2)
```

Maple [A] time = 0.013, size = 211, normalized size = 1.8

$$\frac{1}{f}\left(\frac{da^2\left(fx+e\right)^2}{2f} + 2\,\frac{da^2\left(\left(fx+e\right)\sinh\left(fx+e\right) - \cosh\left(fx+e\right)\right)}{f} + \frac{da^2}{f}\left(\frac{\left(fx+e\right)\cosh\left(fx+e\right)\sinh\left(fx+e\right)}{2} + \frac{(f}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+a*cosh(f*x+e))^2,x)
```

```
[Out] 1/f*(1/2/f*d*a^2*(f*x+e)^2+2/f*d*a^2*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))+1/f*
d*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)
-d*e/f*a^2*(f*x+e)-2*d*e/f*a^2*sinh(f*x+e)-d*e/f*a^2*(1/2*sinh(f*x+e)*cosh(
f*x+e)+1/2*f*x+1/2*e)+c*a^2*(f*x+e)+2*c*a^2*sinh(f*x+e)+c*a^2*(1/2*sinh(f*x
+e)*cosh(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A] time = 1.07958, size = 225, normalized size = 1.91

$$\frac{1}{2}a^2dx^2 + \frac{1}{16}\left(4x^2 + \frac{\left(2fxe^{(2e)} - e^{(2e)}\right)e^{(2fx)}}{f^2} - \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)a^2d + \frac{1}{8}a^2c\left(4x + \frac{e^{(2fx+2e)}}{f} - \frac{e^{(-2fx-2e)}}{f}\right) + a^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2 + 1/16*(4*x^2 + (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 - (2*
f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*d + 1/8*a^2*c*(4*x + e^(2*f*x + 2*e)/f -
 e^(-2*f*x - 2*e)/f) + a^2*c*x + a^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x
```

```
+ 1)*e^(-f*x - e)/f^2) + 2*a^2*c*sinh(f*x + e)/f
```

Fricas [A] time = 2.0235, size = 269, normalized size = 2.28

$$\frac{6\,a^2df^2x^2+12\,a^2cf^2x-a^2d\cosh\left(fx+e\right)^2-a^2d\sinh\left(fx+e\right)^2-16\,a^2d\cosh\left(fx+e\right)+4\left(4\,a^2dfx+4\,a^2cf+\left(a^2dfx\right.\right.}{8\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a^2*d*f^2*x^2 + 12*a^2*c*f^2*x - a^2*d*cosh(f*x + e)^2 - a^2*d*sinh(
f*x + e)^2 - 16*a^2*d*cosh(f*x + e) + 4*(4*a^2*d*f*x + 4*a^2*c*f + (a^2*d*f
*x + a^2*c*f)*cosh(f*x + e))*sinh(f*x + e))/f^2
```

Sympy [A] time = 1.39805, size = 219, normalized size = 1.86

$$\begin{cases}-\frac{a^2cx\sinh^2\left(e+fx\right)}{2}+\frac{a^2cx\cosh^2\left(e+fx\right)}{2}+a^2cx+\frac{a^2c\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{2f}+\frac{2a^2c\sinh\left(e+fx\right)}{f}-\frac{a^2dx^2\sinh^2\left(e+fx\right)}{4}+\frac{a^2dx^2\cosh^2\left(e+fx\right)}{4}+ \\ \left(a\cosh\left(e\right)+a\right)^2\left(cx+\frac{dx^2}{2}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((-a**2*c*x*sinh(e + f*x)**2/2 + a**2*c*x*cosh(e + f*x)**2/2 + a**
2*c*x + a**2*c*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*c*sinh(e + f*x)/f
 - a**2*d*x**2*sinh(e + f*x)**2/4 + a**2*d*x**2*cosh(e + f*x)**2/4 + a**2*d
*x**2/2 + a**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*d*x*sinh(e +
f*x)/f - a**2*d*sinh(e + f*x)**2/(4*f**2) - 2*a**2*d*cosh(e + f*x)/f**2, Ne
(f, 0)), ((a*cosh(e) + a)**2*(c*x + d*x**2/2), True))
```

Giac [A] time = 1.32986, size = 209, normalized size = 1.77

$$\frac{3}{4}\,a^2dx^2+\frac{3}{2}\,a^2cx+\frac{\left(2\,a^2dfx+2\,a^2cf-a^2d\right)e^{\left(2\,fx+2\,e\right)}}{16\,f^2}+\frac{\left(a^2dfx+a^2cf-a^2d\right)e^{\left(fx+e\right)}}{f^2}-\frac{\left(a^2dfx+a^2cf+a^2d\right)e^{\left(-fx-e\right)}}{f^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*d*x^2 + 3/2*a^2*c*x + 1/16*(2*a^2*d*f*x + 2*a^2*c*f - a^2*d)*e^(2*f
*x + 2*e)/f^2 + (a^2*d*f*x + a^2*c*f - a^2*d)*e^(f*x + e)/f^2 - (a^2*d*f*x
+ a^2*c*f + a^2*d)*e^(-f*x - e)/f^2 - 1/16*(2*a^2*d*f*x + 2*a^2*c*f + a^2*d
)*e^(-2*f*x - 2*e)/f^2
```

3.108 $\int \frac{(a+a\cosh(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=145

$$\frac{2a^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{a^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{2a^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a^2\sinh\left(2\right.}{}$$

```
[Out] (2*a^2*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (a^2*Cosh[2*e - (
2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (3*a^2*Log[c + d*x])/(2*
d) + (2*a^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d + (a^2*Sinh[2*
e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rubi [A] time = 0.341507, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3318, 3312, 3303, 3298, 3301}

$$\frac{2a^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{a^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{2a^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a^2\sinh\left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[e + f*x])^2/(c + d*x),x]
```


```
[Out] (2*a^2*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (a^2*Cosh[2*e - (
2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (3*a^2*Log[c + d*x])/(2*
d) + (2*a^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d + (a^2*Sinh[2*
e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cosh(e+fx))^2}{c+dx}\,dx &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)}{c+dx}\,dx\\ &= \left(4a^2\right)\int \left(\frac{3}{8(c+dx)}+\frac{\cosh(e+fx)}{2(c+dx)}+\frac{\cosh(2e+2fx)}{8(c+dx)}\right)dx\\ &= \frac{3a^2\log(c+dx)}{2d}+\frac{1}{2}a^2\int \frac{\cosh(2e+2fx)}{c+dx}\,dx+\left(2a^2\right)\int \frac{\cosh(e+fx)}{c+dx}\,dx\\ &= \frac{3a^2\log(c+dx)}{2d}+\frac{1}{2}\left(a^2\cosh\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\cosh\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx+\left(2a^2\cosh\left(e-\frac{cf}{d}\right)\right)\\ &= \frac{2a^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{3a^2\log(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.197252, size = 113, normalized size = 0.78

$$\frac{a^2\left(4\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\cosh\left(e-\frac{cf}{d}\right)+\text{Chi}\left(\frac{2f(c+dx)}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)+4\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+\sinh\left(2e-\frac{2cf}{d}\right)\text{S}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])^2/(c + d*x),x]
```

```
[Out] (a^2*(4*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + Cosh[2*e - (2*c*f)/d]
*CoshIntegral[(2*f*(c + d*x))/d] + 3*Log[c + d*x] + 4*Sinh[e - (c*f)/d]*Sin
hIntegral[f*(c/d + x)] + Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))
/d]))/(2*d)
```

Maple [A] time = 0.105, size = 191, normalized size = 1.3

$$-\frac{a^2}{d}e^{\frac{cf-de}{d}}\text{Ei}\left(1,fx+e+\frac{cf-de}{d}\right)-\frac{a^2}{d}e^{-\frac{cf-de}{d}}\text{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)+\frac{3a^2\ln(dx+c)}{2d}-\frac{a^2}{4d}e^{2\frac{cf-de}{d}}\text{Ei}\left(1,2fx+2e+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(f*x+e))^2/(d*x+c),x)
```

```
[Out] -a^2/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-a^2/d*exp(-(c*f-d*e)/d)*Ei(
1,-f*x-e-(c*f-d*e)/d)+3/2*a^2*ln(d*x+c)/d-1/4*a^2/d*exp(2*(c*f-d*e)/d)*Ei(1
,2*f*x+2*e+2*(c*f-d*e)/d)-1/4*a^2/d*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(
c*f-d*e)/d)
```

Maxima [A] time = 1.30309, size = 201, normalized size = 1.39

$$-\frac{1}{4}a^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_1\left(-\frac{2(dx+c)f}{d}\right)}{d}-\frac{2\log(dx+c)}{d}\right)-a^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*a^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(2*e
- 2*c*f/d)*exp_integral_e(1, -2*(d*x + c)*f/d)/d - 2*log(d*x + c)/d) - a^2*
(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d + e^(e - c*f/d)*exp_inte
gral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

Fricas [A] time = 2.14107, size = 470, normalized size = 3.24

$$6\,a^2\log\left(dx+c\right)+4\left(a^2\operatorname{Ei}\left(\frac{dfx+cf}{d}\right)+a^2\operatorname{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left(a^2\operatorname{Ei}\left(\frac{2\left(dfx+cf\right)}{d}\right)+a^2\operatorname{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)\right)\cosh\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(6*a^2*log(d*x + c) + 4*(a^2*Ei((d*f*x + c*f)/d) + a^2*Ei(-(d*f*x + c*f
)/d))*cosh(-(d*e - c*f)/d) + (a^2*Ei(2*(d*f*x + c*f)/d) + a^2*Ei(-2*(d*f*x
+ c*f)/d))*cosh(-2*(d*e - c*f)/d) - 4*(a^2*Ei((d*f*x + c*f)/d) - a^2*Ei(-(d
*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) - (a^2*Ei(2*(d*f*x + c*f)/d) - a^2*Ei(
-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int\frac{2\cosh\left(e+fx\right)}{c+dx}\,dx+\int\frac{\cosh^{2}\left(e+fx\right)}{c+dx}\,dx+\int\frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))**2/(d*x+c),x)
```

```
[Out] a**2*(Integral(2*cosh(e + f*x)/(c + d*x), x) + Integral(cosh(e + f*x)**2/(c
 + d*x), x) + Integral(1/(c + d*x), x))
```

Giac [A] time = 1.28038, size = 188, normalized size = 1.3

$$\frac{a^2\operatorname{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2cf}{d}-2e\right)}+4\,a^2\operatorname{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+4\,a^2\operatorname{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+a^2\operatorname{Ei}\left(\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(-\frac{2cf}{d}+2e\right)}+6\,a^2\operatorname{lo}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(a^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*a^2*Ei(-(d*f*x + c*f)
/d)*e^(c*f/d - e) + 4*a^2*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) + a^2*Ei(2*(d*
f*x + c*f)/d)*e^(-2*c*f/d + 2*e) + 6*a^2*log(d*x + c))/d
```

3.109 $\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=157

$$\frac{a^2f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2a^2f\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2a^2f\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}+\frac{a^2f\cosh}{}$$

```
[Out] (-4*a^2*Cosh[e/2 + (f*x)/2]^4)/(d*(c + d*x)) + (a^2*f*CoshIntegral[(2*c*f)/
d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d^2 + (2*a^2*f*CoshIntegral[(c*f)/d + f*x
]*Sinh[e - (c*f)/d])/d^2 + (2*a^2*f*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d
+ f*x])/d^2 + (a^2*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])
/d^2
```

Rubi [A] time = 0.332978, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3318, 3313, 3303, 3298, 3301}

$$\frac{a^2f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2a^2f\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2a^2f\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}+\frac{a^2f\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[e + f*x])^2/(c + d*x)^2,x]
```


```
[Out] (-4*a^2*Cosh[e/2 + (f*x)/2]^4)/(d*(c + d*x)) + (a^2*f*CoshIntegral[(2*c*f)/
d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d^2 + (2*a^2*f*CoshIntegral[(c*f)/d + f*x
]*Sinh[e - (c*f)/d])/d^2 + (2*a^2*f*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d
+ f*x])/d^2 + (a^2*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])
/d^2
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^2}\,dx &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)}{(c+dx)^2}\,dx\\ &= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{\left(8ia^2f\right)\int\left(-\frac{i\sinh(e+fx)}{4(c+dx)}-\frac{i\sinh(2e+2fx)}{8(c+dx)}\right)dx}{d}\\ &= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{\left(a^2f\right)\int\frac{\sinh(2e+2fx)}{c+dx}dx}{d}+\frac{\left(2a^2f\right)\int\frac{\sinh(e+fx)}{c+dx}dx}{d}\\ &= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{\left(a^2f\cosh\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\sinh\left(\frac{2cf}{d}+2fx\right)}{c+dx}dx}{d}+\frac{\left(2a^2f\cosh\left(e-\frac{cf}{d}\right)\right.}{d}\\ &= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{a^2f\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2a^2f\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh}{d^2}\end{aligned}$$

Mathematica [A] time = 0.630069, size = 207, normalized size = 1.32

$$\frac{a^2\left(2f(c+dx)\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)+4f(c+dx)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+4dfx\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (a^2*(-3*d - 4*d*Cosh[e + f*x] - d*Cosh[2*(e + f*x)] + 2*f*(c + d*x)*CoshIn
tegral[(2*f*(c + d*x))/d]*Sinh[2*e - (2*c*f)/d] + 4*f*(c + d*x)*CoshIntegra
l[f*(c/d + x)]*Sinh[e - (c*f)/d] + 4*c*f*Cosh[e - (c*f)/d]*SinhIntegral[f*(
c/d + x)] + 4*d*f*x*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 2*c*f*Cos
h[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] + 2*d*f*x*Cosh[2*e - (2*
c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d]))/(2*d^2*(c + d*x))
```

Maple [A] time = 0.123, size = 308, normalized size = 2.

$$-\frac{fa^2e^{-fx-e}}{d\left(dfx+cf\right)}+\frac{fa^2}{d^2}e^{\frac{cf-de}{d}}\mathrm{Ei}\left(1,fx+e+\frac{cf-de}{d}\right)-\frac{fa^2e^{fx+e}}{d^2}\left(\frac{cf}{d}+fx\right)^{-1}-\frac{fa^2}{d^2}e^{-\frac{cf-de}{d}}\mathrm{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(f*x+e))^2/(d*x+c)^2,x)
```

```
[Out] -a^2*f*exp(-f*x-e)/d/(d*f*x+c*f)+a^2*f/d^2*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f
-d*e)/d)-f*a^2/d^2*exp(f*x+e)/(c*f/d+f*x)-f*a^2/d^2*exp(-(c*f-d*e)/d)*Ei(1,
-f*x-e-(c*f-d*e)/d)-3/2*a^2/d/(d*x+c)-1/4*a^2*f*exp(-2*f*x-2*e)/d/(d*f*x+c*
f)+1/2*a^2*f/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4*f*a^2
/d^2*exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*f*a^2/d^2*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f
*x-2*e-2*(c*f-d*e)/d)
```

Maxima [A] time = 1.32004, size = 246, normalized size = 1.57

$$-\frac{1}{4}\,a^2\left(\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_2\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)d}+\frac{2}{d^2x+cd}\right)-a^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*
d) + e^(2*e - 2*c*f/d)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) +
2/(d^2*x + c*d)) - a^2*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d
*x + c)*d) + e^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d))
 - a^2/(d^2*x + c*d)
```

Fricas [B] time = 2.16994, size = 767, normalized size = 4.89

$$-\frac{a^2d\cosh\left(fx+e\right)^2+a^2d\sinh\left(fx+e\right)^2+4\,a^2d\cosh\left(fx+e\right)+3\,a^2d-2\left(\left(a^2dfx+a^2cf\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-\left(a^2dfx+a^2cf\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*d*cosh(f*x + e)^2 + a^2*d*sinh(f*x + e)^2 + 4*a^2*d*cosh(f*x + e)
 + 3*a^2*d - 2*((a^2*d*f*x + a^2*c*f)*Ei((d*f*x + c*f)/d) - (a^2*d*f*x + a^
2*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - ((a^2*d*f*x + a^2*c*f)*
Ei(2*(d*f*x + c*f)/d) - (a^2*d*f*x + a^2*c*f)*Ei(-2*(d*f*x + c*f)/d))*cosh(
-2*(d*e - c*f)/d) + 2*((a^2*d*f*x + a^2*c*f)*Ei((d*f*x + c*f)/d) + (a^2*d*f
*x + a^2*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + ((a^2*d*f*x + a^
2*c*f)*Ei(2*(d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*Ei(-2*(d*f*x + c*f)/d)
)*sinh(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{2\cosh\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx+\int\frac{\cosh^2\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx+\int\frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*cosh(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(
cosh(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**2 + 2*c*
d*x + d**2*x**2), x))
```

Giac [B] time = 2.31396, size = 485, normalized size = 3.09

$$-\frac{2\,a^2dfx\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2\,cf}{d}-2\,e\right)}+4\,a^2dfx\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}-4\,a^2dfx\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}-2\,a^2dfx\mathrm{Ei}\left(\frac{2\left(dfx+cf\right)}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*a^2*d*f*x*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*a^2*d*f*x*Ei
(-(d*f*x + c*f)/d)*e^(c*f/d - e) - 4*a^2*d*f*x*Ei((d*f*x + c*f)/d)*e^(-c*f/
d + e) - 2*a^2*d*f*x*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e) + 2*a^2*c*f*E
i(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*a^2*c*f*Ei(-(d*f*x + c*f)/d)*e^
(c*f/d - e) - 4*a^2*c*f*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) - 2*a^2*c*f*Ei(2
*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e) + a^2*d*e^(2*f*x + 2*e) + 4*a^2*d*e^(f
*x + e) + 4*a^2*d*e^(-f*x - e) + a^2*d*e^(-2*f*x - 2*e))/(d^3*x + c*d^2) -
3/2*a^2/((d*x + c)*d)
```

3.110 $\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=207

$$\frac{a^2f^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^3}+\frac{a^2f^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d^3}+\frac{a^2f^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^3}+\frac{a^2f^2\sin}{}$$

```
[Out] (-2*a^2*Cosh[e/2 + (f*x)/2]^4)/(d*(c + d*x)^2) + (a^2*f^2*Cosh[e - (c*f)/d]
*CoshIntegral[(c*f)/d + f*x])/d^3 + (a^2*f^2*Cosh[2*e - (2*c*f)/d]*CoshInte
gral[(2*c*f)/d + 2*f*x])/d^3 - (4*a^2*f*Cosh[e/2 + (f*x)/2]^3*Sinh[e/2 + (f
*x)/2])/(d^2*(c + d*x)) + (a^2*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d +
 f*x])/d^3 + (a^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x]
)/d^3
```

Rubi [A] time = 0.504832, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3318, 3314, 3312, 3303, 3298, 3301}

$$\frac{a^2f^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^3}+\frac{a^2f^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d^3}+\frac{a^2f^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^3}+\frac{a^2f^2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[e + f*x])^2/(c + d*x)^3,x]
```


```
[Out] (-2*a^2*Cosh[e/2 + (f*x)/2]^4)/(d*(c + d*x)^2) + (a^2*f^2*Cosh[e - (c*f)/d]
*CoshIntegral[(c*f)/d + f*x])/d^3 + (a^2*f^2*Cosh[2*e - (2*c*f)/d]*CoshInte
gral[(2*c*f)/d + 2*f*x])/d^3 - (4*a^2*f*Cosh[e/2 + (f*x)/2]^3*Sinh[e/2 + (f
*x)/2])/(d^2*(c + d*x)) + (a^2*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d +
 f*x])/d^3 + (a^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x]
)/d^3
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^3}\,dx = \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)}{(c+dx)^3}\,dx$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{\left(6a^2f^2\right)\int \frac{\cosh^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{c+dx}\,dx}{d^2}$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{\left(6a^2f^2\right)\int \left(\frac{1}{2(c+dx)}+\frac{\cosh}{2(c}\right.}{d^2}$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^2(c+dx)} + \frac{\left(a^2f^2\right)\int \frac{\cosh(2e+2fx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^2(c+dx)} + \frac{\left(a^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\right)\int}{d^2}$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2} + \frac{a^2f^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d^3} + \frac{a^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\right.}{d^3}$$

Mathematica [A] time = 1.08188, size = 353, normalized size = 1.71

$$a^2\left(4c^2f^2\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+4c^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2f(c+dx)}{d}\right)+4f^2(c+dx)^2\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\cosh\left(e-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] (a^2*(-3*d^2 - 4*d^2*Cosh[e + f*x] - d^2*Cosh[2*(e + f*x)] + 4*f^2*(c + d*x
)^2*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + 4*f^2*(c + d*x)^2*Cosh[2*
e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] - 4*c*d*f*Sinh[e + f*x] - 4*
d^2*f*x*Sinh[e + f*x] - 2*c*d*f*Sinh[2*(e + f*x)] - 2*d^2*f*x*Sinh[2*(e + f
*x)] + 4*c^2*f^2*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 8*c*d*f^2*x*
Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 4*d^2*f^2*x^2*Sinh[e - (c*f)/
d]*SinhIntegral[f*(c/d + x)] + 4*c^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral
```

```
[(2*f*(c + d*x))/d] + 8*c*d*f^2*x*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(
c + d*x))/d] + 4*d^2*f^2*x^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d
*x))/d]))/(4*d^3*(c + d*x)^2)
```

Maple [B] time = 0.134, size = 618, normalized size = 3.

$$\frac{f^3a^2e^{-fx-e}x}{2d\left(d^2f^2x^2+2cdf^2x+c^2f^2\right)}+\frac{f^3a^2e^{-fx-e}c}{2d^2\left(d^2f^2x^2+2cdf^2x+c^2f^2\right)}-\frac{a^2f^2e^{-fx-e}}{2d\left(d^2f^2x^2+2cdf^2x+c^2f^2\right)}-\frac{a^2f^2}{2d^3}e^{\frac{cf-de}{d}}\mathrm{Ei}\left(1,fx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(f*x+e))^2/(d*x+c)^3,x)
```

```
[Out] 1/2*a^2*f^3*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x+1/2*a^2*f^3*e
xp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/2*a^2*f^2*exp(-f*x-e)/
d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/2*a^2*f^2/d^3*exp((c*f-d*e)/d)*Ei(1,f
*x+e+(c*f-d*e)/d)-1/2*a^2*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/2*a^2*f^2/d^3*
exp(f*x+e)/(c*f/d+f*x)-1/2*a^2*f^2/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d
*e)/d)-3/4*a^2/d/(d*x+c)^2+1/4*a^2*f^3*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d
*f^2*x+c^2*f^2)*x+1/4*a^2*f^3*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+
c^2*f^2)*c-1/8*a^2*f^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-
1/2*a^2*f^2/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/8*a^2*f^
2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)^2-1/4*a^2*f^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*
x)-1/2*a^2*f^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)
```

Maxima [A] time = 1.40956, size = 273, normalized size = 1.32

$$-\frac{1}{4}a^2\left(\frac{1}{d^3x^2+2cd^2x+c^2d}+\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_3\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_3\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-a^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\right.}{(dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) + e^(-2*e + 2*c*f/d)*exp_integral
_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) + e^(2*e - 2*c*f/d)*exp_integral_e(3
, -2*(d*x + c)*f/d)/((d*x + c)^2*d)) - a^2*(e^(-e + c*f/d)*exp_integral_e(3
, (d*x + c)*f/d)/((d*x + c)^2*d) + e^(e - c*f/d)*exp_integral_e(3, -(d*x +
c)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] time = 2.1955, size = 1223, normalized size = 5.91

$$-\frac{a^2d^2\cosh\left(fx+e\right)^2+a^2d^2\sinh\left(fx+e\right)^2+4a^2d^2\cosh\left(fx+e\right)+3a^2d^2-2\left(\left(a^2d^2f^2x^2+2a^2cdf^2x+a^2c^2f^2\right)\mathrm{Ei}\left(\frac{dfx}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*d^2*cosh(f*x + e)^2 + a^2*d^2*sinh(f*x + e)^2 + 4*a^2*d^2*cosh(f*
x + e) + 3*a^2*d^2 - 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*E
i((d*f*x + c*f)/d) + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(-
(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^
2*x + a^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2
*x + a^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(d*e - c*f)/d) + 4*(a^2*d
^2*f*x + a^2*c*d*f + (a^2*d^2*f*x + a^2*c*d*f)*cosh(f*x + e))*sinh(f*x + e)
 + 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei((d*f*x + c*f)/d)
 - (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*
sinh(-(d*e - c*f)/d) + 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)
*Ei(2*(d*f*x + c*f)/d) - (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*
Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*
d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int\frac{2\cosh\left(e+fx\right)}{c^{3}+3c^{2}dx+3cd^{2}x^{2}+d^{3}x^{3}}\,dx+\int\frac{\cosh^{2}\left(e+fx\right)}{c^{3}+3c^{2}dx+3cd^{2}x^{2}+d^{3}x^{3}}\,dx+\int\frac{1}{c^{3}+3c^{2}dx+3cd^{2}x^{2}+d^{3}x^{3}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))**2/(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(2*cosh(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3), x) + Integral(cosh(e + f*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3), x) + Integral(1/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x))
```

Giac [B] time = 1.27554, size = 953, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*a^2*d^2
*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + 4*a^2*d^2*f^2*x^2*Ei((d*f*x +
 c*f)/d)*e^(-c*f/d + e) + 4*a^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f
/d + 2*e) + 8*a^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 8*a^
2*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + 8*a^2*c*d*f^2*x*Ei((d*f*x
+ c*f)/d)*e^(-c*f/d + e) + 8*a^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f/
d + 2*e) + 4*a^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*a^2*c
^2*f^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + 4*a^2*c^2*f^2*Ei((d*f*x + c*f)/
d)*e^(-c*f/d + e) + 4*a^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e)
- 2*a^2*d^2*f*x*e^(2*f*x + 2*e) - 4*a^2*d^2*f*x*e^(f*x + e) + 4*a^2*d^2*f*x
*e^(-f*x - e) + 2*a^2*d^2*f*x*e^(-2*f*x - 2*e) - 2*a^2*c*d*f*e^(2*f*x + 2*e
) - 4*a^2*c*d*f*e^(f*x + e) + 4*a^2*c*d*f*e^(-f*x - e) + 2*a^2*c*d*f*e^(-2*
f*x - 2*e) - a^2*d^2*e^(2*f*x + 2*e) - 4*a^2*d^2*e^(f*x + e) - 4*a^2*d^2*e^
(-f*x - e) - a^2*d^2*e^(-2*f*x - 2*e) - 6*a^2*d^2)/(d^5*x^2 + 2*c*d^4*x + c
^2*d^3)
```

3.111 $\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx$

Optimal. Leaf size=117

$$-\frac{12d^2(c+dx)\text{PolyLog}\left(2,-e^{e+fx}\right)}{af^3}+\frac{12d^3\text{PolyLog}\left(3,-e^{e+fx}\right)}{af^4}-\frac{6d(c+dx)^2\log\left(e^{e+fx}+1\right)}{af^2}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

```
[Out] (c + d*x)^3/(a*f) - (6*d*(c + d*x)^2*Log[1 + E^(e + f*x)])/(a*f^2) - (12*d^
2*(c + d*x)*PolyLog[2, -E^(e + f*x)])/(a*f^3) + (12*d^3*PolyLog[3, -E^(e +
f*x)])/(a*f^4) + ((c + d*x)^3*Tanh[e/2 + (f*x)/2])/(a*f)
```

Rubi [A] time = 0.272983, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3318, 4184, 3718, 2190, 2531, 2282, 6589}

$$-\frac{12d^2(c+dx)\text{PolyLog}\left(2,-e^{e+fx}\right)}{af^3}+\frac{12d^3\text{PolyLog}\left(3,-e^{e+fx}\right)}{af^4}-\frac{6d(c+dx)^2\log\left(e^{e+fx}+1\right)}{af^2}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Cosh[e + f*x]),x]
```

```
[Out] (c + d*x)^3/(a*f) - (6*d*(c + d*x)^2*Log[1 + E^(e + f*x)])/(a*f^2) - (12*d^
2*(c + d*x)*PolyLog[2, -E^(e + f*x)])/(a*f^3) + (12*d^3*PolyLog[3, -E^(e +
f*x)])/(a*f^4) + ((c + d*x)^3*Tanh[e/2 + (f*x)/2])/(a*f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx = \frac{\int (c+dx)^3 \csc^2\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)dx}{2a}$$

$$= \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(3d)\int (c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{af}$$

$$= \frac{(c+dx)^3}{af} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(6d)\int \frac{e^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1+e^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}}dx}{af}$$

$$= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2\log\left(1+e^{e+fx}\right)}{af^2} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{\left(12d^2\right)\int (c+dx)\log}{af^2}$$

$$= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2\log\left(1+e^{e+fx}\right)}{af^2} - \frac{12d^2(c+dx)\text{Li}_2\left(-e^{e+fx}\right)}{af^3} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\right.}{af}$$

$$= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2\log\left(1+e^{e+fx}\right)}{af^2} - \frac{12d^2(c+dx)\text{Li}_2\left(-e^{e+fx}\right)}{af^3} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\right.}{af}$$

$$= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2\log\left(1+e^{e+fx}\right)}{af^2} - \frac{12d^2(c+dx)\text{Li}_2\left(-e^{e+fx}\right)}{af^3} + \frac{12d^3\text{Li}_3\left(-e^{e+fx}\right)}{af^4} +$$

Mathematica [A] time = 2.05863, size = 154, normalized size = 1.32

$$\frac{2\cosh\left(\frac{1}{2}(e+fx)\right)\left(2\cosh\left(\frac{1}{2}(e+fx)\right)\left(6d^2\left(f(c+dx)\text{PolyLog}\left(2,-e^{-e-fx}\right)+d\text{PolyLog}\left(3,-e^{-e-fx}\right)\right)-\frac{f^3(c+dx)^3}{e^e+1}-3\right.\right.}{af^4(\cosh(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Cosh[e + f*x]),x]
```

```
[Out] (2*Cosh[(e + f*x)/2]*(2*Cosh[(e + f*x)/2]*(-((f^3*(c + d*x)^3)/(1 + E^e)) -
 3*d*f^2*(c + d*x)^2*Log[1 + E^(-e - f*x)] + 6*d^2*(f*(c + d*x)*PolyLog[2,
-E^(-e - f*x)] + d*PolyLog[3, -E^(-e - f*x)])) + f^3*(c + d*x)^3*Sech[e/2]*
Sinh[(f*x)/2]))/(a*f^4*(1 + Cosh[e + f*x]))
```

Maple [B] time = 0.119, size = 325, normalized size = 2.8

$$-2\,\frac{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}{fa\left(e^{fx+e}+1\right)}-6\,\frac{c^2d\ln\left(e^{fx+e}+1\right)}{af^2}+6\,\frac{c^2d\ln\left(e^{fx+e}\right)}{af^2}+6\,\frac{d^3e^2\ln\left(e^{fx+e}\right)}{f^4a}+2\,\frac{d^3x^3}{af}-6\,\frac{d^3e^2x}{af^3}-4\,\frac{d^3e^3}{f^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*cosh(f*x+e)),x)
```

```
[Out] -2/f*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/a/(exp(f*x+e)+1)-6*d/f^2/a*c^2*ln(
exp(f*x+e)+1)+6*d/f^2/a*c^2*ln(exp(f*x+e))+6*d^3/f^4/a*e^2*ln(exp(f*x+e))+2
*d^3/f/a*x^3-6*d^3/f^3/a*e^2*x-4*d^3/f^4/a*e^3-6*d^3/f^2/a*ln(exp(f*x+e)+1)
*x^2-12*d^3/f^3/a*polylog(2,-exp(f*x+e))*x+12*d^3*polylog(3,-exp(f*x+e))/a/
f^4-12*d^2/f^3/a*c*e*ln(exp(f*x+e))+6*d^2/f/a*c*x^2+12*d^2/f^2/a*c*e*x+6*d^
2/f^3/a*c*e^2-12*d^2/f^2/a*c*ln(exp(f*x+e)+1)*x-12*d^2/f^3/a*c*polylog(2,-e
xp(f*x+e))
```

Maxima [B] time = 1.61465, size = 308, normalized size = 2.63

$$6\,c^2d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}+af}-\frac{\log\left(\left(e^{(fx+e)}+1\right)e^{(-e)}\right)}{af^2}\right)+\frac{2\,c^3}{\left(ae^{(-fx-e)}+a\right)f}-\frac{2\left(d^3x^3+3\,cd^2x^2\right)}{afe^{(fx+e)}+af}-\frac{12\left(fx\log\left(e^{(fx+e)}+1\right)+\mathrm{Li}_2\right.}{af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 6*c^2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) + a*f) - log((e^(f*x + e) + 1)*e^(-
e))/(a*f^2)) + 2*c^3/((a*e^(-f*x - e) + a)*f) - 2*(d^3*x^3 + 3*c*d^2*x^2)/(
a*f*e^(f*x + e) + a*f) - 12*(f*x*log(e^(f*x + e) + 1) + dilog(-e^(f*x + e))
)*c*d^2/(a*f^3) - 6*(f^2*x^2*log(e^(f*x + e) + 1) + 2*f*x*dilog(-e^(f*x + e
)) - 2*polylog(3, -e^(f*x + e)))*d^3/(a*f^4) + 2*(d^3*f^3*x^3 + 3*c*d^2*f^3
*x^2)/(a*f^4)
```

Fricas [C] time = 2.06006, size = 1008, normalized size = 8.62

$$\frac{2\left(d^3e^3-3\,cd^2e^2f+3\,c^2def^2-c^3f^3+\left(d^3f^3x^3+3\,cd^2f^3x^2+3\,c^2df^3x+d^3e^3-3\,cd^2e^2f+3\,c^2def^2\right)\cosh\left(fx+e\right)-6\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3 + (d^3*f^3*x^3 + 3*c*d
^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2)*cosh(
```

```
f*x + e) - 6*(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + e) + (d^3*
f*x + c*d^2*f)*sinh(f*x + e))*dilog(-cosh(f*x + e) - sinh(f*x + e)) - 3*(d^
3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*
d*f^2)*cosh(f*x + e) + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sinh(f*x +
 e))*log(cosh(f*x + e) + sinh(f*x + e) + 1) + 6*(d^3*cosh(f*x + e) + d^3*si
nh(f*x + e) + d^3)*polylog(3, -cosh(f*x + e) - sinh(f*x + e)) + (d^3*f^3*x^
3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f
^2)*sinh(f*x + e))/(a*f^4*cosh(f*x + e) + a*f^4*sinh(f*x + e) + a*f^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^3}{\cosh(e+fx)+1}\,dx + \int \frac{d^3x^3}{\cosh(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\cosh(e+fx)+1}\,dx + \int \frac{3c^2dx}{\cosh(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*cosh(f*x+e)),x)
```

```
[Out] (Integral(c**3/(cosh(e + f*x) + 1), x) + Integral(d**3*x**3/(cosh(e + f*x)
+ 1), x) + Integral(3*c*d**2*x**2/(cosh(e + f*x) + 1), x) + Integral(3*c**2
*d*x/(cosh(e + f*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{a\cosh(fx+e)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*cosh(f*x + e) + a), x)
```

3.112 $\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx$

Optimal. Leaf size=88

$$-\frac{4d^2\text{PolyLog}\left(2,-e^{e+fx}\right)}{af^3}-\frac{4d(c+dx)\log\left(e^{e+fx}+1\right)}{af^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{(c+dx)^2}{af}$$

```
[Out] (c + d*x)^2/(a*f) - (4*d*(c + d*x)*Log[1 + E^(e + f*x)])/(a*f^2) - (4*d^2*P
olyLog[2, -E^(e + f*x)])/(a*f^3) + ((c + d*x)^2*Tanh[e/2 + (f*x)/2])/(a*f)
```

Rubi [A] time = 0.201384, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3318, 4184, 3718, 2190, 2279, 2391}

$$-\frac{4d^2\text{PolyLog}\left(2,-e^{e+fx}\right)}{af^3}-\frac{4d(c+dx)\log\left(e^{e+fx}+1\right)}{af^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{(c+dx)^2}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Cosh[e + f*x]),x]
```


```
[Out] (c + d*x)^2/(a*f) - (4*d*(c + d*x)*Log[1 + E^(e + f*x)])/(a*f^2) - (4*d^2*P
olyLog[2, -E^(e + f*x)])/(a*f^3) + ((c + d*x)^2*Tanh[e/2 + (f*x)/2])/(a*f)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx = \frac{\int (c+dx)^2\csc^2\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)dx}{2a}$$
$$= \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(2d)\int (c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{af}$$
$$= \frac{(c+dx)^2}{af} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(4d)\int \frac{e^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1+e^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}}dx}{af}$$
$$= \frac{(c+dx)^2}{af} - \frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{af^2} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{\left(4d^2\right)\int \log\left(1+e^{2\left(\frac{e}{2}\right.}\right.}{af^2}$$
$$= \frac{(c+dx)^2}{af} - \frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{af^2} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{\left(4d^2\right)\text{Subst}\left(\int \frac{\log(1+}{x}\right.}{af^3}$$
$$= \frac{(c+dx)^2}{af} - \frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{af^2} - \frac{4d^2\text{Li}_2\left(-e^{e+fx}\right)}{af^3} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

Mathematica [C] time = 6.34555, size = 472, normalized size = 5.36

$$\frac{8d^2\text{csch}\left(\frac{e}{2}\right)\text{sech}\left(\frac{e}{2}\right)\cosh^2\left(\frac{e}{2}+\frac{fx}{2}\right)\left(-\frac{1}{4}f^2x^2e^{-\tanh^{-1}\left(\coth\left(\frac{e}{2}\right)\right)}+\frac{i\coth\left(\frac{e}{2}\right)\left(i\text{PolyLog}\left(2,e^{2i\left(i\tanh^{-1}\left(\coth\left(\frac{e}{2}\right)\right)+\frac{ifx}{2}\right)}\right)-\frac{1}{2}fx\left(-\pi+2i\tan\right.\right.}{}\right.}{f^3\sqrt{\text{csch}^2\left(\frac{e}{2}\right)\left(\sinh^2\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2/(a + a*Cosh[e + f*x]),x]
```

```
[Out] (-8*c*d*Cosh[e/2 + (f*x)/2]^2*Sech[e/2]*(Cosh[e/2]*Log[Cosh[e/2]*Cosh[(f*x)
/2] + Sinh[e/2]*Sinh[(f*x)/2]] - (f*x*Sinh[e/2])/2))/(f^2*(a + a*Cosh[e + f
*x])*(Cosh[e/2]^2 - Sinh[e/2]^2)) + (8*d^2*Cosh[e/2 + (f*x)/2]^2*Csch[e/2]*
(-(f^2*x^2)/(4*E^ArcTanh[Coth[e/2]]) + (I*Coth[e/2]*(-(f*x*(-Pi + (2*I)*Arc
Tanh[Coth[e/2]]))/2 - Pi*Log[1 + E^(f*x)] - 2*((I/2)*f*x + I*ArcTanh[Coth[e
/2]])*Log[1 - E^((2*I)*((I/2)*f*x + I*ArcTanh[Coth[e/2]]))] + Pi*Log[Cosh[(
f*x)/2]] + (2*I)*ArcTanh[Coth[e/2]]*Log[I*Sinh[(f*x)/2 + ArcTanh[Coth[e/2]]
]] + I*PolyLog[2, E^((2*I)*((I/2)*f*x + I*ArcTanh[Coth[e/2]]))]))/Sqrt[1 -
Coth[e/2]^2])*Sech[e/2])/(f^3*(a + a*Cosh[e + f*x])*Sqrt[Csch[e/2]^2*(-Cosh
[e/2]^2 + Sinh[e/2]^2)]) + (2*Cosh[e/2 + (f*x)/2]*Sech[e/2]*(c^2*Sinh[(f*x)
/2] + 2*c*d*x*Sinh[(f*x)/2] + d^2*x^2*Sinh[(f*x)/2]))/(f*(a + a*Cosh[e + f*
```

```
x]))
```

Maple [B] time = 0.045, size = 174, normalized size = 2.

$$-2\frac{d^2x^2+2cdx+c^2}{fa\left(e^{fx+e}+1\right)}-4\frac{cd\ln\left(e^{fx+e}+1\right)}{af^2}+4\frac{cd\ln\left(e^{fx+e}\right)}{af^2}+2\frac{d^2x^2}{af}+4\frac{d^2ex}{af^2}+2\frac{d^2e^2}{f^3a}-4\frac{d^2\ln\left(e^{fx+e}+1\right)x}{af^2}-4\frac{d^2\mathrm{pol}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*cosh(f*x+e)),x)
```

```
[Out] -2/f*(d^2*x^2+2*c*d*x+c^2)/a/(exp(f*x+e)+1)-4*d/f^2/a*c*ln(exp(f*x+e)+1)+4*
d/f^2/a*c*ln(exp(f*x+e))+2*d^2/f/a*x^2+4*d^2/f^2/a*e*x+2*d^2/f^3/a*e^2-4*d^
2/f^2/a*ln(exp(f*x+e)+1)*x-4*d^2*polylog(2,-exp(f*x+e))/a/f^3-4*d^2/f^3/a*e
*ln(exp(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\,d^2\left(\frac{x^2}{afe^{(fx+e)}+af}-2\int\frac{x}{afe^{(fx+e)}+af}\,dx\right)+4\,cd\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}+af}-\frac{\log\left(\left(e^{(fx+e)}+1\right)e^{(-e)}\right)}{af^2}\right)+\frac{2\,c^2}{\left(ae^{(-fx-e)}+a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*d^2*(x^2/(a*f*e^(f*x + e) + a*f) - 2*integrate(x/(a*f*e^(f*x + e) + a*f)
, x)) + 4*c*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) + a*f) - log((e^(f*x + e) + 1
)*e^(-e))/(a*f^2)) + 2*c^2/((a*e^(-f*x - e) + a)*f)
```

Fricas [B] time = 2.1028, size = 590, normalized size = 6.7

$$-\frac{2\left(d^2e^2-2\,cdef+c^2f^2-\left(d^2f^2x^2+2\,cdf^2x-d^2e^2+2\,cdef\right)\cosh\left(fx+e\right)+2\left(d^2\cosh\left(fx+e\right)+d^2\sinh\left(fx+e\right)+d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 +
2*c*d*e*f)*cosh(f*x + e) + 2*(d^2*cosh(f*x + e) + d^2*sinh(f*x + e) + d^2)*
dilog(-cosh(f*x + e) - sinh(f*x + e)) + 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d
*f)*cosh(f*x + e) + (d^2*f*x + c*d*f)*sinh(f*x + e))*log(cosh(f*x + e) + si
nh(f*x + e) + 1) - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*sinh(f
*x + e))/(a*f^3*cosh(f*x + e) + a*f^3*sinh(f*x + e) + a*f^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{c^2}{\cosh\left(e+fx\right)+1}\,dx+\int\frac{d^2x^2}{\cosh\left(e+fx\right)+1}\,dx+\int\frac{2cdx}{\cosh\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*cosh(f*x+e)),x)
```

```
[Out] (Integral(c**2/(cosh(e + f*x) + 1), x) + Integral(d**2*x**2/(cosh(e + f*x)
+ 1), x) + Integral(2*c*d*x/(cosh(e + f*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{a\cosh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*cosh(f*x + e) + a), x)
```

3.113 $\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx$

Optimal. Leaf size=49

$$\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}$$

```
[Out] (-2*d*Log[Cosh[e/2 + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tanh[e/2 + (f*x)/2])/(
a*f)
```

Rubi [A] time = 0.0681504, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3318, 4184, 3475}

$$\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Cosh[e + f*x]),x]
```

```
[Out] (-2*d*Log[Cosh[e/2 + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tanh[e/2 + (f*x)/2])/(
a*f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx &= \frac{\int (c+dx)\operatorname{csc}^2\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)dx}{2a}\\
&= \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{d\int \tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{af}\\
&= -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}
\end{aligned}$$

Mathematica [A] time = 0.257865, size = 70, normalized size = 1.43

$$\frac{2\cosh\left(\frac{1}{2}(e+fx)\right)\left(f(c+dx)\sinh\left(\frac{1}{2}(e+fx)\right)-2d\cosh\left(\frac{1}{2}(e+fx)\right)\log\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{af^2(\cosh(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Cosh[e + f*x]),x]
```

```
[Out] (2*Cosh[(e + f*x)/2]*(-2*d*Cosh[(e + f*x)/2]*Log[Cosh[(e + f*x)/2]] + f*(c
+ d*x)*Sinh[(e + f*x)/2]))/(a*f^2*(1 + Cosh[e + f*x]))
```

Maple [A] time = 0.039, size = 63, normalized size = 1.3

$$2\frac{dx}{af}+2\frac{de}{af^2}-2\frac{dx+c}{fa\left(e^{fx+e}+1\right)}-2\frac{d\ln\left(e^{fx+e}+1\right)}{af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*cosh(f*x+e)),x)
```

```
[Out] 2*d/a/f*x+2*d/a/f^2*e-2/f*(d*x+c)/a/(exp(f*x+e)+1)-2*d/a/f^2*ln(exp(f*x+e)+
1)
```

Maxima [A] time = 1.03576, size = 96, normalized size = 1.96

$$2d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}+af}-\frac{\log\left(\left(e^{(fx+e)}+1\right)e^{(-e)}\right)}{af^2}\right)+\frac{2c}{\left(ae^{(-fx-e)}+a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) + a*f) - log((e^(f*x + e) + 1)*e^(-e))/
(a*f^2)) + 2*c/((a*e^(-f*x - e) + a)*f)
```

Fricas [B] time = 2.10761, size = 251, normalized size = 5.12

$$\frac{2\left(dfx\cosh\left(fx+e\right)+dfx\sinh\left(fx+e\right)-cf-\left(d\cosh\left(fx+e\right)+d\sinh\left(fx+e\right)+d\right)\log\left(\cosh\left(fx+e\right)+\sinh\left(f\right.\right.\right.}{af^2\cosh\left(fx+e\right)+af^2\sinh\left(fx+e\right)+af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2*(d*f*x*cosh(f*x + e) + d*f*x*sinh(f*x + e) - c*f - (d*cosh(f*x + e) + d*s
inh(f*x + e) + d)*log(cosh(f*x + e) + sinh(f*x + e) + 1))/(a*f^2*cosh(f*x +
 e) + a*f^2*sinh(f*x + e) + a*f^2)
```

Sympy [A] time = 2.01085, size = 76, normalized size = 1.55

$$\begin{cases} \frac{c\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{dx\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{dx}{af} + \frac{2d\log\left(\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2} & \text{for } f \neq 0 \\ \frac{cx+\frac{dx^2}{2}}{a\cosh(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e)),x)
```

```
[Out] Piecewise((c*tanh(e/2 + f*x/2)/(a*f) + d*x*tanh(e/2 + f*x/2)/(a*f) - d*x/(a
*f) + 2*d*log(tanh(e/2 + f*x/2) + 1)/(a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)
/(a*cosh(e) + a), True))
```

Giac [A] time = 1.30981, size = 96, normalized size = 1.96

$$\frac{2\left(dfxe^{(fx+e)} - de^{(fx+e)}\log\left(e^{(fx+e)}+1\right) - cf - d\log\left(e^{(fx+e)}+1\right)\right)}{af^2e^{(fx+e)} + af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*(d*f*x*e^(f*x + e) - d*e^(f*x + e)*log(e^(f*x + e) + 1) - c*f - d*log(e^(
f*x + e) + 1))/(a*f^2*e^(f*x + e) + a*f^2)
```

3.114 $\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a\cosh(e+fx)+a)},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + a*Cosh[e + f*x])), x]
```

Rubi [A] time = 0.0604593, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + a*Cosh[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + a*Cosh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx=\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx$$

Mathematica [A] time = 8.87347, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + a*Cosh[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + a*Cosh[e + f*x])), x]
```

Maple [A] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+a\cosh\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*cosh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+a*cosh(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2\,d\int \frac{1}{ad^2fx^2 + 2\,acdfx + ac^2f + \left(ad^2fx^2e^e + 2\,acdfxe^e + ac^2fe^e\right)e^{(fx)}}\,dx - \frac{2}{adfx + acf + \left(adfxe^e + acfe^e\right)e^{(fx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*d*integrate(1/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2*e^e +
2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x)), x) - 2/(a*d*f*x + a*c*f + (a*d*f*x
*e^e + a*c*f*e^e)*e^(f*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{adx + ac + (adx + ac)\cosh\left(fx + e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (a*d*x + a*c)*cosh(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c\cosh\left(e+fx\right)+c+dx\cosh\left(e+fx\right)+dx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e)),x)
```

```
[Out] Integral(1/(c*cosh(e + f*x) + c + d*x*cosh(e + f*x) + d*x), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)\left(a\cosh\left(fx + e\right) + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(a*cosh(f*x + e) + a)), x)
```

3.115 $\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a\cosh(e+fx)+a)},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + a*Cosh[e + f*x])), x]
```

Rubi [A] time = 0.0567993, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + a*Cosh[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Cosh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx$$

Mathematica [A] time = 9.31355, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Cosh[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + a*Cosh[e + f*x])), x]
```

Maple [A] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a+a\cosh\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-4\,d\int \frac{1}{ad^3fx^3+3\,acd^2fx^2+3\,ac^2dfx+ac^3f+\left(ad^3fx^3e^e+3\,acd^2fx^2e^e+3\,ac^2dfxe^e+ac^3fe^e\right)e^{(fx)}}\,dx - \frac{}{ad^2fx^2+2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -4*d*integrate(1/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f +
 (a*d^3*f*x^3*e^e + 3*a*c*d^2*f*x^2*e^e + 3*a*c^2*d*f*x*e^e + a*c^3*f*e^e)*
e^(f*x)), x) - 2/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2*e^e +
2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ad^2x^2+2\,acdx+ac^2+\left(ad^2x^2+2\,acdx+ac^2\right)\cosh\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (a*d^2*x^2 + 2*a*c*d*x + a*c^2)
*cosh(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c^2\cosh\left(e+fx\right)+c^2+2cdx\cosh\left(e+fx\right)+2cdx+d^2x^2\cosh\left(e+fx\right)+d^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*cosh(f*x+e)),x)
```

```
[Out] Integral(1/(c**2*cosh(e + f*x) + c**2 + 2*c*d*x*cosh(e + f*x) + 2*c*d*x + d
**2*x**2*cosh(e + f*x) + d**2*x**2), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a\cosh\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*cosh(f*x + e) + a)), x)
```

3.116 $\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=255

$$-\frac{4d^2(c+dx)\text{PolyLog}\left(2,-e^{e+fx}\right)}{a^2f^3}+\frac{4d^3\text{PolyLog}\left(3,-e^{e+fx}\right)}{a^2f^4}-\frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3}-\frac{2d(c+dx)^2\log\left(e^{e+fx}+\right.}{a^2f^2}$$

```
[Out] (c + d*x)^3/(3*a^2*f) - (2*d*(c + d*x)^2*Log[1 + E^(e + f*x)])/(a^2*f^2) +
(4*d^3*Log[Cosh[e/2 + (f*x)/2]])/(a^2*f^4) - (4*d^2*(c + d*x)*PolyLog[2, -E
^(e + f*x)])/(a^2*f^3) + (4*d^3*PolyLog[3, -E^(e + f*x)])/(a^2*f^4) + (d*(c
 + d*x)^2*Sech[e/2 + (f*x)/2]^2)/(2*a^2*f^2) - (2*d^2*(c + d*x)*Tanh[e/2 +
(f*x)/2])/(a^2*f^3) + ((c + d*x)^3*Tanh[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d
*x)^3*Sech[e/2 + (f*x)/2]^2*Tanh[e/2 + (f*x)/2])/(6*a^2*f)
```

Rubi [A] time = 0.363557, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3318, 4186, 4184, 3475, 3718, 2190, 2531, 2282, 6589}

$$-\frac{4d^2(c+dx)\text{PolyLog}\left(2,-e^{e+fx}\right)}{a^2f^3}+\frac{4d^3\text{PolyLog}\left(3,-e^{e+fx}\right)}{a^2f^4}-\frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3}-\frac{2d(c+dx)^2\log\left(e^{e+fx}+\right.}{a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (c + d*x)^3/(3*a^2*f) - (2*d*(c + d*x)^2*Log[1 + E^(e + f*x)])/(a^2*f^2) +
(4*d^3*Log[Cosh[e/2 + (f*x)/2]])/(a^2*f^4) - (4*d^2*(c + d*x)*PolyLog[2, -E
^(e + f*x)])/(a^2*f^3) + (4*d^3*PolyLog[3, -E^(e + f*x)])/(a^2*f^4) + (d*(c
 + d*x)^2*Sech[e/2 + (f*x)/2]^2)/(2*a^2*f^2) - (2*d^2*(c + d*x)*Tanh[e/2 +
(f*x)/2])/(a^2*f^3) + ((c + d*x)^3*Tanh[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d
*x)^3*Sech[e/2 + (f*x)/2]^2*Tanh[e/2 + (f*x)/2])/(6*a^2*f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx = \frac{\int (c+dx)^3\csc^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)dx}{4a^2}$$

$$= \frac{d(c+dx)^2\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{(c+dx)^3\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)^3\mathrm{sech}}{6a}$$

$$= \frac{d(c+dx)^2\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} - \frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c}{}$$

$$= \frac{(c+dx)^3}{3a^2f} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} + \frac{d(c+dx)^2\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} - \frac{2d^2(c+dx)\tanh\left(\right.}{a^2f^3}$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+e^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} + \frac{d(c+dx)^2\mathrm{sech}^2\left(\right.}{2a^2f^2}$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+e^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4d^2(c+dx)\mathrm{Li}_2\left(-\right.}{a^2f^3}$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+e^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4d^2(c+dx)\mathrm{Li}_2\left(-\right.}{a^2f^3}$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+e^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4d^2(c+dx)\mathrm{Li}_2\left(-\right.}{a^2f^3}$$

Mathematica [A] time = 3.43358, size = 462, normalized size = 1.81

$$\frac{\cosh\left(\frac{1}{2}(e+fx)\right)\left(\mathrm{sech}\left(\frac{e}{2}\right)(c+dx)\left(c^2f^2\sinh\left(e+\frac{3fx}{2}\right)+3c^2f^2\sinh\left(\frac{fx}{2}\right)+2cdf^2x\sinh\left(e+\frac{3fx}{2}\right)+3df(c+dx)\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (Cosh[(e + f*x)/2]*((-8*d*Cosh[(e + f*x)/2]^3*(-6*d^2*x + 3*c^2*f^2*x + 3*c
*d*f^2*x^2 + d^2*f^2*x^3 + 6*c*d*f*x*Log[1 + Cosh[e + f*x] - Sinh[e + f*x]]
*(1 + Cosh[e] + Sinh[e]) + 3*d^2*f*x^2*Log[1 + Cosh[e + f*x] - Sinh[e + f*x
]]*(1 + Cosh[e] + Sinh[e]) - (3*(-2*d^2 + c^2*f^2)*(f*x - Log[1 + Cosh[e +
f*x] + Sinh[e + f*x]])*(1 + Cosh[e] + Sinh[e]))/f - 6*c*d*PolyLog[2, -Cosh[
e + f*x] + Sinh[e + f*x]]*(1 + Cosh[e] + Sinh[e]) - (6*d^2*(f*x*PolyLog[2,
-Cosh[e + f*x] + Sinh[e + f*x]] + PolyLog[3, -Cosh[e + f*x] + Sinh[e + f*x]
])*(1 + Cosh[e] + Sinh[e]))/f))/(1 + Cosh[e] + Sinh[e]) + (c + d*x)*Sech[e/
2]*(3*d*f*(c + d*x)*Cosh[(f*x)/2] + 3*d*f*(c + d*x)*Cosh[e + (f*x)/2] - 12*
d^2*Sinh[(f*x)/2] + 3*c^2*f^2*Sinh[(f*x)/2] + 6*c*d*f^2*x*Sinh[(f*x)/2] + 3
*d^2*f^2*x^2*Sinh[(f*x)/2] + 6*d^2*Sinh[e + (f*x)/2] - 6*d^2*Sinh[e + (3*f*
x)/2] + c^2*f^2*Sinh[e + (3*f*x)/2] + 2*c*d*f^2*x*Sinh[e + (3*f*x)/2] + d^2
*f^2*x^2*Sinh[e + (3*f*x)/2])))/(3*a^2*f^3*(1 + Cosh[e + f*x])^2)
```

Maple [B] time = 0.116, size = 600, normalized size = 2.4

$$-\frac{6 f^2 d^3 x^3 e^{fx+e} + 18 f^2 c d^2 x^2 e^{fx+e} + 2 d^3 f^2 x^3 - 6 d^3 f x^2 e^{2fx+2e} + 18 f^2 c^2 d x e^{fx+e} + 6 c d^2 f^2 x^2 - 12 c d^2 f x e^{2fx+2e} - 6 f d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*cosh(f*x+e))^2,x)
```

```
[Out] -2/3*(3*f^2*d^3*x^3*exp(f*x+e)+9*f^2*c*d^2*x^2*exp(f*x+e)+d^3*f^2*x^3-3*d^3
*f*x^2*exp(2*f*x+2*e)+9*f^2*c^2*d*x*exp(f*x+e)+3*c*d^2*f^2*x^2-6*c*d^2*f*x*
exp(2*f*x+2*e)-3*f*d^3*x^2*exp(f*x+e)+3*f^2*c^3*exp(f*x+e)+3*c^2*d*f^2*x-3*
c^2*d*f*exp(2*f*x+2*e)-6*f*c*d^2*x*exp(f*x+e)-6*d^3*x*exp(2*f*x+2*e)+c^3*f^
2-3*f*c^2*d*exp(f*x+e)-6*c*d^2*exp(2*f*x+2*e)-12*d^3*x*exp(f*x+e)-12*c*d^2*
exp(f*x+e)-6*d^3*x-6*c*d^2)/f^3/a^2/(exp(f*x+e)+1)^3-2*d^3/f^3/a^2*e^2*x+2*
d^2/f/a^2*c*x^2+2*d^2/f^3/a^2*c*e^2-2*d^3/f^2/a^2*ln(exp(f*x+e)+1)*x^2-4*d^
3/f^3/a^2*polylog(2,-exp(f*x+e))*x-4/3*d^3/f^4/a^2*e^3+4*d^3*polylog(3,-exp
(f*x+e))/a^2/f^4+4*d^3/f^4/a^2*ln(exp(f*x+e)+1)-4*d^3/f^4/a^2*ln(exp(f*x+e)
)-4*d^2/f^3/a^2*c*polylog(2,-exp(f*x+e))+4*d^2/f^2/a^2*c*e*x+2/3*d^3/f/a^2*
x^3-2*d/f^2/a^2*c^2*ln(exp(f*x+e)+1)+2*d/f^2/a^2*c^2*ln(exp(f*x+e))+2*d^3/f
^4/a^2*e^2*ln(exp(f*x+e))-4*d^2/f^2/a^2*ln(exp(f*x+e)+1)*c*x-4*d^2/f^3/a^2*
c*e*ln(exp(f*x+e))
```

Maxima [B] time = 1.89542, size = 824, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2*c^2*d*((f*x*e^(3*f*x + 3*e) + (3*f*x*e^(2*e) + e^(2*e))*e^(2*f*x) + e^(f*
x + e))/(a^2*f^2*e^(3*f*x + 3*e) + 3*a^2*f^2*e^(2*f*x + 2*e) + 3*a^2*f^2*e^
(f*x + e) + a^2*f^2) - log((e^(f*x + e) + 1)*e^(-e))/(a^2*f^2)) + 2/3*c^3*(
3*e^(-f*x - e)/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2*f*x - 2*e) + a^2*e^(-3*f*
x - 3*e) + a^2)*f) + 1/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2*f*x - 2*e) + a^2*
e^(-3*f*x - 3*e) + a^2)*f)) - 2/3*(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 - 6*d^3*x
- 6*c*d^2 - 3*(d^3*f*x^2*e^(2*e) + 2*c*d^2*e^(2*e) + 2*(c*d^2*f*e^(2*e) + d
^3*e^(2*e))*x)*e^(2*f*x) + 3*(d^3*f^2*x^3*e^e - 4*c*d^2*e^e + (3*c*d^2*f^2*
e^e - d^3*f*e^e)*x^2 - 2*(c*d^2*f*e^e + 2*d^3*e^e)*x)*e^(f*x))/(a^2*f^3*e^(
3*f*x + 3*e) + 3*a^2*f^3*e^(2*f*x + 2*e) + 3*a^2*f^3*e^(f*x + e) + a^2*f^3)
 - 4*(f*x*log(e^(f*x + e) + 1) + dilog(-e^(f*x + e)))*c*d^2/(a^2*f^3) - 4*d
^3*x/(a^2*f^3) - 2*(f^2*x^2*log(e^(f*x + e) + 1) + 2*f*x*dilog(-e^(f*x + e)
) - 2*polylog(3, -e^(f*x + e)))*d^3/(a^2*f^4) + 4*d^3*log(e^(f*x + e) + 1)/
(a^2*f^4) + 2/3*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/(a^2*f^4)
```

Fricas [C] time = 2.32742, size = 4103, normalized size = 16.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 - 6*d^3*e + (d^3*f^3*x^3 + 3*c*d^2*f
^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - 6*d^3*e + 3*(c^2*d*f^3 -
 2*d^3*f)*x)*cosh(f*x + e)^3 + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3
*c*d^2*e^2*f + 3*c^2*d*e*f^2 - 6*d^3*e + 3*(c^2*d*f^3 - 2*d^3*f)*x)*sinh(f*
x + e)^3 + 3*(d^3*f^3*x^3 + d^3*e^3 - 6*d^3*e + (3*c^2*d*e + c^2*d)*f^2 + (
3*c*d^2*f^3 + d^3*f^2)*x^2 - (3*c*d^2*e^2 - 2*c*d^2)*f + (3*c^2*d*f^3 + 2*c
*d^2*f^2 - 4*d^3*f)*x)*cosh(f*x + e)^2 + 3*(d^3*f^3*x^3 + d^3*e^3 - 6*d^3*e
 + (3*c^2*d*e + c^2*d)*f^2 + (3*c*d^2*f^3 + d^3*f^2)*x^2 - (3*c*d^2*e^2 - 2
*c*d^2)*f + (3*c^2*d*f^3 + 2*c*d^2*f^2 - 4*d^3*f)*x + (d^3*f^3*x^3 + 3*c*d^
2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - 6*d^3*e + 3*(c^2*d*f^
3 - 2*d^3*f)*x)*cosh(f*x + e))*sinh(f*x + e)^2 - 3*(c*d^2*e^2 - 2*c*d^2)*f
 + 3*(d^3*f^2*x^2 + d^3*e^3 - c^3*f^3 - 6*d^3*e + (3*c^2*d*e + c^2*d)*f^2 -
(3*c*d^2*e^2 - 4*c*d^2)*f + 2*(c*d^2*f^2 - d^3*f)*x)*cosh(f*x + e) - 6*(d^3
*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + e)^3 + (d^3*f*x + c*d^2*f)*
sinh(f*x + e)^3 + 3*(d^3*f*x + c*d^2*f)*cosh(f*x + e)^2 + 3*(d^3*f*x + c*d^
2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(d^3*f*x + c*d
^2*f)*cosh(f*x + e) + 3*(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x +
 e)^2 + 2*(d^3*f*x + c*d^2*f)*cosh(f*x + e))*sinh(f*x + e))*dilog(-cosh(f*x
 + e) - sinh(f*x + e)) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*
f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*cosh(f*x + e)^3 + (d^3*f^2*x^2
 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*sinh(f*x + e)^3 - 2*d^3 + 3*(d^3*f^2*
x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*cosh(f*x + e)^2 + 3*(d^3*f^2*x^2 +
 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f
^2 - 2*d^3)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x
 + c^2*d*f^2 - 2*d^3)*cosh(f*x + e) + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*
d*f^2 - 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*cosh(f*x
+ e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*cosh(f*x + e))
*sinh(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e) + 1) + 6*(d^3*cosh(f*x +
e)^3 + d^3*sinh(f*x + e)^3 + 3*d^3*cosh(f*x + e)^2 + 3*d^3*cosh(f*x + e) +
d^3 + 3*(d^3*cosh(f*x + e) + d^3)*sinh(f*x + e)^2 + 3*(d^3*cosh(f*x + e)^2
+ 2*d^3*cosh(f*x + e) + d^3)*sinh(f*x + e))*polylog(3, -cosh(f*x + e) - sin
h(f*x + e)) + 3*(d^3*f^2*x^2 + d^3*e^3 - c^3*f^3 - 6*d^3*e + (3*c^2*d*e + c
^2*d)*f^2 + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^
2*d*e*f^2 - 6*d^3*e + 3*(c^2*d*f^3 - 2*d^3*f)*x)*cosh(f*x + e)^2 - (3*c*d^2
*e^2 - 4*c*d^2)*f + 2*(c*d^2*f^2 - d^3*f)*x + 2*(d^3*f^3*x^3 + d^3*e^3 - 6*
d^3*e + (3*c^2*d*e + c^2*d)*f^2 + (3*c*d^2*f^3 + d^3*f^2)*x^2 - (3*c*d^2*e^
2 - 2*c*d^2)*f + (3*c^2*d*f^3 + 2*c*d^2*f^2 - 4*d^3*f)*x)*cosh(f*x + e))*si
nh(f*x + e))/(a^2*f^4*cosh(f*x + e)^3 + a^2*f^4*sinh(f*x + e)^3 + 3*a^2*f^4
*cosh(f*x + e)^2 + 3*a^2*f^4*cosh(f*x + e) + a^2*f^4 + 3*(a^2*f^4*cosh(f*x
+ e) + a^2*f^4)*sinh(f*x + e)^2 + 3*(a^2*f^4*cosh(f*x + e)^2 + 2*a^2*f^4*co
sh(f*x + e) + a^2*f^4)*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^3}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{d^3x^3}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{3c^2dx}{\cosh^2(e+fx)+2\cosh(}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*cosh(f*x+e))**2,x)
```

```
[Out] (Integral(c**3/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(d**3
*x**3/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(3*c*d**2*x**2
/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(3*c**2*d*x/(cosh(e
 + f*x)**2 + 2*cosh(e + f*x) + 1), x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{\left(a\cosh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*cosh(f*x + e) + a)^2, x)
```

3.117 $\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{4d^2\text{PolyLog}\left(2,-e^{e+fx}\right)}{3a^2f^3}-\frac{4d(c+dx)\log\left(e^{e+fx}+1\right)}{3a^2f^2}+\frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+}{}$$

```
[Out] (c + d*x)^2/(3*a^2*f) - (4*d*(c + d*x)*Log[1 + E^(e + f*x)])/(3*a^2*f^2) -
(4*d^2*PolyLog[2, -E^(e + f*x)])/(3*a^2*f^3) + (d*(c + d*x)*Sech[e/2 + (f*x
)/2]^2)/(3*a^2*f^2) - (2*d^2*Tanh[e/2 + (f*x)/2])/(3*a^2*f^3) + ((c + d*x)^
2*Tanh[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)^2*Sech[e/2 + (f*x)/2]^2*Tanh[
e/2 + (f*x)/2])/(6*a^2*f)
```

Rubi [A] time = 0.252144, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3318, 4186, 3767, 8, 4184, 3718, 2190, 2279, 2391}

$$-\frac{4d^2\text{PolyLog}\left(2,-e^{e+fx}\right)}{3a^2f^3}-\frac{4d(c+dx)\log\left(e^{e+fx}+1\right)}{3a^2f^2}+\frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (c + d*x)^2/(3*a^2*f) - (4*d*(c + d*x)*Log[1 + E^(e + f*x)])/(3*a^2*f^2) -
(4*d^2*PolyLog[2, -E^(e + f*x)])/(3*a^2*f^3) + (d*(c + d*x)*Sech[e/2 + (f*x
)/2]^2)/(3*a^2*f^2) - (2*d^2*Tanh[e/2 + (f*x)/2])/(3*a^2*f^3) + ((c + d*x)^
2*Tanh[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)^2*Sech[e/2 + (f*x)/2]^2*Tanh[
e/2 + (f*x)/2])/(6*a^2*f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx = \frac{\int (c+dx)^2\csc^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)dx}{4a^2}$$

$$= \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)^2\operatorname{sech}^2}{6a^2}$$

$$= \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\right.}{6a^2f}$$

$$= \frac{(c+dx)^2}{3a^2f} + \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}$$

$$= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{3a^2f^2} + \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3}$$

$$= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{3a^2f^2} + \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3}$$

$$= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{3a^2f^2} - \frac{4d^2\operatorname{Li}_2\left(-e^{e+fx}\right)}{3a^2f^3} + \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} -$$

Mathematica [C] time = 6.45686, size = 637, normalized size = 3.18

$$\frac{16d^2\operatorname{csch}\left(\frac{e}{2}\right)\operatorname{sech}\left(\frac{e}{2}\right)\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)\left(-\frac{1}{4}f^2x^2e^{-\tanh^{-1}\left(\coth\left(\frac{e}{2}\right)\right)} + \frac{i\coth\left(\frac{e}{2}\right)\left(i\text{PolyLog}\left(2,e^{2i\left(i\tanh^{-1}\left(\coth\left(\frac{e}{2}\right)\right)+\frac{ifx}{2}\right)}\right)-\frac{1}{2}fx\left(-\pi+2i\tanh\right.\right.}{}\right.}{3f^3\sqrt{\operatorname{csch}^2\left(\frac{e}{2}\right)\left(\sinh^2\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (-16*c*d*Cosh[e/2 + (f*x)/2]^4*Sech[e/2]*(Cosh[e/2]*Log[Cosh[e/2]*Cosh[(f*x
)/2] + Sinh[e/2]*Sinh[(f*x)/2]] - (f*x*Sinh[e/2])/2))/(3*f^2*(a + a*Cosh[e
+ f*x])^2*(Cosh[e/2]^2 - Sinh[e/2]^2)) + (16*d^2*Cosh[e/2 + (f*x)/2]^4*Csch
[e/2]*(-(f^2*x^2)/(4*E^ArcTanh[Coth[e/2]]) + (I*Coth[e/2]*(-(f*x*(-Pi + (2*
I)*ArcTanh[Coth[e/2]]))/2 - Pi*Log[1 + E^(f*x)] - 2*((I/2)*f*x + I*ArcTanh[
Coth[e/2]])*Log[1 - E^((2*I)*((I/2)*f*x + I*ArcTanh[Coth[e/2]]))] + Pi*Log[
Cosh[(f*x)/2]] + (2*I)*ArcTanh[Coth[e/2]]*Log[I*Sinh[(f*x)/2 + ArcTanh[Coth
[e/2]]]] + I*PolyLog[2, E^((2*I)*((I/2)*f*x + I*ArcTanh[Coth[e/2]]))]))/Sqr
t[1 - Coth[e/2]^2])*Sech[e/2])/(3*f^3*(a + a*Cosh[e + f*x])^2*Sqrt[Csch[e/2
]^2*(-Cosh[e/2]^2 + Sinh[e/2]^2)]) + (Cosh[e/2 + (f*x)/2]*Sech[e/2]*(2*c*d*
f*Cosh[(f*x)/2] + 2*d^2*f*x*Cosh[(f*x)/2] + 2*c*d*f*Cosh[e + (f*x)/2] + 2*d
^2*f*x*Cosh[e + (f*x)/2] - 4*d^2*Sinh[(f*x)/2] + 3*c^2*f^2*Sinh[(f*x)/2] +
6*c*d*f^2*x*Sinh[(f*x)/2] + 3*d^2*f^2*x^2*Sinh[(f*x)/2] + 2*d^2*Sinh[e + (f
*x)/2] - 2*d^2*Sinh[e + (3*f*x)/2] + c^2*f^2*Sinh[e + (3*f*x)/2] + 2*c*d*f^
2*x*Sinh[e + (3*f*x)/2] + d^2*f^2*x^2*Sinh[e + (3*f*x)/2]))/(3*f^3*(a + a*C
osh[e + f*x])^2)
```

Maple [A] time = 0.065, size = 313, normalized size = 1.6

$$-\frac{6 f^2 d^2 x^2 e^{fx+e} + 12 f^2 c d x e^{fx+e} + 2 d^2 f^2 x^2 - 4 d^2 f x e^{2fx+2e} + 6 f^2 c^2 e^{fx+e} + 4 c d f^2 x - 4 c d f e^{2fx+2e} - 4 f d^2 x e^{fx+e} + 2 c^2}{3 f^3 a^2 \left(e^{fx+e}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*cosh(f*x+e))^2,x)
```

```
[Out] -2/3*(3*f^2*d^2*x^2*exp(f*x+e)+6*f^2*c*d*x*exp(f*x+e)+d^2*f^2*x^2-2*d^2*f*x
*exp(2*f*x+2*e)+3*f^2*c^2*exp(f*x+e)+2*c*d*f^2*x-2*c*d*f*exp(2*f*x+2*e)-2*f
*d^2*x*exp(f*x+e)+c^2*f^2-2*f*c*d*exp(f*x+e)-2*d^2*exp(2*f*x+2*e)-4*d^2*exp
(f*x+e)-2*d^2)/f^3/a^2/(exp(f*x+e)+1)^3-4/3/f^2*d/a^2*c*ln(exp(f*x+e)+1)+4/
3*d/f^2/a^2*ln(exp(f*x+e))*c+2/3*d^2/f/a^2*x^2+4/3*d^2/f^2/a^2*e*x+2/3*d^2/
f^3/a^2*e^2-4/3/f^2*d^2/a^2*ln(exp(f*x+e)+1)*x-4/3*d^2*polylog(2,-exp(f*x+e
))/a^2/f^3-4/3*d^2/f^3/a^2*e*ln(exp(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{3} d^2 \left(\frac{f^2 x^2 - 2\left(f x e^{(2e)} + e^{(2e)}\right) e^{(2fx)} + \left(3 f^2 x^2 e^e - 2 f x e^e - 4 e^e\right) e^{(fx)} - 2}{a^2 f^3 e^{(3fx+3e)} + 3 a^2 f^3 e^{(2fx+2e)} + 3 a^2 f^3 e^{(fx+e)} + a^2 f^3} - 6 \int \frac{x}{3\left(a^2 f e^{(fx+e)} + a^2 f\right)} dx\right) + \frac{4}{3} c d \left(\frac{fx}{a^2 f^2 e^{(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*d^2*((f^2*x^2 - 2*(f*x*e^(2*e) + e^(2*e))*e^(2*f*x) + (3*f^2*x^2*e^e -
 2*f*x*e^e - 4*e^e)*e^(f*x) - 2)/(a^2*f^3*e^(3*f*x + 3*e) + 3*a^2*f^3*e^(2*
f*x + 2*e) + 3*a^2*f^3*e^(f*x + e) + a^2*f^3) - 6*integrate(1/3*x/(a^2*f*e^
(f*x + e) + a^2*f), x)) + 4/3*c*d*((f*x*e^(3*f*x + 3*e) + (3*f*x*e^(2*e) +
e^(2*e))*e^(2*f*x) + e^(f*x + e))/(a^2*f^2*e^(3*f*x + 3*e) + 3*a^2*f^2*e^(2
*f*x + 2*e) + 3*a^2*f^2*e^(f*x + e) + a^2*f^2) - log((e^(f*x + e) + 1)*e^(-
e))/(a^2*f^2)) + 2/3*c^2*(3*e^(-f*x - e)/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2
*f*x - 2*e) + a^2*e^(-3*f*x - 3*e) + a^2)*f) + 1/((3*a^2*e^(-f*x - e) + 3*a
^2*e^(-2*f*x - 2*e) + a^2*e^(-3*f*x - 3*e) + a^2)*f))
```

Fricas [B] time = 2.19362, size = 2267, normalized size = 11.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2
+ 2*c*d*e*f)*cosh(f*x + e)^3 - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d
*e*f)*sinh(f*x + e)^3 - (3*d^2*f^2*x^2 - 3*d^2*e^2 + 2*d^2 + 2*(3*c*d*e + c
*d)*f + 2*(3*c*d*f^2 + d^2*f)*x)*cosh(f*x + e)^2 - (3*d^2*f^2*x^2 - 3*d^2*e
^2 + 2*d^2 + 2*(3*c*d*e + c*d)*f + 2*(3*c*d*f^2 + d^2*f)*x + 3*(d^2*f^2*x^2
 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e))*sinh(f*x + e)^2 - 2*d^
2 + (3*d^2*e^2 + 3*c^2*f^2 - 2*d^2*f*x - 4*d^2 - 2*(3*c*d*e + c*d)*f)*cosh(
f*x + e) + 2*(d^2*cosh(f*x + e)^3 + d^2*sinh(f*x + e)^3 + 3*d^2*cosh(f*x +
```

```
e)^2 + 3*d^2*cosh(f*x + e) + 3*(d^2*cosh(f*x + e) + d^2)*sinh(f*x + e)^2 +
d^2 + 3*(d^2*cosh(f*x + e)^2 + 2*d^2*cosh(f*x + e) + d^2)*sinh(f*x + e))*di
log(-cosh(f*x + e) - sinh(f*x + e)) + 2*(d^2*f*x + (d^2*f*x + c*d*f)*cosh(f
*x + e)^3 + (d^2*f*x + c*d*f)*sinh(f*x + e)^3 + c*d*f + 3*(d^2*f*x + c*d*f)
*cosh(f*x + e)^2 + 3*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cosh(f*x + e))*si
nh(f*x + e)^2 + 3*(d^2*f*x + c*d*f)*cosh(f*x + e) + 3*(d^2*f*x + c*d*f + (d
^2*f*x + c*d*f)*cosh(f*x + e)^2 + 2*(d^2*f*x + c*d*f)*cosh(f*x + e))*sinh(f
*x + e))*log(cosh(f*x + e) + sinh(f*x + e) + 1) + (3*d^2*e^2 + 3*c^2*f^2 -
2*d^2*f*x - 3*(d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*cosh(f*x +
e)^2 - 4*d^2 - 2*(3*c*d*e + c*d)*f - 2*(3*d^2*f^2*x^2 - 3*d^2*e^2 + 2*d^2 +
 2*(3*c*d*e + c*d)*f + 2*(3*c*d*f^2 + d^2*f)*x)*cosh(f*x + e))*sinh(f*x + e
))/(a^2*f^3*cosh(f*x + e)^3 + a^2*f^3*sinh(f*x + e)^3 + 3*a^2*f^3*cosh(f*x
+ e)^2 + 3*a^2*f^3*cosh(f*x + e) + a^2*f^3 + 3*(a^2*f^3*cosh(f*x + e) + a^2
*f^3)*sinh(f*x + e)^2 + 3*(a^2*f^3*cosh(f*x + e)^2 + 2*a^2*f^3*cosh(f*x + e
) + a^2*f^3)*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^2}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{d^2x^2}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{2cdx}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*cosh(f*x+e))**2,x)
```

```
[Out] (Integral(c**2/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(d**2
*x**2/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(2*c*d*x/(cosh
(e + f*x)**2 + 2*cosh(e + f*x) + 1), x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{\left(a\cosh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*cosh(f*x + e) + a)^2, x)
```

3.118 $\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=123

$$\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{d\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2}$$

```
[Out] (-2*d*Log[Cosh[e/2 + (f*x)/2]])/(3*a^2*f^2) + (d*Sech[e/2 + (f*x)/2]^2)/(6*
a^2*f^2) + ((c + d*x)*Tanh[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)*Sech[e/2
+ (f*x)/2]^2*Tanh[e/2 + (f*x)/2])/(6*a^2*f)
```

Rubi [A] time = 0.0955596, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3318, 4185, 4184, 3475}

$$\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{d\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Cosh[e + f*x])^2,x]
```


```
[Out] (-2*d*Log[Cosh[e/2 + (f*x)/2]])/(3*a^2*f^2) + (d*Sech[e/2 + (f*x)/2]^2)/(6*
a^2*f^2) + ((c + d*x)*Tanh[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)*Sech[e/2
+ (f*x)/2]^2*Tanh[e/2 + (f*x)/2])/(6*a^2*f)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx = \frac{\int (c+dx)\csc^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)dx}{4a^2}$$
$$= \frac{d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2}$$
$$= \frac{d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} -$$
$$= -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2} + \frac{d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)\text{sech}}{}$$

Mathematica [A] time = 0.437002, size = 114, normalized size = 0.93

$$\frac{\cosh\left(\frac{1}{2}(e+fx)\right)\left(f(c+dx)\left(3\sinh\left(\frac{1}{2}(e+fx)\right)+\sinh\left(\frac{3}{2}(e+fx)\right)\right)-2d\cosh\left(\frac{3}{2}(e+fx)\right)\log\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right)+}{3a^2f^2(\cosh(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (Cosh[(e + f*x)/2]*(-2*d*Cosh[(3*(e + f*x))/2]*Log[Cosh[(e + f*x)/2]] + Cos
h[(e + f*x)/2]*(2*d - 6*d*Log[Cosh[(e + f*x)/2]]) + f*(c + d*x)*(3*Sinh[(e
+ f*x)/2] + Sinh[(3*(e + f*x))/2])))/(3*a^2*f^2*(1 + Cosh[e + f*x])^2)
```

Maple [A] time = 0.06, size = 108, normalized size = 0.9

$$\frac{2\,dx}{3\,a^2f}+\frac{2\,de}{3\,a^2f^2}-\frac{6\,dfx\mathrm{e}^{fx+e}+6\,cf\mathrm{e}^{fx+e}+2\,dfx-2\,d\mathrm{e}^{2\,fx+2\,e}+2\,cf-2\,d\mathrm{e}^{fx+e}}{3\,a^2f^2\left(\mathrm{e}^{fx+e}+1\right)^3}-\frac{2\,d\ln\left(\mathrm{e}^{fx+e}+1\right)}{3\,a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*cosh(f*x+e))^2,x)
```

```
[Out] 2/3*d/a^2/f*x+2/3*d/a^2/f^2*e-2/3*(3*d*f*x*exp(f*x+e)+3*c*f*exp(f*x+e)+d*f*
x-d*exp(2*f*x+2*e)+c*f-d*exp(f*x+e))/f^2/a^2/(exp(f*x+e)+1)^3-2/3*d/a^2/f^2
*ln(exp(f*x+e)+1)
```

Maxima [B] time = 1.10989, size = 323, normalized size = 2.63

$$\frac{2}{3}d\left(\frac{fxe^{(3\,fx+3\,e)}+\left(3\,fxe^{(2\,e)}+e^{(2\,e)}\right)e^{(2\,fx)}+e^{(fx+e)}}{a^2f^2e^{(3\,fx+3\,e)}+3\,a^2f^2e^{(2\,fx+2\,e)}+3\,a^2f^2e^{(fx+e)}+a^2f^2}-\frac{\log\left(\left(e^{(fx+e)}+1\right)e^{(-e)}\right)}{a^2f^2}\right)+\frac{2}{3}c\left(\frac{3\,e^{(}}{\left(3\,a^2e^{(-fx-e)}+3\,a^2e^{(-2\,f}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*d*((f*x*e^(3*f*x + 3*e) + (3*f*x*e^(2*e) + e^(2*e))*e^(2*f*x) + e^(f*x
+ e))/(a^2*f^2*e^(3*f*x + 3*e) + 3*a^2*f^2*e^(2*f*x + 2*e) + 3*a^2*f^2*e^(f
*x + e) + a^2*f^2) - log((e^(f*x + e) + 1)*e^(-e))/(a^2*f^2)) + 2/3*c*(3*e^
(-f*x - e)/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2*f*x - 2*e) + a^2*e^(-3*f*x -
3*e) + a^2)*f) + 1/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2*f*x - 2*e) + a^2*e^(-
3*f*x - 3*e) + a^2)*f))
```

Fricas [B] time = 2.11928, size = 1002, normalized size = 8.15

$$\frac{2\left(dfx\cosh\left(fx+e\right)^{3}+dfx\sinh\left(fx+e\right)^{3}+\left(3\,dfx+d\right)\cosh\left(fx+e\right)^{2}+\left(3\,dfx\cosh\left(fx+e\right)+3\,dfx+d\right)\sinh\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(d*f*x*cosh(f*x + e)^3 + d*f*x*sinh(f*x + e)^3 + (3*d*f*x + d)*cosh(f*x
 + e)^2 + (3*d*f*x*cosh(f*x + e) + 3*d*f*x + d)*sinh(f*x + e)^2 - c*f - (3*
c*f - d)*cosh(f*x + e) - (d*cosh(f*x + e)^3 + d*sinh(f*x + e)^3 + 3*d*cosh(
f*x + e)^2 + 3*(d*cosh(f*x + e) + d)*sinh(f*x + e)^2 + 3*d*cosh(f*x + e) +
3*(d*cosh(f*x + e)^2 + 2*d*cosh(f*x + e) + d)*sinh(f*x + e) + d)*log(cosh(f
*x + e) + sinh(f*x + e) + 1) + (3*d*f*x*cosh(f*x + e)^2 - 3*c*f + 2*(3*d*f*
x + d)*cosh(f*x + e) + d)*sinh(f*x + e))/(a^2*f^2*cosh(f*x + e)^3 + a^2*f^2
*sinh(f*x + e)^3 + 3*a^2*f^2*cosh(f*x + e)^2 + 3*a^2*f^2*cosh(f*x + e) + a^
2*f^2 + 3*(a^2*f^2*cosh(f*x + e) + a^2*f^2)*sinh(f*x + e)^2 + 3*(a^2*f^2*co
sh(f*x + e)^2 + 2*a^2*f^2*cosh(f*x + e) + a^2*f^2)*sinh(f*x + e))
```

Sympy [A] time = 2.87245, size = 156, normalized size = 1.27

$$\begin{cases} -\frac{c\tanh^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^{2}f}+\frac{c\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^{2}f}-\frac{dx\tanh^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^{2}f}+\frac{dx\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^{2}f}-\frac{dx}{3a^{2}f}+\frac{2d\log\left(\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{3a^{2}f^{2}}-\frac{d\tanh^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^{2}f^{2}} & \text{for } f\neq 0 \\ \frac{cx+\frac{dx^{2}}{2}}{\left(a\cosh\left(e\right)+a\right)^{2}} & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((-c*tanh(e/2 + f*x/2)**3/(6*a**2*f) + c*tanh(e/2 + f*x/2)/(2*a**2
*f) - d*x*tanh(e/2 + f*x/2)**3/(6*a**2*f) + d*x*tanh(e/2 + f*x/2)/(2*a**2*f
) - d*x/(3*a**2*f) + 2*d*log(tanh(e/2 + f*x/2) + 1)/(3*a**2*f**2) - d*tanh(
e/2 + f*x/2)**2/(6*a**2*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*cosh(e) + a)
**2, True))
```

Giac [B] time = 1.2571, size = 279, normalized size = 2.27

$$\frac{2\left(dfxe^{(3\,fx+3\,e)}+3\,dfxe^{(2\,fx+2\,e)}-3\,cfe^{(fx+e)}-de^{(3\,fx+3\,e)}\log\left(e^{(fx+e)}+1\right)-3\,de^{(2\,fx+2\,e)}\log\left(e^{(fx+e)}+1\right)-3\,de^{(fx+e)}\log\right.}{3\left(a^{2}f^{2}e^{(3\,fx+3\,e)}+3\,a^{2}f^{2}e^{(2\,fx+2\,e)}+3\,a^{2}f^{2}e^{(fx+e)}+a^{2}f^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2/3*(d*f*x*e^(3*f*x + 3*e) + 3*d*f*x*e^(2*f*x + 2*e) - 3*c*f*e^(f*x + e) -
d*e^(3*f*x + 3*e)*log(e^(f*x + e) + 1) - 3*d*e^(2*f*x + 2*e)*log(e^(f*x + e
) + 1) - 3*d*e^(f*x + e)*log(e^(f*x + e) + 1) - c*f + d*e^(2*f*x + 2*e) + d
*e^(f*x + e) - d*log(e^(f*x + e) + 1))/(a^2*f^2*e^(3*f*x + 3*e) + 3*a^2*f^2
*e^(2*f*x + 2*e) + 3*a^2*f^2*e^(f*x + e) + a^2*f^2)
```

3.119 $\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a\cosh(e+fx)+a)^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + a*Cosh[e + f*x])^2), x]
```

Rubi [A] time = 0.0543908, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + a*Cosh[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + a*Cosh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx$$

Mathematica [A] time = 30.1044, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + a*Cosh[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + a*Cosh[e + f*x])^2), x]
```

Maple [A] time = 0.347, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+a\cosh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x)
```

MaximA [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(d^2f^2x^2+2\,cdf^2x+c^2f^2-2\,d^2+\left(d^2fxe^{(2\,e)}+cdfe^{(2}\right.\right.}{3\left(a^2d^3f^3x^3+3\,a^2cd^2f^3x^2+3\,a^2c^2df^3x+a^2c^3f^3+\left(a^2d^3f^3x^3e^{(3\,e)}+3\,a^2cd^2f^3x^2e^{(3\,e)}+3\,a^2c^2df^3xe^{(3\,e)}+a^2c^3f^3e^{(3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 - 2*d^2 + (d^2*f*x*e^(2*e) + c*d*
f*e^(2*e) - 2*d^2*e^(2*e))*e^(2*f*x) + (3*d^2*f^2*x^2*e^e + 3*c^2*f^2*e^e +
 c*d*f*e^e - 4*d^2*e^e + (6*c*d*f^2*e^e + d^2*f*e^e)*x)*e^(f*x))/(a^2*d^3*f
^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + (a^2*d^3*f
^3*x^3*e^(3*e) + 3*a^2*c*d^2*f^3*x^2*e^(3*e) + 3*a^2*c^2*d*f^3*x*e^(3*e) +
a^2*c^3*f^3*e^(3*e))*e^(3*f*x) + 3*(a^2*d^3*f^3*x^3*e^(2*e) + 3*a^2*c*d^2*f
^3*x^2*e^(2*e) + 3*a^2*c^2*d*f^3*x*e^(2*e) + a^2*c^3*f^3*e^(2*e))*e^(2*f*x)
 + 3*(a^2*d^3*f^3*x^3*e^e + 3*a^2*c*d^2*f^3*x^2*e^e + 3*a^2*c^2*d*f^3*x*e^e
 + a^2*c^3*f^3*e^e)*e^(f*x)) - integrate(2/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x +
 c^2*d*f^2 - 6*d^3)/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*
f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4*e^e + 4*a^2*c*
d^3*f^3*x^3*e^e + 6*a^2*c^2*d^2*f^3*x^2*e^e + 4*a^2*c^3*d*f^3*x*e^e + a^2*c
^4*f^3*e^e)*e^(f*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2dx+a^2c+\left(a^2dx+a^2c\right)\cosh\left(fx+e\right)^2+2\left(a^2dx+a^2c\right)\cosh\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (a^2*d*x + a^2*c)*cosh(f*x + e)^2 + 2*(a^2*d*
x + a^2*c)*cosh(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c\cosh^2\left(e+fx\right)+2c\cosh\left(e+fx\right)+c+dx\cosh^2\left(e+fx\right)+2dx\cosh\left(e+fx\right)+dx}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Integral(1/(c*cosh(e + f*x)**2 + 2*c*cosh(e + f*x) + c + d*x*cosh(e + f*x)*
*2 + 2*d*x*cosh(e + f*x) + d*x), x)/a**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a\cosh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(a*cosh(f*x + e) + a)^2), x)
```

3.120 $\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a\cosh(e+fx)+a)^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + a*Cosh[e + f*x])^2), x]
```

Rubi [A] time = 0.0524199, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + a*Cosh[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Cosh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx=\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx$$

Mathematica [A] time = 31.2858, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Cosh[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + a*Cosh[e + f*x])^2), x]
```

Maple [A] time = 0.465, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a+a\cosh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 - 6*d^2 + 2*(d^2*f*x*e^(2*e) + c*
d*f*e^(2*e) - 3*d^2*e^(2*e))*e^(2*f*x) + (3*d^2*f^2*x^2*e^e + 3*c^2*f^2*e^e
 + 2*c*d*f*e^e - 12*d^2*e^e + 2*(3*c*d*f^2*e^e + d^2*f*e^e)*x)*e^(f*x))/(a^
2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f
^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4*e^(3*e) + 4*a^2*c*d^3*f^3*x^3*e^(3*e)
 + 6*a^2*c^2*d^2*f^3*x^2*e^(3*e) + 4*a^2*c^3*d*f^3*x*e^(3*e) + a^2*c^4*f^3*
e^(3*e))*e^(3*f*x) + 3*(a^2*d^4*f^3*x^4*e^(2*e) + 4*a^2*c*d^3*f^3*x^3*e^(2*
e) + 6*a^2*c^2*d^2*f^3*x^2*e^(2*e) + 4*a^2*c^3*d*f^3*x*e^(2*e) + a^2*c^4*f^
3*e^(2*e))*e^(2*f*x) + 3*(a^2*d^4*f^3*x^4*e^e + 4*a^2*c*d^3*f^3*x^3*e^e + 6
*a^2*c^2*d^2*f^3*x^2*e^e + 4*a^2*c^3*d*f^3*x*e^e + a^2*c^4*f^3*e^e)*e^(f*x)
) - integrate(4/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 12*d^3)/(a^2*d
^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*
f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3 + (a^2*d^5*f^3*x^5*e^e + 5*a^2*c*
d^4*f^3*x^4*e^e + 10*a^2*c^2*d^3*f^3*x^3*e^e + 10*a^2*c^3*d^2*f^3*x^2*e^e +
 5*a^2*c^4*d*f^3*x*e^e + a^2*c^5*f^3*e^e)*e^(f*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2d^2x^2 + 2\,a^2cdx + a^2c^2 + \left(a^2d^2x^2 + 2\,a^2cdx + a^2c^2\right)\cosh\left(fx+e\right)^2 + 2\left(a^2d^2x^2 + 2\,a^2cdx + a^2c^2\right)\cosh\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (a^2*d^2*x^2 + 2*a^2*c*d*
x + a^2*c^2)*cosh(f*x + e)^2 + 2*(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*cosh
(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a\cosh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*cosh(f*x + e) + a)^2), x)
```

3.121 $\int x^3\sqrt{a+a\cosh(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{12x^2\sqrt{a\cosh(c+dx)+a}}{d^2}-\frac{96\sqrt{a\cosh(c+dx)+a}}{d^4}+\frac{48x\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}}{d^3}+\frac{2x^3\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a}}{d}$$

```
[Out] (-96*Sqrt[a + a*Cosh[c + d*x]])/d^4 - (12*x^2*Sqrt[a + a*Cosh[c + d*x]])/d^
2 + (48*x*Sqrt[a + a*Cosh[c + d*x]]*Tanh[c/2 + (d*x)/2])/d^3 + (2*x^3*Sqrt[
a + a*Cosh[c + d*x]]*Tanh[c/2 + (d*x)/2])/d
```

Rubi [A] time = 0.147767, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3319, 3296, 2638}

$$-\frac{12x^2\sqrt{a\cosh(c+dx)+a}}{d^2}-\frac{96\sqrt{a\cosh(c+dx)+a}}{d^4}+\frac{48x\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}}{d^3}+\frac{2x^3\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (-96*Sqrt[a + a*Cosh[c + d*x]])/d^4 - (12*x^2*Sqrt[a + a*Cosh[c + d*x]])/d^
2 + (48*x*Sqrt[a + a*Cosh[c + d*x]]*Tanh[c/2 + (d*x)/2])/d^3 + (2*x^3*Sqrt[
a + a*Cosh[c + d*x]]*Tanh[c/2 + (d*x)/2])/d
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^3\sqrt{a+a\cosh(c+dx)}\,dx = \left(\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int x^3\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)$$

$$= \frac{2x^3\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(6\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\right.\right.}{d}$$

$$= -\frac{12x^2\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{2x^3\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} + \frac{\left(24\sqrt{a+a\cos\right.}{}$$

$$= -\frac{12x^2\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{48x\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3} + \frac{2x^3\sqrt{a+a\cos}}{}$$

$$= -\frac{96\sqrt{a+a\cosh(c+dx)}}{d^4} - \frac{12x^2\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{48x\sqrt{a+a\cosh(c+dx)}\tanh}{d^3}$$

Mathematica [A] time = 0.197331, size = 53, normalized size = 0.48

$$\frac{2\left(dx\left(d^2x^2+24\right)\tanh\left(\frac{1}{2}(c+dx)\right)-6\left(d^2x^2+8\right)\right)\sqrt{a(\cosh(c+dx)+1)}}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cosh[c + d*x])]*(-6*(8 + d^2*x^2) + d*x*(24 + d^2*x^2)*Tanh[
(c + d*x)/2]))/d^4
```

Maple [A] time = 0.106, size = 108, normalized size = 1.

$$\frac{\sqrt{2}\left(d^3x^3e^{dx+c}-d^3x^3-6\,d^2x^2e^{dx+c}-6\,d^2x^2+24\,dxe^{dx+c}-24\,dx-48\,e^{dx+c}-48\right)}{\left(e^{dx+c}+1\right)d^4}\sqrt{a\left(e^{dx+c}+1\right)^2e^{-dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] 2^(1/2)*(a*(exp(d*x+c)+1)^2*exp(-d*x-c))^(1/2)/(exp(d*x+c)+1)*(d^3*x^3*exp(
d*x+c)-d^3*x^3-6*d^2*x^2*exp(d*x+c)-6*d^2*x^2+24*d*x*exp(d*x+c)-24*d*x-48*e
xp(d*x+c)-48)/d^4
```

Maxima [A] time = 1.7096, size = 162, normalized size = 1.47

$$-\frac{\left(\sqrt{2}\sqrt{a}d^3x^3+6\sqrt{2}\sqrt{a}d^2x^2+24\sqrt{2}\sqrt{a}dx-\left(\sqrt{2}\sqrt{a}d^3x^3e^c-6\sqrt{2}\sqrt{a}d^2x^2e^c+24\sqrt{2}\sqrt{a}dxe^c-48\sqrt{2}\sqrt{a}e^c\right)e^{(dx)}+48\sqrt{}\right.}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*d^3*x^3 + 6*sqrt(2)*sqrt(a)*d^2*x^2 + 24*sqrt(2)*sqrt(a)*
d*x - (sqrt(2)*sqrt(a)*d^3*x^3*e^c - 6*sqrt(2)*sqrt(a)*d^2*x^2*e^c + 24*sqr
```

```
t(2)*sqrt(a)*d*x*e^c - 48*sqrt(2)*sqrt(a)*e^c)*e^(d*x) + 48*sqrt(2)*sqrt(a)
)*e^(-1/2*d*x - 1/2*c)/d^4
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(cosh(c + d*x) + 1)), x)
```

Giac [A] time = 1.30485, size = 198, normalized size = 1.8

$$\frac{\sqrt{2}\left(\sqrt{a}d^3x^3e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)} - \sqrt{a}d^3x^3e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)} - 6\sqrt{a}d^2x^2e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)} - 6\sqrt{a}d^2x^2e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)} + 24\sqrt{a}dxe^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)} - 24\sqrt{a}dxe^{\left(-\right.}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(a)*d^3*x^3*e^(1/2*d*x + 1/2*c) - sqrt(a)*d^3*x^3*e^(-1/2*d*x
- 1/2*c) - 6*sqrt(a)*d^2*x^2*e^(1/2*d*x + 1/2*c) - 6*sqrt(a)*d^2*x^2*e^(-1/
2*d*x - 1/2*c) + 24*sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) - 24*sqrt(a)*d*x*e^(-1/
2*d*x - 1/2*c) - 48*sqrt(a)*e^(1/2*d*x + 1/2*c) - 48*sqrt(a)*e^(-1/2*d*x -
1/2*c))/d^4
```

3.122 $\int x^2\sqrt{a+a\cosh(c+dx)}\,dx$

Optimal. Leaf size=88

$$-\frac{8x\sqrt{a\cosh(c+dx)+a}}{d^2}+\frac{16\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}}{d^3}+\frac{2x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}}{d}$$

```
[Out] (-8*x*Sqrt[a + a*Cosh[c + d*x]])/d^2 + (16*Sqrt[a + a*Cosh[c + d*x]]*Tanh[c
/2 + (d*x)/2])/d^3 + (2*x^2*Sqrt[a + a*Cosh[c + d*x]]*Tanh[c/2 + (d*x)/2])/
d
```

Rubi [A] time = 0.113328, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3319, 3296, 2637}

$$-\frac{8x\sqrt{a\cosh(c+dx)+a}}{d^2}+\frac{16\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}}{d^3}+\frac{2x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (-8*x*Sqrt[a + a*Cosh[c + d*x]])/d^2 + (16*Sqrt[a + a*Cosh[c + d*x]]*Tanh[c
/2 + (d*x)/2])/d^3 + (2*x^2*Sqrt[a + a*Cosh[c + d*x]]*Tanh[c/2 + (d*x)/2])/
d
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2\sqrt{a+a\cosh(c+dx)}\,dx = \left(\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int x^2\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)dx$$

$$= \frac{2x^2\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(4\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{id}{2}\right.\right.}{d}$$

$$= -\frac{8x\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{2x^2\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} + \frac{\left(8\sqrt{a+a\cosh(c+\right.}{}$$

$$= -\frac{8x\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{16\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3} + \frac{2x^2\sqrt{a+a\cosh(c+}{d}$$

Mathematica [A] time = 0.149346, size = 44, normalized size = 0.5

$$\frac{2\left(\left(d^2x^2+8\right)\tanh\left(\frac{1}{2}(c+dx)\right)-4dx\right)\sqrt{a(\cosh(c+dx)+1)}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cosh[c + d*x])]*(-4*d*x + (8 + d^2*x^2)*Tanh[(c + d*x)/2]))/
d^3
```

Maple [A] time = 0.056, size = 86, normalized size = 1.

$$\frac{\sqrt{2}\left(d^2x^2\mathrm{e}^{dx+c}-d^2x^2-4\,dx\mathrm{e}^{dx+c}-4\,dx+8\,\mathrm{e}^{dx+c}-8\right)}{\left(\mathrm{e}^{dx+c}+1\right)d^3}\sqrt{a\left(\mathrm{e}^{dx+c}+1\right)^2\mathrm{e}^{-dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] 2^(1/2)*(a*(exp(d*x+c)+1)^2*exp(-d*x-c))^(1/2)/(exp(d*x+c)+1)*(d^2*x^2*exp(
d*x+c)-d^2*x^2-4*d*x*exp(d*x+c)-4*d*x+8*exp(d*x+c)-8)/d^3
```

Maxima [A] time = 1.65905, size = 122, normalized size = 1.39

$$-\frac{\left(\sqrt{2}\sqrt{a}d^2x^2+4\sqrt{2}\sqrt{a}dx-\left(\sqrt{2}\sqrt{a}d^2x^2e^c-4\sqrt{2}\sqrt{a}dxe^c+8\sqrt{2}\sqrt{a}e^c\right)e^{(dx)}+8\sqrt{2}\sqrt{a}\right)e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*d^2*x^2 + 4*sqrt(2)*sqrt(a)*d*x - (sqrt(2)*sqrt(a)*d^2*x^
2*e^c - 4*sqrt(2)*sqrt(a)*d*x*e^c + 8*sqrt(2)*sqrt(a)*e^c)*e^(d*x) + 8*sqrt
(2)*sqrt(a))*e^(-1/2*d*x - 1/2*c)/d^3
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(cosh(c + d*x) + 1)), x)
```

Giac [A] time = 1.33322, size = 144, normalized size = 1.64

$$\frac{\sqrt{2}\left(\sqrt{a}d^2x^2e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\sqrt{a}d^2x^2e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-4\,\sqrt{a}dxe^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-4\,\sqrt{a}dxe^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+8\,\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-8\,\sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(a)*d^2*x^2*e^(1/2*d*x + 1/2*c) - sqrt(a)*d^2*x^2*e^(-1/2*d*x
- 1/2*c) - 4*sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) - 4*sqrt(a)*d*x*e^(-1/2*d*x -
1/2*c) + 8*sqrt(a)*e^(1/2*d*x + 1/2*c) - 8*sqrt(a)*e^(-1/2*d*x - 1/2*c))/d^
3
```

3.123 $\int x\sqrt{a+a\cosh(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{2x\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}}{d}-\frac{4\sqrt{a\cosh(c+dx)+a}}{d^2}$$

```
[Out] (-4*Sqrt[a + a*Cosh[c + d*x]])/d^2 + (2*x*Sqrt[a + a*Cosh[c + d*x]]*Tanh[c/
2 + (d*x)/2])/d
```

Rubi [A] time = 0.0621659, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3319, 3296, 2638}

$$\frac{2x\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}}{d}-\frac{4\sqrt{a\cosh(c+dx)+a}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (-4*Sqrt[a + a*Cosh[c + d*x]])/d^2 + (2*x*Sqrt[a + a*Cosh[c + d*x]]*Tanh[c/
2 + (d*x)/2])/d
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+a\cosh(c+dx)}\,dx &= \left(\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int x\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)dx\\ &= \frac{2x\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{\left(2\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right.}{d}\\ &= -\frac{4\sqrt{a+a\cosh(c+dx)}}{d^2}+\frac{2x\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.119905, size = 34, normalized size = 0.64

$$\frac{2\left(dx\tanh\left(\frac{1}{2}(c+dx)\right)-2\right)\sqrt{a(\cosh(c+dx)+1)}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cosh[c + d*x])]*(-2 + d*x*Tanh[(c + d*x)/2]))/d^2
```

Maple [A] time = 0.055, size = 64, normalized size = 1.2

$$\frac{\sqrt{2}\left(dx\mathrm{e}^{dx+c}-dx-2\,\mathrm{e}^{dx+c}-2\right)}{\left(\mathrm{e}^{dx+c}+1\right)d^2}\sqrt{a\left(\mathrm{e}^{dx+c}+1\right)^2\mathrm{e}^{-dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] 2^(1/2)*(a*(exp(d*x+c)+1)^2*exp(-d*x-c))^(1/2)/(exp(d*x+c)+1)*(d*x*exp(d*x+
c)-d*x-2*exp(d*x+c)-2)/d^2
```

Maxima [A] time = 1.76643, size = 81, normalized size = 1.53

$$-\frac{\left(\sqrt{2}\sqrt{a}dx-\left(\sqrt{2}\sqrt{a}dxe^c-2\,\sqrt{2}\sqrt{a}e^c\right)e^{(dx)}+2\,\sqrt{2}\sqrt{a}\right)e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*d*x - (sqrt(2)*sqrt(a)*d*x*e^c - 2*sqrt(2)*sqrt(a)*e^c)*e
^(d*x) + 2*sqrt(2)*sqrt(a))*e^(-1/2*d*x - 1/2*c)/d^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*(cosh(c + d*x) + 1)), x)
```

Giac [A] time = 1.29389, size = 90, normalized size = 1.7

$$\frac{\sqrt{2}\left(\sqrt{a}dxe^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}-\sqrt{a}dxe^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}-2\sqrt{a}e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}-2\sqrt{a}e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) - sqrt(a)*d*x*e^(-1/2*d*x - 1/2*c)
 - 2*sqrt(a)*e^(1/2*d*x + 1/2*c) - 2*sqrt(a)*e^(-1/2*d*x - 1/2*c))/d^2
```

3.124 $\int \frac{\sqrt{a+a\cosh(c+dx)}}{x}\,dx$

Optimal. Leaf size=83

$$\cosh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}+\sinh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}$$

```
[Out] Cosh[c/2]*Sqrt[a + a*Cosh[c + d*x]]*CoshIntegral[(d*x)/2]*Sech[c/2 + (d*x)/
2] + Sqrt[a + a*Cosh[c + d*x]]*Sech[c/2 + (d*x)/2]*Sinh[c/2]*SinhIntegral[(
d*x)/2]
```

Rubi [A] time = 0.131785, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3319, 3303, 3298, 3301}

$$\cosh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}+\sinh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[c + d*x]]/x,x]
```

```
[Out] Cosh[c/2]*Sqrt[a + a*Cosh[c + d*x]]*CoshIntegral[(d*x)/2]*Sech[c/2 + (d*x)/
2] + Sqrt[a + a*Cosh[c + d*x]]*Sech[c/2 + (d*x)/2]*Sinh[c/2]*SinhIntegral[(
d*x)/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x}\,dx = \left(\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)}{x}\,dx$$
$$= \left(\cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int \frac{\cosh\left(\frac{dx}{2}\right)}{x}\,dx + \left(\sqrt{a+a\,\mathrm{co}\right.$$
$$= \cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\mathrm{Chi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)+\sqrt{a+a\cosh(c+dx)}\mathrm{sech}\left(\frac{c}{2}+\frac{d}{2}\right.$$

Mathematica [A] time = 0.0750043, size = 54, normalized size = 0.65

$$\mathrm{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cosh(c+dx)+1)}\left(\cosh\left(\frac{c}{2}\right)\mathrm{Chi}\left(\frac{dx}{2}\right)+\sinh\left(\frac{c}{2}\right)\mathrm{Shi}\left(\frac{dx}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[c + d*x]]/x,x]
```

```
[Out] Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(Cosh[c/2]*CoshIntegral[(d*x)
/2] + Sinh[c/2]*SinhIntegral[(d*x)/2])
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{a+a\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(1/2)/x,x)
```

```
[Out] int((a+a*cosh(d*x+c))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cosh(dx+c)+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(d*x + c) + a)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*(cosh(c + d*x) + 1))/x, x)
```

Giac [A] time = 1.29986, size = 43, normalized size = 0.52

$$\frac{1}{2}\,\sqrt{2}\left(\sqrt{a}\operatorname{Ei}\left(\frac{1}{2}\,dx\right)e^{\left(\frac{1}{2}\,c\right)}+\sqrt{a}\operatorname{Ei}\left(-\frac{1}{2}\,dx\right)e^{\left(-\frac{1}{2}\,c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(a)*Ei(1/2*d*x)*e^(1/2*c) + sqrt(a)*Ei(-1/2*d*x)*e^(-1/2*c
))
```

3.125 $\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^2}\,dx$

Optimal. Leaf size=110

$$\frac{1}{2}d\sinh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}+\frac{1}{2}d\cosh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}$$

```
[Out] -(Sqrt[a + a*Cosh[c + d*x]]/x) + (d*Sqrt[a + a*Cosh[c + d*x]]*CoshIntegral[
(d*x)/2]*Sech[c/2 + (d*x)/2]*Sinh[c/2])/2 + (d*Cosh[c/2]*Sqrt[a + a*Cosh[c
+ d*x]]*Sech[c/2 + (d*x)/2]*SinhIntegral[(d*x)/2])/2
```

Rubi [A] time = 0.140128, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 3297, 3303, 3298, 3301}

$$\frac{1}{2}d\sinh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}+\frac{1}{2}d\cosh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + a*Cosh[c + d*x]]/x) + (d*Sqrt[a + a*Cosh[c + d*x]]*CoshIntegral[
(d*x)/2]*Sech[c/2 + (d*x)/2]*Sinh[c/2])/2 + (d*Cosh[c/2]*Sqrt[a + a*Cosh[c
+ d*x]]*Sech[c/2 + (d*x)/2]*SinhIntegral[(d*x)/2])/2
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^2}\,dx &= \left(\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)}{x^2}\,dx \\
&= -\frac{\sqrt{a+a\cosh(c+dx)}}{x}+\frac{1}{2}\left(d\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int \text{sinh} \\
&= -\frac{\sqrt{a+a\cosh(c+dx)}}{x}+\frac{1}{2}\left(d\cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right.\right. \\
&= -\frac{\sqrt{a+a\cosh(c+dx)}}{x}+\frac{1}{2}d\sqrt{a+a\cosh(c+dx)}\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}\right)+\frac{1}{2}
\end{aligned}
$$

Mathematica [A] time = 0.136294, size = 75, normalized size = 0.68

$$
\frac{\sqrt{a(\cosh(c+dx)+1)}\left(dx\sinh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{1}{2}(c+dx)\right)+dx\cosh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{1}{2}(c+dx)\right)-2\right)}{2x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[c + d*x]]/x^2,x]
```

```
[Out] (Sqrt[a*(1 + Cosh[c + d*x])]*(-2 + d*x*CoshIntegral[(d*x)/2]*Sech[(c + d*x)
/2]*Sinh[c/2] + d*x*Cosh[c/2]*Sech[(c + d*x)/2]*SinhIntegral[(d*x)/2]))/(2*
x)
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$
\int \frac{1}{x^2}\sqrt{a+a\cosh(dx+c)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(1/2)/x^2,x)
```

```
[Out] int((a+a*cosh(d*x+c))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{a\cosh(dx+c)+a}}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(d*x + c) + a)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*(cosh(c + d*x) + 1))/x**2, x)
```

Giac [A] time = 1.21604, size = 92, normalized size = 0.84

$$\frac{\sqrt{2}\left(\sqrt{a}dx\mathrm{Ei}\left(\frac{1}{2}\,dx\right)e^{\left(\frac{1}{2}\,c\right)}-\sqrt{a}dx\mathrm{Ei}\left(-\frac{1}{2}\,dx\right)e^{\left(-\frac{1}{2}\,c\right)}-2\,\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-2\,\sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}\right)}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(sqrt(a)*d*x*Ei(1/2*d*x)*e^(1/2*c) - sqrt(a)*d*x*Ei(-1/2*d*x)*e
^(-1/2*c) - 2*sqrt(a)*e^(1/2*d*x + 1/2*c) - 2*sqrt(a)*e^(-1/2*d*x - 1/2*c))
/x
```

3.126 $\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^3} \, dx$

Optimal. Leaf size=151

$$\frac{1}{8}d^2\cosh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}+\frac{1}{8}d^2\sinh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx}$$

```
[Out] -Sqrt[a + a*Cosh[c + d*x]]/(2*x^2) + (d^2*Cosh[c/2]*Sqrt[a + a*Cosh[c + d*x
]]*CoshIntegral[(d*x)/2]*Sech[c/2 + (d*x)/2])/8 + (d^2*Sqrt[a + a*Cosh[c +
d*x]]*Sech[c/2 + (d*x)/2]*Sinh[c/2]*SinhIntegral[(d*x)/2])/8 - (d*Sqrt[a +
a*Cosh[c + d*x]]*Tanh[c/2 + (d*x)/2])/(4*x)
```

Rubi [A] time = 0.169583, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 3297, 3303, 3298, 3301}

$$\frac{1}{8}d^2\cosh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}+\frac{1}{8}d^2\sinh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[c + d*x]]/x^3,x]
```


```
[Out] -Sqrt[a + a*Cosh[c + d*x]]/(2*x^2) + (d^2*Cosh[c/2]*Sqrt[a + a*Cosh[c + d*x
]]*CoshIntegral[(d*x)/2]*Sech[c/2 + (d*x)/2])/8 + (d^2*Sqrt[a + a*Cosh[c +
d*x]]*Sech[c/2 + (d*x)/2]*Sinh[c/2]*SinhIntegral[(d*x)/2])/8 - (d*Sqrt[a +
a*Cosh[c + d*x]]*Tanh[c/2 + (d*x)/2])/(4*x)
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^3}\,dx = \left(\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)}{x^3}\,dx$$

$$= -\frac{\sqrt{a+a\cosh(c+dx)}}{2x^2}+\frac{1}{4}\left(d\sqrt{a+a\cosh(c+dx)}\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int \frac{\sinh\left(\frac{c}{2}\right.}{x^2}$$

$$= -\frac{\sqrt{a+a\cosh(c+dx)}}{2x^2}-\frac{d\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x}+\frac{1}{8}\left(d^2\sqrt{a+a\cosh(c+dx)}\right.$$

$$= -\frac{\sqrt{a+a\cosh(c+dx)}}{2x^2}-\frac{d\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x}+\frac{1}{8}\left(d^2\cosh\left(\frac{c}{2}\right)\sqrt{a+a\cos\right.$$

$$= -\frac{\sqrt{a+a\cosh(c+dx)}}{2x^2}+\frac{1}{8}d^2\cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{1}{8}d$$

Mathematica [A] time = 0.205298, size = 97, normalized size = 0.64

$$\frac{\sqrt{a(\cosh(c+dx)+1)}\left(d^2x^2\cosh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{1}{2}(c+dx)\right)+d^2x^2\sinh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{1}{2}(c+dx)\right)-2dx\tanh\left(\frac{1}{2}\right.\right.}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[c + d*x]]/x^3,x]
```

```
[Out] (Sqrt[a*(1 + Cosh[c + d*x])]*(-4 + d^2*x^2*Cosh[c/2]*CoshIntegral[(d*x)/2]*
Sech[(c + d*x)/2] + d^2*x^2*Sech[(c + d*x)/2]*Sinh[c/2]*SinhIntegral[(d*x)/
2] - 2*d*x*Tanh[(c + d*x)/2]))/(8*x^2)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{a+a\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(1/2)/x^3,x)
```

```
[Out] int((a+a*cosh(d*x+c))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cosh(dx+c)+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(d*x + c) + a)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a(\cosh(c+dx)+1)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*(cosh(c + d*x) + 1))/x**3, x)
```

Giac [A] time = 1.19754, size = 144, normalized size = 0.95

$$\frac{\sqrt{2}\left(\sqrt{a}d^2x^2\mathrm{Ei}\left(\frac{1}{2}dx\right)e^{\left(\frac{1}{2}c\right)} + \sqrt{a}d^2x^2\mathrm{Ei}\left(-\frac{1}{2}dx\right)e^{\left(-\frac{1}{2}c\right)} - 2\sqrt{a}dxe^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)} + 2\sqrt{a}dxe^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)} - 4\sqrt{a}e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)} - 4\sqrt{}\right.}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(sqrt(a)*d^2*x^2*Ei(1/2*d*x)*e^(1/2*c) + sqrt(a)*d^2*x^2*Ei(-1
/2*d*x)*e^(-1/2*c) - 2*sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) + 2*sqrt(a)*d*x*e^(-
1/2*d*x - 1/2*c) - 4*sqrt(a)*e^(1/2*d*x + 1/2*c) - 4*sqrt(a)*e^(-1/2*d*x -
1/2*c))/x^2
```

3.127 $\int x^3\sqrt{a+a\cosh(x)}\,dx$

Optimal. Leaf size=68

$$-12x^2\sqrt{a\cosh(x)+a}+2x^3\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-96\sqrt{a\cosh(x)+a}+48x\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

```
[Out] -96*Sqrt[a + a*Cosh[x]] - 12*x^2*Sqrt[a + a*Cosh[x]] + 48*x*Sqrt[a + a*Cosh
[x]]*Tanh[x/2] + 2*x^3*Sqrt[a + a*Cosh[x]]*Tanh[x/2]
```

Rubi [A] time = 0.11683, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3319, 3296, 2638}

$$-12x^2\sqrt{a\cosh(x)+a}+2x^3\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-96\sqrt{a\cosh(x)+a}+48x\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + a*Cosh[x]],x]
```

```
[Out] -96*Sqrt[a + a*Cosh[x]] - 12*x^2*Sqrt[a + a*Cosh[x]] + 48*x*Sqrt[a + a*Cosh
[x]]*Tanh[x/2] + 2*x^3*Sqrt[a + a*Cosh[x]]*Tanh[x/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{a+a\cosh(x)}\,dx &= \left(\sqrt{a+a\cosh(x)}\,\textbf{sech}\left(\frac{x}{2}\right)\right)\int x^3\cosh\left(\frac{x}{2}\right)dx\\
&= 2x^3\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)-\left(6\sqrt{a+a\cosh(x)}\,\textbf{sech}\left(\frac{x}{2}\right)\right)\int x^2\sinh\left(\frac{x}{2}\right)dx\\
&= -12x^2\sqrt{a+a\cosh(x)}+2x^3\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)+\left(24\sqrt{a+a\cosh(x)}\,\textbf{sech}\left(\frac{x}{2}\right)\right)\int x\cosh\\
&= -12x^2\sqrt{a+a\cosh(x)}+48x\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)+2x^3\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)-\left(48\sqrt{a}\right.\\
&= -96\sqrt{a+a\cosh(x)}-12x^2\sqrt{a+a\cosh(x)}+48x\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)+2x^3\sqrt{a+a\cosh(x
\end{aligned}$$

Mathematica [A] time = 0.0524797, size = 33, normalized size = 0.49

$$2\left(x\left(x^2+24\right)\tanh\left(\frac{x}{2}\right)-6\left(x^2+8\right)\right)\sqrt{a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + a*Cosh[x]],x]
```

```
[Out] 2*Sqrt[a*(1 + Cosh[x])]*(-6*(8 + x^2) + x*(24 + x^2)*Tanh[x/2])
```

Maple [A] time = 0.051, size = 62, normalized size = 0.9

$$\frac{\sqrt{2}\left(x^3e^x-x^3-6x^2e^x-6x^2+24xe^x-24x-48e^x-48\right)}{e^x+1}\sqrt{a\left(e^x+1\right)^2e^{-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*cosh(x))^(1/2),x)
```

```
[Out] 2^(1/2)*(a*(exp(x)+1)^2*exp(-x))^(1/2)/(exp(x)+1)*(x^3*exp(x)-x^3-6*x^2*exp
(x)-6*x^2+24*x*exp(x)-24*x-48*exp(x)-48)
```

Maxima [A] time = 1.73962, size = 119, normalized size = 1.75

$$-\left(\sqrt{2}\sqrt{a}x^3+6\sqrt{2}\sqrt{a}x^2+24\sqrt{2}\sqrt{a}x-\left(\sqrt{2}\sqrt{a}x^3-6\sqrt{2}\sqrt{a}x^2+24\sqrt{2}\sqrt{a}x-48\sqrt{2}\sqrt{a}\right)e^x+48\sqrt{2}\sqrt{a}\right)e^{\left(-\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*x^3 + 6*sqrt(2)*sqrt(a)*x^2 + 24*sqrt(2)*sqrt(a)*x - (sqr
t(2)*sqrt(a)*x^3 - 6*sqrt(2)*sqrt(a)*x^2 + 24*sqrt(2)*sqrt(a)*x - 48*sqrt(2
)*sqrt(a))*e^x + 48*sqrt(2)*sqrt(a))*e^(-1/2*x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{a\left(\cosh\left(x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cosh(x))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(cosh(x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cosh(x) + a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)*x^3, x)
```

3.128 $\int x^2\sqrt{a+a\cosh(x)}\,dx$

Optimal. Leaf size=53

$$2x^2\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-8x\sqrt{a\cosh(x)+a}+16\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

```
[Out] -8*x*Sqrt[a + a*Cosh[x]] + 16*Sqrt[a + a*Cosh[x]]*Tanh[x/2] + 2*x^2*Sqrt[a
+ a*Cosh[x]]*Tanh[x/2]
```

Rubi [A] time = 0.0973149, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3319, 3296, 2637}

$$2x^2\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-8x\sqrt{a\cosh(x)+a}+16\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + a*Cosh[x]],x]
```

```
[Out] -8*x*Sqrt[a + a*Cosh[x]] + 16*Sqrt[a + a*Cosh[x]]*Tanh[x/2] + 2*x^2*Sqrt[a
+ a*Cosh[x]]*Tanh[x/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+a\cosh(x)}\,dx &= \left(\sqrt{a+a\cosh(x)}\mathbf{sech}\left(\frac{x}{2}\right)\right)\int x^2\cosh\left(\frac{x}{2}\right)\,dx\\ &= 2x^2\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)-\left(4\sqrt{a+a\cosh(x)}\mathbf{sech}\left(\frac{x}{2}\right)\right)\int x\sinh\left(\frac{x}{2}\right)\,dx\\ &= -8x\sqrt{a+a\cosh(x)}+2x^2\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)+\left(8\sqrt{a+a\cosh(x)}\mathbf{sech}\left(\frac{x}{2}\right)\right)\int \cosh\left(\right.\\ &= -8x\sqrt{a+a\cosh(x)}+16\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)+2x^2\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0435416, size = 31, normalized size = 0.58

$$8\left(\frac{1}{4}\left(x^2+8\right)\tanh\left(\frac{x}{2}\right)-x\right)\sqrt{a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + a*Cosh[x]],x]
```

```
[Out] 8*Sqrt[a*(1 + Cosh[x])]*(-x + ((8 + x^2)*Tanh[x/2])/4)
```

Maple [A] time = 0.038, size = 50, normalized size = 0.9

$$\frac{\sqrt{2}\left(x^{2}\mathrm{e}^{x}-x^{2}-4\,x\mathrm{e}^{x}-4\,x+8\,\mathrm{e}^{x}-8\right)}{\mathrm{e}^{x}+1}\sqrt{a\left(\mathrm{e}^{x}+1\right)^{2}\mathrm{e}^{-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*cosh(x))^(1/2),x)
```

```
[Out] 2^(1/2)*(a*(exp(x)+1)^2*exp(-x))^(1/2)/(exp(x)+1)*(x^2*exp(x)-x^2-4*x*exp(x
)-4*x+8*exp(x)-8)
```

Maxima [A] time = 1.69466, size = 89, normalized size = 1.68

$$-\left(\sqrt{2}\sqrt{a}x^{2}+4\,\sqrt{2}\sqrt{a}x-\left(\sqrt{2}\sqrt{a}x^{2}-4\,\sqrt{2}\sqrt{a}x+8\,\sqrt{2}\sqrt{a}\right)e^{x}+8\,\sqrt{2}\sqrt{a}\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*x^2 + 4*sqrt(2)*sqrt(a)*x - (sqrt(2)*sqrt(a)*x^2 - 4*sqrt
(2)*sqrt(a)*x + 8*sqrt(2)*sqrt(a))*e^x + 8*sqrt(2)*sqrt(a))*e^(-1/2*x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2}\sqrt{a\left(\cosh\left(x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*cosh(x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(cosh(x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cosh(x) + a}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)*x^2, x)
```

3.129 $\int x\sqrt{a+a\cosh(x)}\,dx$

Optimal. Leaf size=32

$$2x\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-4\sqrt{a\cosh(x)+a}$$

```
[Out] -4*Sqrt[a + a*Cosh[x]] + 2*x*Sqrt[a + a*Cosh[x]]*Tanh[x/2]
```

Rubi [A] time = 0.0508111, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3319, 3296, 2638}

$$2x\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-4\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + a*Cosh[x]],x]
```

```
[Out] -4*Sqrt[a + a*Cosh[x]] + 2*x*Sqrt[a + a*Cosh[x]]*Tanh[x/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{a+a\cosh(x)}\,dx &= \left(\sqrt{a+a\cosh(x)}\operatorname{sech}\left(\frac{x}{2}\right)\right)\int x\cosh\left(\frac{x}{2}\right)\,dx\\
&= 2x\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)-\left(2\sqrt{a+a\cosh(x)}\operatorname{sech}\left(\frac{x}{2}\right)\right)\int \sinh\left(\frac{x}{2}\right)\,dx\\
&= -4\sqrt{a+a\cosh(x)}+2x\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0201894, size = 22, normalized size = 0.69

$$2\left(x\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + a*Cosh[x]],x]

[Out] 2*Sqrt[a*(1 + Cosh[x])]*(-2 + x*Tanh[x/2])
```

Maple [A] time = 0.036, size = 38, normalized size = 1.2

$$\frac{\sqrt{2}\,(x\mathrm{e}^{x} - x - 2\,\mathrm{e}^{x} - 2)}{\mathrm{e}^{x} + 1}\sqrt{a\,(\mathrm{e}^{x} + 1)^{2}\,\mathrm{e}^{-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*cosh(x))^(1/2),x)

[Out] 2^(1/2)*(a*(exp(x)+1)^2*exp(-x))^(1/2)/(exp(x)+1)*(x*exp(x)-x-2*exp(x)-2)
```

Maxima [A] time = 1.68724, size = 59, normalized size = 1.84

$$-\left(\sqrt{2}\sqrt{a}x - \left(\sqrt{2}\sqrt{a}x - 2\,\sqrt{2}\sqrt{a}\right)e^{x} + 2\,\sqrt{2}\sqrt{a}\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(1/2),x, algorithm="maxima")

[Out] -(sqrt(2)*sqrt(a)*x - (sqrt(2)*sqrt(a)*x - 2*sqrt(2)*sqrt(a))*e^x + 2*sqrt(
2)*sqrt(a))*e^(-1/2*x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a\,(\cosh{(x)} + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))**(1/2),x)

[Out] Integral(x*sqrt(a*(cosh(x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \cosh{(x)} + a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)*x, x)
```

3.130 $\int \frac{\sqrt{a+a\cosh(x)}}{x}\,dx$

Optimal. Leaf size=23

$$\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

```
[Out] Sqrt[a + a*Cosh[x]]*CoshIntegral[x/2]*Sech[x/2]
```

Rubi [A] time = 0.0832048, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3319, 3301}

$$\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[x]]/x,x]
```

```
[Out] Sqrt[a + a*Cosh[x]]*CoshIntegral[x/2]*Sech[x/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cosh(x)}}{x}\,dx &= \left(\sqrt{a+a\cosh(x)}\,\text{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh\left(\frac{x}{2}\right)}{x}\,dx\\ &= \sqrt{a+a\cosh(x)}\,\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0072812, size = 23, normalized size = 1.

$$\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[x]]/x,x]
```

```
[Out] Sqrt[a*(1 + Cosh[x])]*CoshIntegral[x/2]*Sech[x/2]
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{a + a\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(1/2)/x,x)
```

```
[Out] int((a+a*cosh(x))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cosh(x) + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a(\cosh(x) + 1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*(cosh(x) + 1))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cosh(x) + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)/x, x)
```

3.131 $\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\,dx$

Optimal. Leaf size=42

$$\frac{1}{2}\mathrm{Shi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{\sqrt{a\cosh(x)+a}}{x}$$

```
[Out] -(Sqrt[a + a*Cosh[x]]/x) + (Sqrt[a + a*Cosh[x]]*Sech[x/2]*SinhIntegral[x/2]
)/2
```

Rubi [A] time = 0.0903072, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3319, 3297, 3298}

$$\frac{1}{2}\mathrm{Shi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{\sqrt{a\cosh(x)+a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[x]]/x^2,x]
```

```
[Out] -(Sqrt[a + a*Cosh[x]]/x) + (Sqrt[a + a*Cosh[x]]*Sech[x/2]*SinhIntegral[x/2]
)/2
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\,dx &= \left(\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh\left(\frac{x}{2}\right)}{x^2}\,dx\\ &= -\frac{\sqrt{a+a\cosh(x)}}{x}+\frac{1}{2}\left(\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)\int \frac{\sinh\left(\frac{x}{2}\right)}{x}\,dx\\ &= -\frac{\sqrt{a+a\cosh(x)}}{x}+\frac{1}{2}\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\mathrm{Shi}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0531439, size = 33, normalized size = 0.79

$$\frac{\sqrt{a(\cosh(x)+1)}\left(x\text{Shi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)-2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[x]]/x^2,x]
```

```
[Out] (Sqrt[a*(1 + Cosh[x])]*(-2 + x*Sech[x/2]*SinhIntegral[x/2]))/(2*x)
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{a+a\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(1/2)/x^2,x)
```

```
[Out] int((a+a*cosh(x))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cosh(x)+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a(\cosh(x)+1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*(cosh(x) + 1))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cosh(x) + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)/x^2, x)
```

3.132 $\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx$

Optimal. Leaf size=67

$$\frac{1}{8}\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{\sqrt{a\cosh(x)+a}}{2x^2}-\frac{\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}}{4x}$$

```
[Out] -Sqrt[a + a*Cosh[x]]/(2*x^2) + (Sqrt[a + a*Cosh[x]]*CoshIntegral[x/2]*Sech[
x/2])/8 - (Sqrt[a + a*Cosh[x]]*Tanh[x/2])/(4*x)
```

Rubi [A] time = 0.10618, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3319, 3297, 3301}

$$\frac{1}{8}\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{\sqrt{a\cosh(x)+a}}{2x^2}-\frac{\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[x]]/x^3,x]
```


```
[Out] -Sqrt[a + a*Cosh[x]]/(2*x^2) + (Sqrt[a + a*Cosh[x]]*CoshIntegral[x/2]*Sech[
x/2])/8 - (Sqrt[a + a*Cosh[x]]*Tanh[x/2])/(4*x)
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx = \left(\sqrt{a+a\cosh(x)}\operatorname{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh\left(\frac{x}{2}\right)}{x^3}\,dx$$
$$= -\frac{\sqrt{a+a\cosh(x)}}{2x^2} + \frac{1}{4}\left(\sqrt{a+a\cosh(x)}\operatorname{sech}\left(\frac{x}{2}\right)\right)\int \frac{\sinh\left(\frac{x}{2}\right)}{x^2}\,dx$$
$$= -\frac{\sqrt{a+a\cosh(x)}}{2x^2} - \frac{\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)}{4x} + \frac{1}{8}\left(\sqrt{a+a\cosh(x)}\operatorname{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh\left(\frac{x}{2}\right)}{x}\,dx$$
$$= -\frac{\sqrt{a+a\cosh(x)}}{2x^2} + \frac{1}{8}\sqrt{a+a\cosh(x)}\operatorname{Chi}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right) - \frac{\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)}{4x}$$

Mathematica [A] time = 0.0704671, size = 44, normalized size = 0.66

$$\frac{\sqrt{a(\cosh(x)+1)}\left(x^2\operatorname{Chi}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)-2x\tanh\left(\frac{x}{2}\right)-4\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[x]]/x^3,x]
```

```
[Out] (Sqrt[a*(1 + Cosh[x])]*(-4 + x^2*CoshIntegral[x/2]*Sech[x/2] - 2*x*Tanh[x/2
]))/(8*x^2)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{a+a\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(1/2)/x^3,x)
```

```
[Out] int((a+a*cosh(x))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cosh(x)+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\cosh\left(x\right)+1\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*(cosh(x) + 1))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cosh\left(x\right)+a}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)/x^3, x)
```

3.133 $\int x^3(a + a\cosh(x))^{3/2}\,dx$

Optimal. Leaf size=185

$$-\frac{8}{3}ax^2\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a} - 16ax^2\sqrt{a\cosh(x)+a} + \frac{4}{3}ax^3\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a} + \frac{8}{3}ax^3\tanh\left(\frac{x}{2}\right)\sqrt{a}$$

```
[Out] (-1280*a*Sqrt[a + a*Cosh[x]])/9 - 16*a*x^2*Sqrt[a + a*Cosh[x]] - (64*a*Cosh
[x/2]^2*Sqrt[a + a*Cosh[x]])/27 - (8*a*x^2*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])
/3 + (32*a*x*Cosh[x/2]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/9 + (4*a*x^3*Cosh[x/2
]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/3 + (640*a*x*Sqrt[a + a*Cosh[x]]*Tanh[x/2]
)/9 + (8*a*x^3*Sqrt[a + a*Cosh[x]]*Tanh[x/2])/3
```

Rubi [A] time = 0.19477, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3319, 3311, 3296, 2638, 3310}

$$-\frac{8}{3}ax^2\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a} - 16ax^2\sqrt{a\cosh(x)+a} + \frac{4}{3}ax^3\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a} + \frac{8}{3}ax^3\tanh\left(\frac{x}{2}\right)\sqrt{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (-1280*a*Sqrt[a + a*Cosh[x]])/9 - 16*a*x^2*Sqrt[a + a*Cosh[x]] - (64*a*Cosh
[x/2]^2*Sqrt[a + a*Cosh[x]])/27 - (8*a*x^2*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])
/3 + (32*a*x*Cosh[x/2]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/9 + (4*a*x^3*Cosh[x/2
]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/3 + (640*a*x*Sqrt[a + a*Cosh[x]]*Tanh[x/2]
)/9 + (8*a*x^3*Sqrt[a + a*Cosh[x]]*Tanh[x/2])/3
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int x^3(a+a\cosh(x))^{3/2}\,dx = \left(2a\sqrt{a+a\cosh(x)}\,\mathbf{sech}\left(\frac{x}{2}\right)\right)\int x^3\cosh^3\left(\frac{x}{2}\right)dx$$
$$= -\frac{8}{3}ax^2\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax^3\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\sinh\left(\frac{x}{2}\right)+\frac{1}{3}\left(4a\sqrt{a\right.$$
$$= -\frac{64}{27}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}-\frac{8}{3}ax^2\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{32}{9}ax\cosh\left(\frac{x}{2}\right)\sqrt{a\right.$$
$$= -16ax^2\sqrt{a+a\cosh(x)}-\frac{64}{27}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}-\frac{8}{3}ax^2\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(\right.$$
$$= -\frac{128}{9}a\sqrt{a+a\cosh(x)}-16ax^2\sqrt{a+a\cosh(x)}-\frac{64}{27}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}-\frac{8}{3}ax^2\,c$$
$$= -\frac{1280}{9}a\sqrt{a+a\cosh(x)}-16ax^2\sqrt{a+a\cosh(x)}-\frac{64}{27}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}-\frac{8}{3}ax^2$$

Mathematica [A] time = 0.278957, size = 70, normalized size = 0.38

$$\frac{2}{27}a\sqrt{a(\cosh(x)+1)}\left(-2\left(117x^2+968\right)+3x\left(15x^2+328\right)\tanh\left(\frac{x}{2}\right)+\cosh(x)\left(3x\left(3x^2+8\right)\tanh\left(\frac{x}{2}\right)-2\left(9x^2+8\right)\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cosh[x])]*(-2*(968 + 117*x^2) + 3*x*(328 + 15*x^2)*Tanh[x/
2] + Cosh[x]*(-2*(8 + 9*x^2) + 3*x*(8 + 3*x^2)*Tanh[x/2])))/27
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x^3(a+a\cosh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(x^3*(a+a*cosh(x))^(3/2),x)
```

Maxima [A] time = 1.67038, size = 243, normalized size = 1.31

$$-\frac{1}{54}\left(9\sqrt{2}a^{\frac{3}{2}}x^3+18\sqrt{2}a^{\frac{3}{2}}x^2+24\sqrt{2}a^{\frac{3}{2}}x+16\sqrt{2}a^{\frac{3}{2}}-\left(9\sqrt{2}a^{\frac{3}{2}}x^3-18\sqrt{2}a^{\frac{3}{2}}x^2+24\sqrt{2}a^{\frac{3}{2}}x-16\sqrt{2}a^{\frac{3}{2}}\right)e^{(3x)}-81\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/54*(9*sqrt(2)*a^(3/2)*x^3 + 18*sqrt(2)*a^(3/2)*x^2 + 24*sqrt(2)*a^(3/2)*
x + 16*sqrt(2)*a^(3/2) - (9*sqrt(2)*a^(3/2)*x^3 - 18*sqrt(2)*a^(3/2)*x^2 +
24*sqrt(2)*a^(3/2)*x - 16*sqrt(2)*a^(3/2))*e^(3*x) - 81*(sqrt(2)*a^(3/2)*x^
3 - 6*sqrt(2)*a^(3/2)*x^2 + 24*sqrt(2)*a^(3/2)*x - 48*sqrt(2)*a^(3/2))*e^(2
*x) + 81*(sqrt(2)*a^(3/2)*x^3 + 6*sqrt(2)*a^(3/2)*x^2 + 24*sqrt(2)*a^(3/2)*
x + 48*sqrt(2)*a^(3/2))*e^x)*e^(-3/2*x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cosh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18834, size = 240, normalized size = 1.3

$$\frac{1}{54}\sqrt{2}\left(9\,a^{\frac{3}{2}}x^{3}e^{\left(\frac{3}{2}\,x\right)}+81\,a^{\frac{3}{2}}x^{3}e^{\left(\frac{1}{2}\,x\right)}-81\,a^{\frac{3}{2}}x^{3}e^{\left(-\frac{1}{2}\,x\right)}-9\,a^{\frac{3}{2}}x^{3}e^{\left(-\frac{3}{2}\,x\right)}-18\,a^{\frac{3}{2}}x^{2}e^{\left(\frac{3}{2}\,x\right)}-486\,a^{\frac{3}{2}}x^{2}e^{\left(\frac{1}{2}\,x\right)}-486\,a^{\frac{3}{2}}x^{2}e^{\left(-\frac{1}{2}\,x\right)}-18\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/54*sqrt(2)*(9*a^(3/2)*x^3*e^(3/2*x) + 81*a^(3/2)*x^3*e^(1/2*x) - 81*a^(3/
2)*x^3*e^(-1/2*x) - 9*a^(3/2)*x^3*e^(-3/2*x) - 18*a^(3/2)*x^2*e^(3/2*x) - 4
86*a^(3/2)*x^2*e^(1/2*x) - 486*a^(3/2)*x^2*e^(-1/2*x) - 18*a^(3/2)*x^2*e^(-
3/2*x) + 24*a^(3/2)*x*e^(3/2*x) + 1944*a^(3/2)*x*e^(1/2*x) - 1944*a^(3/2)*x
*e^(-1/2*x) - 24*a^(3/2)*x*e^(-3/2*x) - 16*a^(3/2)*e^(3/2*x) - 3888*a^(3/2)
*e^(1/2*x) - 3888*a^(3/2)*e^(-1/2*x) - 16*a^(3/2)*e^(-3/2*x))
```

3.134 $\int x^2(a + a\cosh(x))^{3/2}\,dx$

Optimal. Leaf size=145

$$\frac{4}{3}ax^2\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{8}{3}ax^2\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{32}{3}ax\sqrt{}$$

```
[Out] (-32*a*x*Sqrt[a + a*Cosh[x]])/3 - (16*a*x*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])/
9 + (4*a*x^2*Cosh[x/2]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/3 + (224*a*Sqrt[a + a
*Cosh[x]]*Tanh[x/2])/9 + (8*a*x^2*Sqrt[a + a*Cosh[x]]*Tanh[x/2])/3 + (32*a*
Sqrt[a + a*Cosh[x]]*Sinh[x/2]^2*Tanh[x/2])/27
```

Rubi [A] time = 0.149125, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3319, 3311, 3296, 2637, 2633}

$$\frac{4}{3}ax^2\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{8}{3}ax^2\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{32}{3}ax\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + a*Cosh[x])^(3/2),x]
```


```
[Out] (-32*a*x*Sqrt[a + a*Cosh[x]])/3 - (16*a*x*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])/
9 + (4*a*x^2*Cosh[x/2]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/3 + (224*a*Sqrt[a + a
*Cosh[x]]*Tanh[x/2])/9 + (8*a*x^2*Sqrt[a + a*Cosh[x]]*Tanh[x/2])/3 + (32*a*
Sqrt[a + a*Cosh[x]]*Sinh[x/2]^2*Tanh[x/2])/27
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int x^2(a+a\cosh(x))^{3/2}\,dx &= \left(2a\sqrt{a+a\cosh(x)}\operatorname{sech}\left(\frac{x}{2}\right)\right)\int x^2\cosh^3\left(\frac{x}{2}\right)dx\\ &= -\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax^2\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\sinh\left(\frac{x}{2}\right)+\frac{1}{3}\left(4a\sqrt{a+a\right.\\ &= -\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax^2\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\sinh\left(\frac{x}{2}\right)+\frac{8}{3}ax^2\sqrt{a+a\right.\\ &= -\frac{32}{3}ax\sqrt{a+a\cosh(x)}-\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax^2\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\,\mathrm{si}\\ &= -\frac{32}{3}ax\sqrt{a+a\cosh(x)}-\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax^2\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\,\mathrm{si}\end{aligned}$$

Mathematica [A] time = 0.21464, size = 54, normalized size = 0.37

$$\frac{2}{27}a\sqrt{a(\cosh(x)+1)}\left(\left(45x^2+328\right)\tanh\left(\frac{x}{2}\right)+\cosh(x)\left(\left(9x^2+8\right)\tanh\left(\frac{x}{2}\right)-12x\right)-156x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cosh[x])]*(-156*x + (328 + 45*x^2)*Tanh[x/2] + Cosh[x]*(-1
2*x + (8 + 9*x^2)*Tanh[x/2])))/27
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^2\left(a+a\cosh(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(x^2*(a+a*cosh(x))^(3/2),x)
```

Maxima [A] time = 1.68322, size = 184, normalized size = 1.27

$$-\frac{1}{54}\left(9\sqrt{2}a^{\frac{3}{2}}x^2+12\sqrt{2}a^{\frac{3}{2}}x+8\sqrt{2}a^{\frac{3}{2}}-\left(9\sqrt{2}a^{\frac{3}{2}}x^2-12\sqrt{2}a^{\frac{3}{2}}x+8\sqrt{2}a^{\frac{3}{2}}\right)e^{(3x)}-81\left(\sqrt{2}a^{\frac{3}{2}}x^2-4\sqrt{2}a^{\frac{3}{2}}x+8\sqrt{2}a^{\frac{3}{2}}\right)e^{(2x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/54*(9*sqrt(2)*a^(3/2)*x^2 + 12*sqrt(2)*a^(3/2)*x + 8*sqrt(2)*a^(3/2) - (
9*sqrt(2)*a^(3/2)*x^2 - 12*sqrt(2)*a^(3/2)*x + 8*sqrt(2)*a^(3/2))*e^(3*x) -
 81*(sqrt(2)*a^(3/2)*x^2 - 4*sqrt(2)*a^(3/2)*x + 8*sqrt(2)*a^(3/2))*e^(2*x)
```

```
 + 81*(sqrt(2)*a^(3/2)*x^2 + 4*sqrt(2)*a^(3/2)*x + 8*sqrt(2)*a^(3/2))*e^x)*
e^(-3/2*x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*cosh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20157, size = 176, normalized size = 1.21

$$\frac{1}{54}\sqrt{2}\left(9\,a^{\frac{3}{2}}x^2e^{\left(\frac{3}{2}\,x\right)} + 81\,a^{\frac{3}{2}}x^2e^{\left(\frac{1}{2}\,x\right)} - 81\,a^{\frac{3}{2}}x^2e^{\left(-\frac{1}{2}\,x\right)} - 9\,a^{\frac{3}{2}}x^2e^{\left(-\frac{3}{2}\,x\right)} - 12\,a^{\frac{3}{2}}xe^{\left(\frac{3}{2}\,x\right)} - 324\,a^{\frac{3}{2}}xe^{\left(\frac{1}{2}\,x\right)} - 324\,a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}\,x\right)} - 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/54*sqrt(2)*(9*a^(3/2)*x^2*e^(3/2*x) + 81*a^(3/2)*x^2*e^(1/2*x) - 81*a^(3/
2)*x^2*e^(-1/2*x) - 9*a^(3/2)*x^2*e^(-3/2*x) - 12*a^(3/2)*x*e^(3/2*x) - 324
*a^(3/2)*x*e^(1/2*x) - 324*a^(3/2)*x*e^(-1/2*x) - 12*a^(3/2)*x*e^(-3/2*x) +
 8*a^(3/2)*e^(3/2*x) + 648*a^(3/2)*e^(1/2*x) - 648*a^(3/2)*e^(-1/2*x) - 8*a
^(3/2)*e^(-3/2*x))
```

3.135 $\int x(a + a\cosh(x))^{3/2}\,dx$

Optimal. Leaf size=89

$$-\frac{8}{9}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{16}{3}a\sqrt{a\cosh(x)+a}+\frac{4}{3}ax\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{8}{3}ax\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh$$

```
[Out] (-16*a*Sqrt[a + a*Cosh[x]])/3 - (8*a*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])/9 + (
4*a*x*Cosh[x/2]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/3 + (8*a*x*Sqrt[a + a*Cosh[x
]]*Tanh[x/2])/3
```

Rubi [A] time = 0.0749648, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3319, 3310, 3296, 2638}

$$-\frac{8}{9}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{16}{3}a\sqrt{a\cosh(x)+a}+\frac{4}{3}ax\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{8}{3}ax\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + a*Cosh[x])^(3/2),x]
```


```
[Out] (-16*a*Sqrt[a + a*Cosh[x]])/3 - (8*a*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])/9 + (
4*a*x*Cosh[x/2]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/3 + (8*a*x*Sqrt[a + a*Cosh[x
]]*Tanh[x/2])/3
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x(a+a\cosh(x))^{3/2}\,dx = \left(2a\sqrt{a+a\cosh(x)}\mathbf{sech}\left(\frac{x}{2}\right)\right)\int x\cosh^3\left(\frac{x}{2}\right)dx$$
$$= -\frac{8}{9}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)} + \frac{4}{3}ax\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\sinh\left(\frac{x}{2}\right) + \frac{1}{3}\left(4a\sqrt{a+a\,\mathrm{c}\right.$$
$$= -\frac{8}{9}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)} + \frac{4}{3}ax\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\sinh\left(\frac{x}{2}\right) + \frac{8}{3}ax\sqrt{a+a\,\mathrm{co}}$$
$$= -\frac{16}{3}a\sqrt{a+a\cosh(x)} - \frac{8}{9}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)} + \frac{4}{3}ax\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\,\mathrm{sinh}$$

Mathematica [A] time = 0.0838374, size = 56, normalized size = 0.63

$$\frac{1}{9}a\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a(\cosh(x)+1)}\left(3x\left(9\sinh\left(\frac{x}{2}\right)+\sinh\left(\frac{3x}{2}\right)\right)-54\cosh\left(\frac{x}{2}\right)-2\cosh\left(\frac{3x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cosh[x])]*Sech[x/2]*(-54*Cosh[x/2] - 2*Cosh[(3*x)/2] + 3*x*(
9*Sinh[x/2] + Sinh[(3*x)/2])))/9
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x\,(a+a\cosh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(x*(a+a*cosh(x))^(3/2),x)
```

Maxima [A] time = 1.7426, size = 124, normalized size = 1.39

$$-\frac{1}{18}\left(3\sqrt{2}a^{\frac{3}{2}}x+2\sqrt{2}a^{\frac{3}{2}}-\left(3\sqrt{2}a^{\frac{3}{2}}x-2\sqrt{2}a^{\frac{3}{2}}\right)e^{(3x)}-27\left(\sqrt{2}a^{\frac{3}{2}}x-2\sqrt{2}a^{\frac{3}{2}}\right)e^{(2x)}+27\left(\sqrt{2}a^{\frac{3}{2}}x+2\sqrt{2}a^{\frac{3}{2}}\right)e^{x}\right)e^{\left(-\frac{3}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/18*(3*sqrt(2)*a^(3/2)*x + 2*sqrt(2)*a^(3/2) - (3*sqrt(2)*a^(3/2)*x - 2*s
qrt(2)*a^(3/2))*e^(3*x) - 27*(sqrt(2)*a^(3/2)*x - 2*sqrt(2)*a^(3/2))*e^(2*x
) + 27*(sqrt(2)*a^(3/2)*x + 2*sqrt(2)*a^(3/2))*e^x)*e^(-3/2*x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21066, size = 111, normalized size = 1.25

$$\frac{1}{18}\sqrt{2}\left(3a^{\frac{3}{2}}xe^{\left(\frac{3}{2}x\right)} + 27a^{\frac{3}{2}}xe^{\left(\frac{1}{2}x\right)} - 27a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}x\right)} - 3a^{\frac{3}{2}}xe^{\left(-\frac{3}{2}x\right)} - 2a^{\frac{3}{2}}e^{\left(\frac{3}{2}x\right)} - 54a^{\frac{3}{2}}e^{\left(\frac{1}{2}x\right)} - 54a^{\frac{3}{2}}e^{\left(-\frac{1}{2}x\right)} - 2a^{\frac{3}{2}}e^{\left(-\frac{3}{2}x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(2)*(3*a^(3/2)*x*e^(3/2*x) + 27*a^(3/2)*x*e^(1/2*x) - 27*a^(3/2)*x
*e^(-1/2*x) - 3*a^(3/2)*x*e^(-3/2*x) - 2*a^(3/2)*e^(3/2*x) - 54*a^(3/2)*e^(
1/2*x) - 54*a^(3/2)*e^(-1/2*x) - 2*a^(3/2)*e^(-3/2*x))
```

3.136 $\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx$

Optimal. Leaf size=55

$$\frac{3}{2}a\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{1}{2}a\mathrm{Chi}\left(\frac{3x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

```
[Out] (3*a*Sqrt[a + a*Cosh[x]]*CoshIntegral[x/2]*Sech[x/2])/2 + (a*Sqrt[a + a*Cos
h[x]]*CoshIntegral[(3*x)/2]*Sech[x/2])/2
```

Rubi [A] time = 0.128222, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3319, 3312, 3301}

$$\frac{3}{2}a\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{1}{2}a\mathrm{Chi}\left(\frac{3x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x])^(3/2)/x,x]
```

```
[Out] (3*a*Sqrt[a + a*Cosh[x]]*CoshIntegral[x/2]*Sech[x/2])/2 + (a*Sqrt[a + a*Cos
h[x]]*CoshIntegral[(3*x)/2]*Sech[x/2])/2
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+a\cosh(x))^{3/2}}{x}dx = \left(2a\sqrt{a+a\cosh(x)}\text{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh^3\left(\frac{x}{2}\right)}{x}dx$$
$$= \left(2a\sqrt{a+a\cosh(x)}\text{sech}\left(\frac{x}{2}\right)\right)\int \left(\frac{3\cosh\left(\frac{x}{2}\right)}{4x}+\frac{\cosh\left(\frac{3x}{2}\right)}{4x}\right)dx$$
$$= \frac{1}{2}\left(a\sqrt{a+a\cosh(x)}\text{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh\left(\frac{3x}{2}\right)}{x}dx+\frac{1}{2}\left(3a\sqrt{a+a\cosh(x)}\text{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh\left(\frac{x}{2}\right)}{x}$$
$$= \frac{3}{2}a\sqrt{a+a\cosh(x)}\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)+\frac{1}{2}a\sqrt{a+a\cosh(x)}\text{Chi}\left(\frac{3x}{2}\right)\text{sech}\left(\frac{x}{2}\right)$$

Mathematica [A] time = 0.0196958, size = 36, normalized size = 0.65

$$\frac{1}{2}a\left(3\text{Chi}\left(\frac{x}{2}\right)+\text{Chi}\left(\frac{3x}{2}\right)\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x])^(3/2)/x,x]
```

```
[Out] (a*Sqrt[a*(1 + Cosh[x])]*(3*CoshIntegral[x/2] + CoshIntegral[(3*x)/2])*Sech
[x/2])/2
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{x}(a+a\cosh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(3/2)/x,x)
```

```
[Out] int((a+a*cosh(x))^(3/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cosh(x)+a)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + a)^(3/2)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17945, size = 54, normalized size = 0.98

$$\frac{1}{4}\sqrt{2}\left(a^{\frac{3}{2}}\mathrm{Ei}\left(\frac{3}{2}x\right)+3a^{\frac{3}{2}}\mathrm{Ei}\left(\frac{1}{2}x\right)+3a^{\frac{3}{2}}\mathrm{Ei}\left(-\frac{1}{2}x\right)+a^{\frac{3}{2}}\mathrm{Ei}\left(-\frac{3}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a^(3/2)*Ei(3/2*x) + 3*a^(3/2)*Ei(1/2*x) + 3*a^(3/2)*Ei(-1/2*x)
 + a^(3/2)*Ei(-3/2*x))
```

3.137 $\int \frac{(a+a\cosh(x))^{3/2}}{x^2}\,dx$

Optimal. Leaf size=79

$$\frac{3}{4}a\text{Shi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{3}{4}a\text{Shi}\left(\frac{3x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{2a\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}}{x}$$

```
[Out] (-2*a*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])/x + (3*a*Sqrt[a + a*Cosh[x]]*Sech[x/
2]*SinhIntegral[x/2])/4 + (3*a*Sqrt[a + a*Cosh[x]]*Sech[x/2]*SinhIntegral[(
3*x)/2])/4
```

Rubi [A] time = 0.13079, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3319, 3313, 3298}

$$\frac{3}{4}a\text{Shi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{3}{4}a\text{Shi}\left(\frac{3x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{2a\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x])^(3/2)/x^2,x]
```

```
[Out] (-2*a*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])/x + (3*a*Sqrt[a + a*Cosh[x]]*Sech[x/
2]*SinhIntegral[x/2])/4 + (3*a*Sqrt[a + a*Cosh[x]]*Sech[x/2]*SinhIntegral[(
3*x)/2])/4
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^2}\,dx = \left(2a\sqrt{a+a\cosh(x)}\text{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh^3\left(\frac{x}{2}\right)}{x^2}\,dx$$

$$= -\frac{2a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x} + \left(3ia\sqrt{a+a\cosh(x)}\text{sech}\left(\frac{x}{2}\right)\right)\int \left(-\frac{i\sinh\left(\frac{x}{2}\right)}{4x} - \frac{i\sinh\left(\frac{3}{2}\right.}{4x}\right.$$

$$= -\frac{2a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x} + \frac{1}{4}\left(3a\sqrt{a+a\cosh(x)}\text{sech}\left(\frac{x}{2}\right)\right)\int \frac{\sinh\left(\frac{x}{2}\right)}{x}\,dx + \frac{1}{4}\left(3a\sqrt{\right.$$

$$= -\frac{2a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x} + \frac{3}{4}a\sqrt{a+a\cosh(x)}\text{sech}\left(\frac{x}{2}\right)\text{Shi}\left(\frac{x}{2}\right) + \frac{3}{4}a\sqrt{a+a\cosh(x)}$$

Mathematica [A] time = 0.0838189, size = 53, normalized size = 0.67

$$-\frac{a\text{sech}\left(\frac{x}{2}\right)\sqrt{a(\cosh(x)+1)}\left(-3x\text{Shi}\left(\frac{x}{2}\right) - 3x\text{Shi}\left(\frac{3x}{2}\right) + 8\cosh^3\left(\frac{x}{2}\right)\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x])^(3/2)/x^2,x]
```

```
[Out] -(a*Sqrt[a*(1 + Cosh[x])]*Sech[x/2]*(8*Cosh[x/2]^3 - 3*x*SinhIntegral[x/2]
- 3*x*SinhIntegral[(3*x)/2]))/(4*x)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}(a+a\cosh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(3/2)/x^2,x)
```

```
[Out] int((a+a*cosh(x))^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cosh(x)+a)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + a)^(3/2)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26839, size = 115, normalized size = 1.46

$$\frac{\sqrt{2}\left(3\,a^{\frac{3}{2}}x\mathrm{Ei}\left(\frac{3}{2}\,x\right)+3\,a^{\frac{3}{2}}x\mathrm{Ei}\left(\frac{1}{2}\,x\right)-3\,a^{\frac{3}{2}}x\mathrm{Ei}\left(-\frac{1}{2}\,x\right)-3\,a^{\frac{3}{2}}x\mathrm{Ei}\left(-\frac{3}{2}\,x\right)-2\,a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)}-6\,a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)}-6\,a^{\frac{3}{2}}e^{\left(-\frac{1}{2}\,x\right)}-2\,a^{\frac{3}{2}}e^{\left(-\frac{3}{2}\,x\right)}\right)}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*a^(3/2)*x*Ei(3/2*x) + 3*a^(3/2)*x*Ei(1/2*x) - 3*a^(3/2)*x*Ei
(-1/2*x) - 3*a^(3/2)*x*Ei(-3/2*x) - 2*a^(3/2)*e^(3/2*x) - 6*a^(3/2)*e^(1/2*
x) - 6*a^(3/2)*e^(-1/2*x) - 2*a^(3/2)*e^(-3/2*x))/x
```

3.138 $\int \frac{(a+a\cosh(x))^{3/2}}{x^3}\,dx$

Optimal. Leaf size=109

$$\frac{3}{16}a\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{9}{16}a\mathrm{Chi}\left(\frac{3x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}}{x^2}-\frac{3a\,\mathrm{si}}{}$$

```
[Out] -((a*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])/x^2) + (3*a*Sqrt[a + a*Cosh[x]]*CoshI
ntegral[x/2]*Sech[x/2])/16 + (9*a*Sqrt[a + a*Cosh[x]]*CoshIntegral[(3*x)/2]
*Sech[x/2])/16 - (3*a*Cosh[x/2]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/(2*x)
```

Rubi [A] time = 0.172993, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3319, 3314, 3301, 3312}

$$\frac{3}{16}a\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{9}{16}a\mathrm{Chi}\left(\frac{3x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}}{x^2}-\frac{3a\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x])^(3/2)/x^3,x]
```


```
[Out] -((a*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])/x^2) + (3*a*Sqrt[a + a*Cosh[x]]*CoshI
ntegral[x/2]*Sech[x/2])/16 + (9*a*Sqrt[a + a*Cosh[x]]*CoshIntegral[(3*x)/2]
*Sech[x/2])/16 - (3*a*Cosh[x/2]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/(2*x)
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^3}\,dx = \left(2a\sqrt{a+a\cosh(x)}\operatorname{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh^3\left(\frac{x}{2}\right)}{x^3}\,dx$$
$$= -\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x^2} - \frac{3a\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\sinh\left(\frac{x}{2}\right)}{2x} - \frac{1}{2}\left(3a\sqrt{a+a\cosh(x)}\text{se}\right.$$
$$= -\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x^2} - \frac{3}{2}a\sqrt{a+a\cosh(x)}\text{Chi}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right) - \frac{3a\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cos}}{2x}$$
$$= -\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x^2} - \frac{3}{2}a\sqrt{a+a\cosh(x)}\text{Chi}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right) - \frac{3a\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cos}}{2x}$$
$$= -\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x^2} + \frac{3}{16}a\sqrt{a+a\cosh(x)}\text{Chi}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right) + \frac{9}{16}a\sqrt{a+a\cosh(x)}\text{C}$$

Mathematica [A] time = 0.0646642, size = 69, normalized size = 0.63

$$\frac{(a(\cosh(x)+1))^{3/2}\left(3x^2\text{Chi}\left(\frac{x}{2}\right)\operatorname{sech}^3\left(\frac{x}{2}\right)+9x^2\text{Chi}\left(\frac{3x}{2}\right)\operatorname{sech}^3\left(\frac{x}{2}\right)-8\left(3x\tanh\left(\frac{x}{2}\right)+2\right)\right)}{32x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x])^(3/2)/x^3,x]
```

```
[Out] ((a*(1 + Cosh[x]))^(3/2)*(3*x^2*CoshIntegral[x/2]*Sech[x/2]^3 + 9*x^2*CoshI
ntegral[(3*x)/2]*Sech[x/2]^3 - 8*(2 + 3*x*Tanh[x/2])))/(32*x^2)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(a+a\cosh(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(3/2)/x^3,x)
```

```
[Out] int((a+a*cosh(x))^(3/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cosh(x)+a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + a)^(3/2)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30933, size = 180, normalized size = 1.65

$$\frac{\sqrt{2}\left(9\,a^{\frac{3}{2}}x^{2}\mathrm{Ei}\left(\frac{3}{2}\,x\right) + 3\,a^{\frac{3}{2}}x^{2}\mathrm{Ei}\left(\frac{1}{2}\,x\right) + 3\,a^{\frac{3}{2}}x^{2}\mathrm{Ei}\left(-\frac{1}{2}\,x\right) + 9\,a^{\frac{3}{2}}x^{2}\mathrm{Ei}\left(-\frac{3}{2}\,x\right) - 6\,a^{\frac{3}{2}}xe^{\left(\frac{3}{2}\,x\right)} - 6\,a^{\frac{3}{2}}xe^{\left(\frac{1}{2}\,x\right)} + 6\,a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}\,x\right)} + \right.}{32\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/32*sqrt(2)*(9*a^(3/2)*x^2*Ei(3/2*x) + 3*a^(3/2)*x^2*Ei(1/2*x) + 3*a^(3/2)
*x^2*Ei(-1/2*x) + 9*a^(3/2)*x^2*Ei(-3/2*x) - 6*a^(3/2)*x*e^(3/2*x) - 6*a^(3
/2)*x*e^(1/2*x) + 6*a^(3/2)*x*e^(-1/2*x) + 6*a^(3/2)*x*e^(-3/2*x) - 4*a^(3/
2)*e^(3/2*x) - 12*a^(3/2)*e^(1/2*x) - 12*a^(3/2)*e^(-1/2*x) - 4*a^(3/2)*e^(
-3/2*x))/x^2
```

3.139 $\int \frac{x^3}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=383

$$-\frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{48ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(\right.}{d^3\sqrt{a\cosh(c+dx)+}}$$

```
[Out] (4*x^3*ArcTan[E^(c/2 + (d*x)/2)]*Cosh[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cosh[c
+ d*x]]) - ((12*I)*x^2*Cosh[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^(c/2 + (d*x)/2
)])/(d^2*Sqrt[a + a*Cosh[c + d*x]]) + ((12*I)*x^2*Cosh[c/2 + (d*x)/2]*PolyL
og[2, I*E^(c/2 + (d*x)/2)])/(d^2*Sqrt[a + a*Cosh[c + d*x]]) + ((48*I)*x*Cos
h[c/2 + (d*x)/2]*PolyLog[3, (-I)*E^(c/2 + (d*x)/2)])/(d^3*Sqrt[a + a*Cosh[c
 + d*x]]) - ((48*I)*x*Cosh[c/2 + (d*x)/2]*PolyLog[3, I*E^(c/2 + (d*x)/2)])/
(d^3*Sqrt[a + a*Cosh[c + d*x]]) - ((96*I)*Cosh[c/2 + (d*x)/2]*PolyLog[4, (-
I)*E^(c/2 + (d*x)/2)])/(d^4*Sqrt[a + a*Cosh[c + d*x]]) + ((96*I)*Cosh[c/2 +
 (d*x)/2]*PolyLog[4, I*E^(c/2 + (d*x)/2)])/(d^4*Sqrt[a + a*Cosh[c + d*x]])
```

Rubi [A] time = 0.208099, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3319, 4180, 2531, 6609, 2282, 6589}

$$-\frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{48ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(\right.}{d^3\sqrt{a\cosh(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (4*x^3*ArcTan[E^(c/2 + (d*x)/2)]*Cosh[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cosh[c
+ d*x]]) - ((12*I)*x^2*Cosh[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^(c/2 + (d*x)/2
)])/(d^2*Sqrt[a + a*Cosh[c + d*x]]) + ((12*I)*x^2*Cosh[c/2 + (d*x)/2]*PolyL
og[2, I*E^(c/2 + (d*x)/2)])/(d^2*Sqrt[a + a*Cosh[c + d*x]]) + ((48*I)*x*Cos
h[c/2 + (d*x)/2]*PolyLog[3, (-I)*E^(c/2 + (d*x)/2)])/(d^3*Sqrt[a + a*Cosh[c
 + d*x]]) - ((48*I)*x*Cosh[c/2 + (d*x)/2]*PolyLog[3, I*E^(c/2 + (d*x)/2)])/
(d^3*Sqrt[a + a*Cosh[c + d*x]]) - ((96*I)*Cosh[c/2 + (d*x)/2]*PolyLog[4, (-
I)*E^(c/2 + (d*x)/2)])/(d^4*Sqrt[a + a*Cosh[c + d*x]]) + ((96*I)*Cosh[c/2 +
 (d*x)/2]*PolyLog[4, I*E^(c/2 + (d*x)/2)])/(d^4*Sqrt[a + a*Cosh[c + d*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\int x^3\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)dx}{\sqrt{a+a\cosh(c+dx)}}$$

$$= \frac{4x^3\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{\left(6i\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int x^2\log\left(1-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d\sqrt{a+a\cosh(c+dx)}}$$

$$= \frac{4x^3\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{12ix^2\cosh\left(\frac{c}{2}\right.}{d^2\sqrt{a+a}}$$

$$= \frac{4x^3\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{12ix^2\cosh\left(\frac{c}{2}\right.}{d^2\sqrt{a+a}}$$

$$= \frac{4x^3\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{12ix^2\cosh\left(\frac{c}{2}\right.}{d^2\sqrt{a+a}}$$

$$= \frac{4x^3\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{12ix^2\cosh\left(\frac{c}{2}\right.}{d^2\sqrt{a+a}}$$

Mathematica [A] time = 1.72405, size = 213, normalized size = 0.56

$$\frac{2i\cosh\left(\frac{1}{2}(c+dx)\right)\left(-6d^2x^2\text{PolyLog}\left(2,-ie^{\frac{1}{2}(c+dx)}\right)+6d^2x^2\text{PolyLog}\left(2,ie^{\frac{1}{2}(c+dx)}\right)+24dx\text{PolyLog}\left(3,-ie^{\frac{1}{2}(c+dx)}\right)-2\right.}{d^4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] ((2*I)*Cosh[(c + d*x)/2]*(d^3*x^3*Log[1 - I*E^((c + d*x)/2)] - d^3*x^3*Log[
1 + I*E^((c + d*x)/2)] - 6*d^2*x^2*PolyLog[2, (-I)*E^((c + d*x)/2)] + 6*d^2
*x^2*PolyLog[2, I*E^((c + d*x)/2)] + 24*d*x*PolyLog[3, (-I)*E^((c + d*x)/2)
] - 24*d*x*PolyLog[3, I*E^((c + d*x)/2)] - 48*PolyLog[4, (-I)*E^((c + d*x)/
2)] + 48*PolyLog[4, I*E^((c + d*x)/2)]))/(d^4*Sqrt[a*(1 + Cosh[c + d*x])])
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int x^{3}\frac{1}{\sqrt{a+a\cosh\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] int(x^3/(a+a*cosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\sqrt{2}d^{3}\int\frac{x^{3}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{a}d^{3}e^{(2\,dx+2\,c)}+2\,\sqrt{a}d^{3}e^{(dx+c)}+\sqrt{a}d^{3}}\,dx+12\,\sqrt{2}d^{2}\int\frac{x^{2}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{a}d^{3}e^{(2\,dx+2\,c)}+2\,\sqrt{a}d^{3}e^{(dx+c)}+\sqrt{a}d^{3}}\,dx+48\,\sqrt{2}d\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*d^3*integrate(x^3*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d^3*e^(2*d*x + 2*c
) + 2*sqrt(a)*d^3*e^(d*x + c) + sqrt(a)*d^3), x) + 12*sqrt(2)*d^2*integrate
(x^2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d^3*e^(2*d*x + 2*c) + 2*sqrt(a)*d^3*e^(d*
x + c) + sqrt(a)*d^3), x) + 48*sqrt(2)*d*integrate(x*e^(1/2*d*x + 1/2*c)/(s
qrt(a)*d^3*e^(2*d*x + 2*c) + 2*sqrt(a)*d^3*e^(d*x + c) + sqrt(a)*d^3), x) +
 96*sqrt(2)*(e^(1/2*d*x + 1/2*c)/((sqrt(a)*d^3*e^(d*x + c) + sqrt(a)*d^3)*d
) + arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d^4)) - 2*(sqrt(2)*sqrt(a)*d^3*x^3
*e^(1/2*c) + 6*sqrt(2)*sqrt(a)*d^2*x^2*e^(1/2*c) + 24*sqrt(2)*sqrt(a)*d*x*e
^(1/2*c) + 48*sqrt(2)*sqrt(a)*e^(1/2*c))*e^(1/2*d*x)/(a*d^4*e^(d*x + c) + a
*d^4)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{3}}{\sqrt{a\cosh\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(a*cosh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*(cosh(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a\cosh\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(a*cosh(d*x + c) + a), x)
```

3.140 $\int \frac{x^2}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=269

$$-\frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{16i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(3,-ie\right.}{d^3\sqrt{a\cosh(c+dx)+a}}$$

```
[Out] (4*x^2*ArcTan[E^(c/2 + (d*x)/2)]*Cosh[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cosh[c
+ d*x]]) - ((8*I)*x*Cosh[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^(c/2 + (d*x)/2)])
/(d^2*Sqrt[a + a*Cosh[c + d*x]]) + ((8*I)*x*Cosh[c/2 + (d*x)/2]*PolyLog[2,
I*E^(c/2 + (d*x)/2)])/(d^2*Sqrt[a + a*Cosh[c + d*x]]) + ((16*I)*Cosh[c/2 +
(d*x)/2]*PolyLog[3, (-I)*E^(c/2 + (d*x)/2)])/(d^3*Sqrt[a + a*Cosh[c + d*x]]
) - ((16*I)*Cosh[c/2 + (d*x)/2]*PolyLog[3, I*E^(c/2 + (d*x)/2)])/(d^3*Sqrt[
a + a*Cosh[c + d*x]])
```

Rubi [A] time = 0.163233, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3319, 4180, 2531, 2282, 6589}

$$-\frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{16i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(3,-ie\right.}{d^3\sqrt{a\cosh(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (4*x^2*ArcTan[E^(c/2 + (d*x)/2)]*Cosh[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cosh[c
+ d*x]]) - ((8*I)*x*Cosh[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^(c/2 + (d*x)/2)])
/(d^2*Sqrt[a + a*Cosh[c + d*x]]) + ((8*I)*x*Cosh[c/2 + (d*x)/2]*PolyLog[2,
I*E^(c/2 + (d*x)/2)])/(d^2*Sqrt[a + a*Cosh[c + d*x]]) + ((16*I)*Cosh[c/2 +
(d*x)/2]*PolyLog[3, (-I)*E^(c/2 + (d*x)/2)])/(d^3*Sqrt[a + a*Cosh[c + d*x]]
) - ((16*I)*Cosh[c/2 + (d*x)/2]*PolyLog[3, I*E^(c/2 + (d*x)/2)])/(d^3*Sqrt[
a + a*Cosh[c + d*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{a+a\cosh(c+dx)}}\,dx &= \frac{\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\int x^2\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)dx}{\sqrt{a+a\cosh(c+dx)}}\\
&= \frac{4x^2\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{\left(4i\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int x\log\left(1-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d\sqrt{a+a\cosh(c+dx)}}\\
&= \frac{4x^2\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right.}{d^2\sqrt{a+a\cos}}\\
&= \frac{4x^2\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right.}{d^2\sqrt{a+a\cos}}\\
&= \frac{4x^2\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right.}{d^2\sqrt{a+a\cos}}
\end{aligned}
$$

Mathematica [A] time = 1.70702, size = 163, normalized size = 0.61

$$
\frac{2i\cosh\left(\frac{1}{2}(c+dx)\right)\left(-4dx\text{PolyLog}\left(2,-ie^{\frac{1}{2}(c+dx)}\right)+4dx\text{PolyLog}\left(2,ie^{\frac{1}{2}(c+dx)}\right)+8\text{PolyLog}\left(3,-ie^{\frac{1}{2}(c+dx)}\right)-8\text{PolyLo}\right.}{d^3\sqrt{a(\cosh(c+dx)+1)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] ((2*I)*Cosh[(c + d*x)/2]*(d^2*x^2*Log[1 - I*E^((c + d*x)/2)] - d^2*x^2*Log[
1 + I*E^((c + d*x)/2)] - 4*d*x*PolyLog[2, (-I)*E^((c + d*x)/2)] + 4*d*x*Pol
yLog[2, I*E^((c + d*x)/2)] + 8*PolyLog[3, (-I)*E^((c + d*x)/2)] - 8*PolyLog
[3, I*E^((c + d*x)/2)]))/(d^3*Sqrt[a*(1 + Cosh[c + d*x])])
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$
\int x^2\frac{1}{\sqrt{a+a\cosh(dx+c)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] int(x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\sqrt{2}d^2\int \frac{x^2e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{\sqrt{a}d^2e^{(2dx+2c)}+2\sqrt{a}d^2e^{(dx+c)}+\sqrt{a}d^2}\,dx+8\sqrt{2}d\int \frac{xe^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{\sqrt{a}d^2e^{(2dx+2c)}+2\sqrt{a}d^2e^{(dx+c)}+\sqrt{a}d^2}\,dx+16\sqrt{2}\left(\frac{\quad}{(\sqrt{a}d^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*d^2*integrate(x^2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d^2*e^(2*d*x + 2*c
) + 2*sqrt(a)*d^2*e^(d*x + c) + sqrt(a)*d^2), x) + 8*sqrt(2)*d*integrate(x*
e^(1/2*d*x + 1/2*c)/(sqrt(a)*d^2*e^(2*d*x + 2*c) + 2*sqrt(a)*d^2*e^(d*x + c
) + sqrt(a)*d^2), x) + 16*sqrt(2)*(e^(1/2*d*x + 1/2*c)/((sqrt(a)*d^2*e^(d*x
 + c) + sqrt(a)*d^2)*d) + arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d^3)) - 2*(s
qrt(2)*d^2*x^2*e^(1/2*c) + 4*sqrt(2)*d*x*e^(1/2*c) + 8*sqrt(2)*e^(1/2*c))*e
^(1/2*d*x)/(sqrt(a)*d^3*e^(d*x + c) + sqrt(a)*d^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{a\cosh(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(a*cosh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a(\cosh(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*(cosh(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a\cosh(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a*cosh(d*x + c) + a), x)
```

3.141 $\int \frac{x}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=157

$$-\frac{4i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{4i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{4x\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a\cosh(c+dx)+a}}$$

```
[Out] (4*x*ArcTan[E^(c/2 + (d*x)/2)]*Cosh[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cosh[c +
d*x]]) - ((4*I)*Cosh[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^(c/2 + (d*x)/2)])/(d^
2*Sqrt[a + a*Cosh[c + d*x]]) + ((4*I)*Cosh[c/2 + (d*x)/2]*PolyLog[2, I*E^(c
/2 + (d*x)/2)])/(d^2*Sqrt[a + a*Cosh[c + d*x]])
```

Rubi [A] time = 0.0821126, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3319, 4180, 2279, 2391}

$$-\frac{4i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{4i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{4x\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a\cosh(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (4*x*ArcTan[E^(c/2 + (d*x)/2)]*Cosh[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cosh[c +
d*x]]) - ((4*I)*Cosh[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^(c/2 + (d*x)/2)])/(d^
2*Sqrt[a + a*Cosh[c + d*x]]) + ((4*I)*Cosh[c/2 + (d*x)/2]*PolyLog[2, I*E^(c
/2 + (d*x)/2)])/(d^2*Sqrt[a + a*Cosh[c + d*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\int x\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)dx}{\sqrt{a+a\cosh(c+dx)}}$$

$$= \frac{4x\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{\left(2i\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int\log\left(1-ie^{\frac{c}{2}+\frac{dx}{2}}\right)dx}{d\sqrt{a+a\cosh(c+dx)}}$$

$$= \frac{4x\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{\left(4i\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,$$

$$= \frac{4x\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{4i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{4i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}}{d^2\sqrt{a+a\cosh(}$$

Mathematica [A] time = 0.665434, size = 117, normalized size = 0.75

$$\frac{4\cosh\left(\frac{1}{2}(c+dx)\right)\left(-i\text{PolyLog}\left(2,-i\left(\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\right)\right)+i\text{PolyLog}\left(2,i\left(\sinh\left(\frac{1}{2}(c+dx)\right)+\text{co}\right.\right.\right.}{d^2\sqrt{a(\cosh(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (4*Cosh[(c + d*x)/2]*(d*x*ArcTan[Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]] - I
*PolyLog[2, (-I)*(Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] + I*PolyLog[2, I*
(Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])]))/(d^2*Sqrt[a*(1 + Cosh[c + d*x])]
)
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] int(x/(a+a*cosh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\sqrt{2}d\int\frac{xe^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{\sqrt{a}de^{(2dx+2c)}+2\sqrt{a}de^{(dx+c)}+\sqrt{a}d}\,dx+4\sqrt{2}\left(\frac{e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{\left(\sqrt{a}de^{(dx+c)}+\sqrt{a}d\right)d}+\frac{\arctan\left(e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}\right)}{\sqrt{a}d^2}\right)-\frac{2\left(\sqrt{2}\sqrt{a}dxe^{(}\right.}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*d*integrate(x*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d*e^(2*d*x + 2*c) + 2*
sqrt(a)*d*e^(d*x + c) + sqrt(a)*d), x) + 4*sqrt(2)*(e^(1/2*d*x + 1/2*c)/((s
qrt(a)*d*e^(d*x + c) + sqrt(a)*d)*d) + arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)
*d^2)) - 2*(sqrt(2)*sqrt(a)*d*x*e^(1/2*c) + 2*sqrt(2)*sqrt(a)*e^(1/2*c))*e^
(1/2*d*x)/(a*d^2*e^(d*x + c) + a*d^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{a\cosh\left(dx+c\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(a*cosh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*(cosh(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a\cosh\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(a*cosh(d*x + c) + a), x)
```

3.142 $\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{a\cosh(c+dx)+a}},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[a + a*Cosh[c + d*x]]), x]
```

Rubi [A] time = 0.0680483, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[a + a*Cosh[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[a + a*Cosh[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx=\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx$$

Mathematica [A] time = 2.66364, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[a + a*Cosh[c + d*x]]),x]
```

```
[Out] Integrate[1/(x*Sqrt[a + a*Cosh[c + d*x]]), x]
```

Maple [A] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{a+a\cosh(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/x/(a+a*cosh(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cosh\left(dx+c\right)+a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*cosh(d*x + c) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cosh\left(dx+c\right)+a}}{ax\cosh\left(dx+c\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(d*x + c) + a)/(a*x*cosh(d*x + c) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*(cosh(c + d*x) + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cosh\left(dx+c\right)+a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cosh(d*x + c) + a)*x), x)
```

3.143 $\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a\cosh(c+dx)+a}},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[a + a*Cosh[c + d*x]]), x]
```

Rubi [A] time = 0.0664644, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[a + a*Cosh[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[a + a*Cosh[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx$$

Mathematica [A] time = 1.64585, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[a + a*Cosh[c + d*x]]),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[a + a*Cosh[c + d*x]]), x]
```

Maple [A] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{a+a\cosh(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cosh\left(dx+c\right)+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*cosh(d*x + c) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cosh\left(dx+c\right)+a}}{ax^2\cosh\left(dx+c\right)+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(d*x + c) + a)/(a*x^2*cosh(d*x + c) + a*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*(cosh(c + d*x) + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cosh\left(dx+c\right)+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cosh(d*x + c) + a)*x^2), x)
```

3.144 $\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=402

$$-\frac{3ix^2\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{3ix^2\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{12ix\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(3,-ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}-\frac{12ix\text{ co}}{}$$

```
[Out] (3*x^2)/(a*Sqrt[a + a*Cosh[x]]) - (24*x*ArcTan[E^(x/2)]*Cosh[x/2])/(a*Sqrt[
a + a*Cosh[x]]) + (x^3*ArcTan[E^(x/2)]*Cosh[x/2])/(a*Sqrt[a + a*Cosh[x]]) +
 ((24*I)*Cosh[x/2]*PolyLog[2, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((3*
I)*x^2*Cosh[x/2]*PolyLog[2, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((24*I
)*Cosh[x/2]*PolyLog[2, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((3*I)*x^2*Cos
h[x/2]*PolyLog[2, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((12*I)*x*Cosh[x/2]
*PolyLog[3, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((12*I)*x*Cosh[x/2]*Po
lyLog[3, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((24*I)*Cosh[x/2]*PolyLog[4,
 (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((24*I)*Cosh[x/2]*PolyLog[4, I*E^
(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + (x^3*Tanh[x/2])/(2*a*Sqrt[a + a*Cosh[x]])
```

Rubi [A] time = 0.257355, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3319, 4186, 4180, 2279, 2391, 2531, 6609, 2282, 6589}

$$-\frac{3ix^2\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{3ix^2\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{12ix\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(3,-ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}-\frac{12ix\text{ co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + a*Cosh[x])^(3/2),x]
```


```
[Out] (3*x^2)/(a*Sqrt[a + a*Cosh[x]]) - (24*x*ArcTan[E^(x/2)]*Cosh[x/2])/(a*Sqrt[
a + a*Cosh[x]]) + (x^3*ArcTan[E^(x/2)]*Cosh[x/2])/(a*Sqrt[a + a*Cosh[x]]) +
 ((24*I)*Cosh[x/2]*PolyLog[2, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((3*
I)*x^2*Cosh[x/2]*PolyLog[2, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((24*I
)*Cosh[x/2]*PolyLog[2, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((3*I)*x^2*Cos
h[x/2]*PolyLog[2, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((12*I)*x*Cosh[x/2]
*PolyLog[3, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((12*I)*x*Cosh[x/2]*Po
lyLog[3, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((24*I)*Cosh[x/2]*PolyLog[4,
 (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((24*I)*Cosh[x/2]*PolyLog[4, I*E^
(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + (x^3*Tanh[x/2])/(2*a*Sqrt[a + a*Cosh[x]])
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
```

```
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx = \frac{\cosh\left(\frac{x}{2}\right)\int x^3\text{sech}^3\left(\frac{x}{2}\right)dx}{2a\sqrt{a+a\cosh(x)}}$$
$$= \frac{3x^2}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}} + \frac{\cosh\left(\frac{x}{2}\right)\int x^3\text{sech}\left(\frac{x}{2}\right)dx}{4a\sqrt{a+a\cosh(x)}} - \frac{\left(6\cosh\left(\frac{x}{2}\right)\right)\int x\text{sech}}{a\sqrt{a+a\cosh(}}$$
$$= \frac{3x^2}{a\sqrt{a+a\cosh(x)}} - \frac{24x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(}}$$
$$= \frac{3x^2}{a\sqrt{a+a\cosh(x)}} - \frac{24x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{3ix^2\cosh\left(\frac{x}{2}\right)\text{L}}{a\sqrt{a+a\text{co}}}$$
$$= \frac{3x^2}{a\sqrt{a+a\cosh(x)}} - \frac{24x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{24i\cosh\left(\frac{x}{2}\right)\text{Li}}{a\sqrt{a+a\text{co}}}$$
$$= \frac{3x^2}{a\sqrt{a+a\cosh(x)}} - \frac{24x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{24i\cosh\left(\frac{x}{2}\right)\text{Li}}{a\sqrt{a+a\text{co}}}$$
$$= \frac{3x^2}{a\sqrt{a+a\cosh(x)}} - \frac{24x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{24i\cosh\left(\frac{x}{2}\right)\text{Li}}{a\sqrt{a+a\text{co}}}$$

Mathematica [A] time = 2.92329, size = 716, normalized size = 1.78

$$-\frac{i\cosh\left(\frac{x}{2}\right)\left(48x^2\cosh^2\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{x/2}\right)-48\left(-x^2-2i\pi x+\pi^2+8\right)\cosh^2\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{-x/2}\right)+96i\pi x\,\text{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] ((-I/8)*Cosh[x/2]*((48*I)*x^2*Cosh[x/2] + 7*Pi^4*Cosh[x/2]^2 + (4*I)*Pi^3*x
*Cosh[x/2]^2 + 6*Pi^2*x^2*Cosh[x/2]^2 - (4*I)*Pi*x^3*Cosh[x/2]^2 - x^4*Cosh
[x/2]^2 - 192*x*Cosh[x/2]^2*Log[1 - I/E^(x/2)] + (8*I)*Pi^3*Cosh[x/2]^2*Log
[1 + I/E^(x/2)] + 192*x*Cosh[x/2]^2*Log[1 + I/E^(x/2)] + 24*Pi^2*x*Cosh[x/2
]^2*Log[1 + I/E^(x/2)] - (24*I)*Pi*x^2*Cosh[x/2]^2*Log[1 + I/E^(x/2)] - 8*x
^3*Cosh[x/2]^2*Log[1 + I/E^(x/2)] - 24*Pi^2*x*Cosh[x/2]^2*Log[1 - I*E^(x/2)
] + (24*I)*Pi*x^2*Cosh[x/2]^2*Log[1 - I*E^(x/2)] - (8*I)*Pi^3*Cosh[x/2]^2*L
og[1 + I*E^(x/2)] + 8*x^3*Cosh[x/2]^2*Log[1 + I*E^(x/2)] + (8*I)*Pi^3*Cosh[
x/2]^2*Log[Tan[(Pi + I*x)/4]] - 48*(8 + Pi^2 - (2*I)*Pi*x - x^2)*Cosh[x/2]^
2*PolyLog[2, (-I)/E^(x/2)] + 384*Cosh[x/2]^2*PolyLog[2, I/E^(x/2)] + 48*x^2
*Cosh[x/2]^2*PolyLog[2, (-I)*E^(x/2)] - 48*Pi^2*Cosh[x/2]^2*PolyLog[2, I*E^
(x/2)] + (96*I)*Pi*x*Cosh[x/2]^2*PolyLog[2, I*E^(x/2)] + (192*I)*Pi*Cosh[x/
2]^2*PolyLog[3, (-I)/E^(x/2)] + 192*x*Cosh[x/2]^2*PolyLog[3, (-I)/E^(x/2)]
- 192*x*Cosh[x/2]^2*PolyLog[3, (-I)*E^(x/2)] - (192*I)*Pi*Cosh[x/2]^2*PolyL
og[3, I*E^(x/2)] + 384*Cosh[x/2]^2*PolyLog[4, (-I)/E^(x/2)] + 384*Cosh[x/2]
^2*PolyLog[4, (-I)*E^(x/2)] + (8*I)*x^3*Sinh[x/2]))/(a*(1 + Cosh[x]))^(3/2)
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int x^3\left(a+a\cosh(x)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(x^3/(a+a*cosh(x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{8}{27}\sqrt{2}\left(\frac{3\,e^{\left(\frac{5}{2}\,x\right)}+8\,e^{\left(\frac{3}{2}\,x\right)}-3\,e^{\left(\frac{1}{2}\,x\right)}}{a^{\frac{3}{2}}e^{(3\,x)}+3\,a^{\frac{3}{2}}e^{(2\,x)}+3\,a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}+\frac{3\,\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{a^{\frac{3}{2}}}\right)+36\,\sqrt{2}\int\frac{x^{3}e^{\left(\frac{3}{2}\,x\right)}}{9\left(a^{\frac{3}{2}}e^{(4\,x)}+4\,a^{\frac{3}{2}}e^{(3\,x)}+6\,a^{\frac{3}{2}}e^{(2\,x)}+4\,a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 8/27*sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) +
3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2))
 + 36*sqrt(2)*integrate(1/9*x^3*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3
*x) + 6*a^(3/2)*e^(2*x) + 4*a^(3/2)*e^x + a^(3/2)), x) + 72*sqrt(2)*integra
te(1/9*x^2*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3*x) + 6*a^(3/2)*e^(2*
x) + 4*a^(3/2)*e^x + a^(3/2)), x) + 96*sqrt(2)*integrate(1/9*x*e^(3/2*x)/(a
^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3*x) + 6*a^(3/2)*e^(2*x) + 4*a^(3/2)*e^x + a^
(3/2)), x) - 4/27*(9*sqrt(2)*sqrt(a)*x^3 + 18*sqrt(2)*sqrt(a)*x^2 + 24*sqrt
(2)*sqrt(a)*x + 16*sqrt(2)*sqrt(a))*e^(3/2*x)/(a^2*e^(3*x) + 3*a^2*e^(2*x)
+ 3*a^2*e^x + a^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cosh\left(x\right)+a}\,x^{3}}{a^{2}\cosh\left(x\right)^{2}+2\,a^{2}\cosh\left(x\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + a)*x^3/(a^2*cosh(x)^2 + 2*a^2*cosh(x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{3}}{\left(a\left(\cosh\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral(x**3/(a*(cosh(x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{3}}{\left(a\cosh\left(x\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(a*cosh(x) + a)^(3/2), x)
```

3.145 $\int \frac{x^2}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{2ix\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{2ix\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{4i\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(3,-ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}-\frac{4i\cosh\left(\frac{x}{2}\right)\text{Po}}{a\sqrt{a\cos}}$$

```
[Out] (2*x)/(a*Sqrt[a + a*Cosh[x]]) + (x^2*ArcTan[E^(x/2)]*Cosh[x/2])/(a*Sqrt[a +
 a*Cosh[x]]) - (4*ArcTan[Sinh[x/2]]*Cosh[x/2])/(a*Sqrt[a + a*Cosh[x]]) - ((
2*I)*x*Cosh[x/2]*PolyLog[2, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((2*I)
*x*Cosh[x/2]*PolyLog[2, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((4*I)*Cosh[x
/2]*PolyLog[3, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((4*I)*Cosh[x/2]*Po
lyLog[3, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + (x^2*Tanh[x/2])/(2*a*Sqrt[a
+ a*Cosh[x]])
```

Rubi [A] time = 0.191267, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3319, 4186, 3770, 4180, 2531, 2282, 6589}

$$-\frac{2ix\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{2ix\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{4i\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(3,-ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}-\frac{4i\cosh\left(\frac{x}{2}\right)\text{Po}}{a\sqrt{a\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + a*Cosh[x])^(3/2),x]
```


```
[Out] (2*x)/(a*Sqrt[a + a*Cosh[x]]) + (x^2*ArcTan[E^(x/2)]*Cosh[x/2])/(a*Sqrt[a +
 a*Cosh[x]]) - (4*ArcTan[Sinh[x/2]]*Cosh[x/2])/(a*Sqrt[a + a*Cosh[x]]) - ((
2*I)*x*Cosh[x/2]*PolyLog[2, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((2*I)
*x*Cosh[x/2]*PolyLog[2, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((4*I)*Cosh[x
/2]*PolyLog[3, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((4*I)*Cosh[x/2]*Po
lyLog[3, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + (x^2*Tanh[x/2])/(2*a*Sqrt[a
+ a*Cosh[x]])
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{(a+a\cosh(x))^{3/2}}\,dx = \frac{\cosh\left(\frac{x}{2}\right)\int x^2\operatorname{sech}^3\left(\frac{x}{2}\right)dx}{2a\sqrt{a+a\cosh(x)}}$$

$$= \frac{2x}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}} + \frac{\cosh\left(\frac{x}{2}\right)\int x^2\operatorname{sech}\left(\frac{x}{2}\right)dx}{4a\sqrt{a+a\cosh(x)}} - \frac{\left(2\cosh\left(\frac{x}{2}\right)\right)\int \operatorname{sech}}{a\sqrt{a+a\cosh(x}}$$

$$= \frac{2x}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\tanh(}{2a\sqrt{a+a\cos}}$$

$$= \frac{2x}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{2ix\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a}}$$

$$= \frac{2x}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{2ix\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a}}$$

$$= \frac{2x}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{2ix\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a}}$$

Mathematica [A] time = 0.879235, size = 214, normalized size = 0.86

$$\cosh\left(\frac{x}{2}\right)\left(-4ix\cosh^2\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-i\left(\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\right)\right)+4ix\cosh^2\left(\frac{x}{2}\right)\text{PolyLog}\left(2,i\left(\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\right)\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (Cosh[x/2]*(4*x*Cosh[x/2] - 16*ArcTan[Cosh[x/2] + Sinh[x/2]]*Cosh[x/2]^2 +
2*x^2*ArcTan[Cosh[x/2] + Sinh[x/2]]*Cosh[x/2]^2 - (4*I)*x*Cosh[x/2]^2*PolyL
og[2, (-I)*(Cosh[x/2] + Sinh[x/2])] + (4*I)*x*Cosh[x/2]^2*PolyLog[2, I*(Cos
h[x/2] + Sinh[x/2])] + (8*I)*Cosh[x/2]^2*PolyLog[3, (-I)*(Cosh[x/2] + Sinh[
x/2])] - (8*I)*Cosh[x/2]^2*PolyLog[3, I*(Cosh[x/2] + Sinh[x/2])] + x^2*Sinh
[x/2]))/(a*(1 + Cosh[x]))^(3/2)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x^2 (a + a\cosh(x))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(x^2/(a+a*cosh(x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4}{27}\sqrt{2}\left(\frac{3e^{\left(\frac{5}{2}x\right)} + 8e^{\left(\frac{3}{2}x\right)} - 3e^{\left(\frac{1}{2}x\right)}}{a^{\frac{3}{2}}e^{(3x)} + 3a^{\frac{3}{2}}e^{(2x)} + 3a^{\frac{3}{2}}e^{x} + a^{\frac{3}{2}}} + \frac{3\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{a^{\frac{3}{2}}}\right) + 36\sqrt{2}\int \frac{x^2 e^{\left(\frac{3}{2}x\right)}}{9\left(a^{\frac{3}{2}}e^{(4x)} + 4a^{\frac{3}{2}}e^{(3x)} + 6a^{\frac{3}{2}}e^{(2x)} + 4a^{\frac{3}{2}}e^{x} + a^{\frac{3}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 4/27*sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) +
3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2))
 + 36*sqrt(2)*integrate(1/9*x^2*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3
*x) + 6*a^(3/2)*e^(2*x) + 4*a^(3/2)*e^x + a^(3/2)), x) + 48*sqrt(2)*integra
te(1/9*x*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3*x) + 6*a^(3/2)*e^(2*x)
 + 4*a^(3/2)*e^x + a^(3/2)), x) - 4/27*(9*sqrt(2)*x^2 + 12*sqrt(2)*x + 8*sq
rt(2))*e^(3/2*x)/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(
3/2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cosh(x) + a}x^2}{a^2\cosh(x)^2 + 2a^2\cosh(x) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + a)*x^2/(a^2*cosh(x)^2 + 2*a^2*cosh(x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a(\cosh(x)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral(x**2/(a*(cosh(x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a*cosh(x) + a)^(3/2), x)
```

3.146 $\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{i\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{i\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{1}{a\sqrt{a\cosh(x)+a}}+\frac{x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{x\tan}{2a\sqrt{a\cosh}}$$

```
[Out] 1/(a*Sqrt[a + a*Cosh[x]]) + (x*ArcTan[E^(x/2)]*Cosh[x/2])/(a*Sqrt[a + a*Cos
h[x]]) - (I*Cosh[x/2]*PolyLog[2, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + (
I*Cosh[x/2]*PolyLog[2, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + (x*Tanh[x/2])/
(2*a*Sqrt[a + a*Cosh[x]])
```

Rubi [A] time = 0.101197, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3319, 4185, 4180, 2279, 2391}

$$-\frac{i\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{i\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{1}{a\sqrt{a\cosh(x)+a}}+\frac{x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{x\tan}{2a\sqrt{a\cosh}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] 1/(a*Sqrt[a + a*Cosh[x]]) + (x*ArcTan[E^(x/2)]*Cosh[x/2])/(a*Sqrt[a + a*Cos
h[x]]) - (I*Cosh[x/2]*PolyLog[2, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + (
I*Cosh[x/2]*PolyLog[2, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + (x*Tanh[x/2])/
(2*a*Sqrt[a + a*Cosh[x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx &= \frac{\cosh\left(\frac{x}{2}\right)\int x\mathrm{sech}^3\left(\frac{x}{2}\right)dx}{2a\sqrt{a+a\cosh(x)}} \\
&= \frac{1}{a\sqrt{a+a\cosh(x)}}+\frac{x\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}}+\frac{\cosh\left(\frac{x}{2}\right)\int x\mathrm{sech}\left(\frac{x}{2}\right)dx}{4a\sqrt{a+a\cosh(x)}} \\
&= \frac{1}{a\sqrt{a+a\cosh(x)}}+\frac{x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}+\frac{x\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}}-\frac{\left(i\cosh\left(\frac{x}{2}\right)\right)\int\log\left(1-i\right.}{2a\sqrt{a+a\cosh(x}} \\
&= \frac{1}{a\sqrt{a+a\cosh(x)}}+\frac{x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}+\frac{x\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}}-\frac{\left(i\cosh\left(\frac{x}{2}\right)\right)\mathrm{Subst}\left(\int\frac{\mathrm{lo}}{}\right.}{a\sqrt{a+a\cos}} \\
&= \frac{1}{a\sqrt{a+a\cosh(x)}}+\frac{x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}-\frac{i\cosh\left(\frac{x}{2}\right)\mathrm{Li}_2\left(-ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}}+\frac{i\cosh\left(\frac{x}{2}\right)\mathrm{Li}_2\left(ie^{x/2}\right.}{a\sqrt{a+a\cosh(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.101678, size = 137, normalized size = 0.98

$$
\frac{2\cosh^3\left(\frac{x}{2}\right)\left(-i\left(\mathrm{PolyLog}\left(2,-ie^{-x/2}\right)-\mathrm{PolyLog}\left(2,ie^{-x/2}\right)\right)-\frac{1}{2}ix\left(\log\left(1-ie^{-x/2}\right)-\log\left(1+ie^{-x/2}\right)\right)\right)}{(a(\cosh(x)+1))^{3/2}}+\frac{2\cosh^2\left(\right.}{(a(\cosh(x)+}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (2*Cosh[x/2]^2)/(a*(1 + Cosh[x]))^(3/2) + (2*Cosh[x/2]^3*((-I/2)*x*(Log[1 -
 I/E^(x/2)] - Log[1 + I/E^(x/2)]) - I*(PolyLog[2, (-I)/E^(x/2)] - PolyLog[2
, I/E^(x/2)])))/(a*(1 + Cosh[x]))^(3/2) + (x*Cosh[x/2]*Sinh[x/2])/(a*(1 + C
osh[x]))^(3/2)
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int x\left(a+a\cosh(x)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(x/(a+a*cosh(x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{9}\sqrt{2}\left(\frac{3e^{\left(\frac{5}{2}x\right)}+8e^{\left(\frac{3}{2}x\right)}-3e^{\left(\frac{1}{2}x\right)}}{a^{\frac{3}{2}}e^{(3x)}+3a^{\frac{3}{2}}e^{(2x)}+3a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}+\frac{3\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{a^{\frac{3}{2}}}\right)+12\sqrt{2}\int\frac{xe^{\left(\frac{3}{2}x\right)}}{3\left(a^{\frac{3}{2}}e^{(4x)}+4a^{\frac{3}{2}}e^{(3x)}+6a^{\frac{3}{2}}e^{(2x)}+4a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) + 3
*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2))
+ 12*sqrt(2)*integrate(1/3*x*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3*x)
 + 6*a^(3/2)*e^(2*x) + 4*a^(3/2)*e^x + a^(3/2)), x) - 4/9*(3*sqrt(2)*sqrt(a
)*x + 2*sqrt(2)*sqrt(a))*e^(3/2*x)/(a^2*e^(3*x) + 3*a^2*e^(2*x) + 3*a^2*e^x
 + a^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cosh(x)+a}\,x}{a^2\cosh(x)^2+2a^2\cosh(x)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + a)*x/(a^2*cosh(x)^2 + 2*a^2*cosh(x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{(a(\cosh(x)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral(x/(a*(cosh(x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{(a\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(a*cosh(x) + a)^(3/2), x)
```

3.147 $\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x(a\cosh(x)+a)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(x*(a + a*Cosh[x])^(3/2)), x]
```

Rubi [A] time = 0.0774498, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + a*Cosh[x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*(a + a*Cosh[x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx = \int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx$$

Mathematica [A] time = 8.37268, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + a*Cosh[x])^(3/2)),x]
```

```
[Out] Integrate[1/(x*(a + a*Cosh[x])^(3/2)), x]
```

Maple [A] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+a\cosh(x)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(1/x/(a+a*cosh(x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x)+a)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cosh(x) + a)^(3/2)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\cosh(x)+a}}{a^2x\cosh(x)^2+2\,a^2x\cosh(x)+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + a)/(a^2*x*cosh(x)^2 + 2*a^2*x*cosh(x) + a^2*x), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x)+a)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cosh(x) + a)^(3/2)*x), x)
```

3.148 $\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x^2(a\cosh(x)+a)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + a*Cosh[x])^(3/2)), x]
```

Rubi [A] time = 0.0744424, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + a*Cosh[x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(x^2*(a + a*Cosh[x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx=\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx$$

Mathematica [A] time = 10.1783, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + a*Cosh[x])^(3/2)),x]
```

```
[Out] Integrate[1/(x^2*(a + a*Cosh[x])^(3/2)), x]
```

Maple [A] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+a\cosh(x)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(1/x^2/(a+a*cosh(x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x)+a)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cosh(x) + a)^(3/2)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\cosh(x)+a}}{a^2x^2\cosh(x)^2+2\,a^2x^2\cosh(x)+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + a)/(a^2*x^2*cosh(x)^2 + 2*a^2*x^2*cosh(x) + a^2*x
^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+a*cosh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x)+a)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cosh(x) + a)^(3/2)*x^2), x)
```

3.149 $\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{\sqrt[3]{a\cosh(c+dx)+a}}{x},x\right)$$

```
[Out] Unintegrable[(a + a*Cosh[c + d*x])^(1/3)/x, x]
```

Rubi [A] time = 0.0629971, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + a*Cosh[c + d*x])^(1/3)/x,x]
```

```
[Out] Defer[Int][(a + a*Cosh[c + d*x])^(1/3)/x, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx = \int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx$$

Mathematica [A] time = 2.51328, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(1/3)/x,x]
```

```
[Out] Integrate[(a + a*Cosh[c + d*x])^(1/3)/x, x]
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(1/3)/x,x)
```

```
[Out] int((a+a*cosh(d*x+c))^(1/3)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cosh(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(d*x + c) + a)^(1/3)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a(\cosh(c+dx)+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(1/3)/x,x)
```

```
[Out] Integral((a*(cosh(c + d*x) + 1))**(1/3)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cosh(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((a*cosh(d*x + c) + a)^(1/3)/x, x)
```

3.150 $\int (c+dx)^m (a+a\cosh(e+fx))^n\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((c+dx)^m(a\cosh(e+fx)+a)^n,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*(a + a*Cosh[e + f*x])^n, x]
```

Rubi [A] time = 0.0452379, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + a*Cosh[e + f*x])^n,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(a + a*Cosh[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx = \int (c+dx)^m(a+a\cosh(e+fx))^n\,dx$$

Mathematica [A] time = 5.87462, size = 0, normalized size = 0.

$$\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x])^n,x]
```

```
[Out] Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x])^n, x]
```

Maple [A] time = 0.082, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+a\cosh\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*cosh(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(a+a*cosh(f*x+e))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a\cosh\left(fx+e\right)+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*(a*cosh(f*x + e) + a)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m\left(a\cosh\left(fx+e\right)+a\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*(a*cosh(f*x + e) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*cosh(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a\cosh\left(fx+e\right)+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*(a*cosh(f*x + e) + a)^n, x)
```

3.151 $\int (c+dx)^m (a+a\cosh(e+fx))^3\,dx$

Optimal. Leaf size=402

$$\frac{a^3 3^{-m-1} e^{3e-\frac{3cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1,-\frac{3f(c+dx)}{d}\right)}{8f}+\frac{3a^3 2^{-m-3} e^{2e-\frac{2cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(\right.}{f}$$

```
[Out] (5*a^3*(c + d*x)^(1 + m))/(2*d*(1 + m)) + (3^(-1 - m)*a^3*E^(3*e - (3*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(c + d*x))/d])/(8*f*(-((f*(c + d*x))/d))^
m) + (3*2^(-3 - m)*a^3*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(
c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (15*a^3*E^(e - (c*f)/d)*(c + d*x
)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(8*f*(-((f*(c + d*x))/d))^m) - (15*a^
3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(8*f*((f*(c +
 d*x))/d)^m) - (3*2^(-3 - m)*a^3*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 +
 m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - (3^(-1 - m)*a^3*E^(-3*e +
 (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/d])/(8*f*((f*(c + d*x)
)/d)^m)
```

Rubi [A] time = 0.552822, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3318, 3312, 3307, 2181}

$$\frac{a^3 3^{-m-1} e^{3e-\frac{3cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1,-\frac{3f(c+dx)}{d}\right)}{8f}+\frac{3a^3 2^{-m-3} e^{2e-\frac{2cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + a*Cosh[e + f*x])^3,x]
```

```
[Out] (5*a^3*(c + d*x)^(1 + m))/(2*d*(1 + m)) + (3^(-1 - m)*a^3*E^(3*e - (3*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(c + d*x))/d])/(8*f*(-((f*(c + d*x))/d))^
m) + (3*2^(-3 - m)*a^3*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(
c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (15*a^3*E^(e - (c*f)/d)*(c + d*x
)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(8*f*(-((f*(c + d*x))/d))^m) - (15*a^
3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(8*f*((f*(c +
 d*x))/d)^m) - (3*2^(-3 - m)*a^3*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 +
 m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - (3^(-1 - m)*a^3*E^(-3*e +
 (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/d])/(8*f*((f*(c + d*x)
)/d)^m)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m(a+a\cosh(e+fx))^3\,dx &= \left(8a^3\right)\int (c+dx)^m\sin^6\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)dx\\ &= \left(8a^3\right)\int\left(\frac{5}{16}(c+dx)^m+\frac{15}{32}(c+dx)^m\cosh(e+fx)+\frac{3}{16}(c+dx)^m\cosh(2e+2fx)\right.\\ &= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}+\frac{1}{4}a^3\int (c+dx)^m\cosh(3e+3fx)\,dx+\frac{1}{2}\left(3a^3\right)\int (c+dx)^m\cosh\\ &= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}+\frac{1}{8}a^3\int e^{-i(3ie+3ifx)}(c+dx)^m\,dx+\frac{1}{8}a^3\int e^{i(3ie+3ifx)}(c+dx)^m\,dx\\ &= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}+\frac{3^{-1-m}a^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f}+\end{aligned}$$

Mathematica [A] time = 2.28873, size = 429, normalized size = 1.07

$$-\frac{a^3 2^{-m-6}3^{-m-1}e^{-3\left(\frac{cf}{d}+e\right)}(c+dx)^m(\cosh(e+fx)+1)^3\operatorname{sech}^6\left(\frac{1}{2}(e+fx)\right)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(2^m e^{\frac{3cf}{d}}\left(d(m+1)e^{\frac{3cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^m\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x])^3,x]
```

```
[Out] -((2^(-6 - m)*3^(-1 - m)*a^3*(c + d*x)^m*(1 + Cosh[e + f*x])^3*(-(2^m*d*E^(
6*e)*(1 + m)*((f*(c + d*x))/d)^m*Gamma[1 + m, (-3*f*(c + d*x))/d]) - 3^(2 +
 m)*d*E^(5*e + (c*f)/d)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, (-2*f*(c + d*x
))/d] - 5*2^m*3^(2 + m)*d*E^(4*e + (2*c*f)/d)*(1 + m)*((f*(c + d*x))/d)^m*G
amma[1 + m, -((f*(c + d*x))/d)] + 5*2^m*3^(2 + m)*d*E^(2*e + (4*c*f)/d)*(1
+ m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (f*(c + d*x))/d] + 3^(2 + m)*d*E^(
e + (5*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (2*f*(c + d*x))/
d] + 2^m*E^((3*c*f)/d)*(-20*3^(1 + m)*E^(3*e)*f*(c + d*x)*(-((f^2*(c + d*x)
^2)/d^2))^m + d*E^((3*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (
3*f*(c + d*x))/d]))*Sech[(e + f*x)/2]^6)/(d*E^(3*(e + (c*f)/d))*f*(1 + m)*(
-((f^2*(c + d*x)^2)/d^2))^m))
```

Maple [F] time = 0.15, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+a\cosh\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*cosh(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m*(a+a*cosh(f*x+e))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.40286, size = 1665, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/24*((a^3*d*m + a^3*d)*cosh((d*m*log(3*f/d) + 3*d*e - 3*c*f)/d)*gamma(m +
 1, 3*(d*f*x + c*f)/d) + 9*(a^3*d*m + a^3*d)*cosh((d*m*log(2*f/d) + 2*d*e -
 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) + 45*(a^3*d*m + a^3*d)*cosh((d*m
*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 45*(a^3*d*m + a^3
*d)*cosh((d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - 9*
(a^3*d*m + a^3*d)*cosh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2
*(d*f*x + c*f)/d) - (a^3*d*m + a^3*d)*cosh((d*m*log(-3*f/d) - 3*d*e + 3*c*f
)/d)*gamma(m + 1, -3*(d*f*x + c*f)/d) - (a^3*d*m + a^3*d)*gamma(m + 1, 3*(d
*f*x + c*f)/d)*sinh((d*m*log(3*f/d) + 3*d*e - 3*c*f)/d) - 9*(a^3*d*m + a^3*
d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)
 - 45*(a^3*d*m + a^3*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*log(f/d) +
d*e - c*f)/d) + 45*(a^3*d*m + a^3*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d
*m*log(-f/d) - d*e + c*f)/d) + 9*(a^3*d*m + a^3*d)*gamma(m + 1, -2*(d*f*x +
 c*f)/d)*sinh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d) + (a^3*d*m + a^3*d)*gamm
a(m + 1, -3*(d*f*x + c*f)/d)*sinh((d*m*log(-3*f/d) - 3*d*e + 3*c*f)/d) - 60
*(a^3*d*f*x + a^3*c*f)*cosh(m*log(d*x + c)) - 60*(a^3*d*f*x + a^3*c*f)*sinh
(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*cosh(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cosh\left(fx+e\right)+a\right)^{3}(dx+c)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*cosh(f*x + e) + a)^3*(d*x + c)^m, x)
```

3.152 $\int (c+dx)^m (a+a\cosh(e+fx))^2\,dx$

Optimal. Leaf size=263

$$\frac{a^2 2^{-m-3} e^{2e-\frac{2cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{a^2 e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{f}$$

```
[Out] (3*a^2*(c + d*x)^(1 + m))/(2*d*(1 + m)) + (2^(-3 - m)*a^2*E^(2*e - (2*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m)
 + (a^2*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-
((f*(c + d*x))/d))^m) - (a^2*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(
c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - (2^(-3 - m)*a^2*E^(-2*e + (2*c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.341582, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3318, 3312, 3307, 2181}

$$\frac{a^2 2^{-m-3} e^{2e-\frac{2cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{a^2 e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (3*a^2*(c + d*x)^(1 + m))/(2*d*(1 + m)) + (2^(-3 - m)*a^2*E^(2*e - (2*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m)
 + (a^2*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-
((f*(c + d*x))/d))^m) - (a^2*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(
c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - (2^(-3 - m)*a^2*E^(-2*e + (2*c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m(a+a\cosh(e+fx))^2\,dx = \left(4a^2\right)\int (c+dx)^m\sin^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)dx$$
$$= \left(4a^2\right)\int\left(\frac{3}{8}(c+dx)^m+\frac{1}{2}(c+dx)^m\cosh(e+fx)+\frac{1}{8}(c+dx)^m\cosh(2e+2fx)\right) d$$
$$= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)}+\frac{1}{2}a^2\int (c+dx)^m\cosh(2e+2fx)\,dx+\left(2a^2\right)\int (c+dx)^m\cosh(e$$
$$= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)}+\frac{1}{4}a^2\int e^{-i(2ie+2ifx)}(c+dx)^m\,dx+\frac{1}{4}a^2\int e^{i(2ie+2ifx)}(c+dx)^m\,dx$$
$$= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)}+\frac{2^{-3-m}a^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f}+\; a^2$$

Mathematica [A] time = 1.0451, size = 302, normalized size = 1.15

$$-\frac{a^2 2^{-m-5}e^{-2\left(\frac{cf}{d}+e\right)}(c+dx)^m(\cosh(e+fx)+1)^2\text{sech}^4\left(\frac{1}{2}(e+fx)\right)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(-de^{4e}(m+1)\left(f\left(\frac{c}{d}+x\right)\right)^m\text{Gamma}\left(m\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] -((2^(-5 - m)*a^2*(c + d*x)^m*(1 + Cosh[e + f*x])^2*(-3*2^(2 + m)*E^(2*(e +
 (c*f)/d))*f*(c + d*x)*(-((f^2*(c + d*x)^2)/d^2))^m - d*E^(4*e)*(1 + m)*(f*
(c/d + x))^m*Gamma[1 + m, (-2*f*(c + d*x))/d] - 2^(3 + m)*d*E^(3*e + (c*f)/
d)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, -((f*(c + d*x))/d)] + 2^(3 + m)*d*E
^(e + (3*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (f*(c + d*x))/
d] + d*E^((4*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (2*f*(c +
d*x))/d])*Sech[(e + f*x)/2]^4)/(d*E^(2*(e + (c*f)/d))*f*(1 + m)*(-((f^2*(c
+ d*x)^2)/d^2))^m))
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+a\cosh\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*cosh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+a*cosh(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.29729, size = 1139, normalized size = 4.33

$$-\frac{\left(a^{2}dm+a^{2}d\right)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)\Gamma\left(m+1,\frac{2\left(dfx+cf\right)}{d}\right)+8\left(a^{2}dm+a^{2}d\right)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/8*((a^2*d*m + a^2*d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m +
1, 2*(d*f*x + c*f)/d) + 8*(a^2*d*m + a^2*d)*cosh((d*m*log(f/d) + d*e - c*f)
/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8*(a^2*d*m + a^2*d)*cosh((d*m*log(-f/d)
 - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (a^2*d*m + a^2*d)*cosh((d
*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) - (a^2*
d*m + a^2*d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e -
 2*c*f)/d) - 8*(a^2*d*m + a^2*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*lo
g(f/d) + d*e - c*f)/d) + 8*(a^2*d*m + a^2*d)*gamma(m + 1, -(d*f*x + c*f)/d)
*sinh((d*m*log(-f/d) - d*e + c*f)/d) + (a^2*d*m + a^2*d)*gamma(m + 1, -2*(d
*f*x + c*f)/d)*sinh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d) - 12*(a^2*d*f*x +
a^2*c*f)*cosh(m*log(d*x + c)) - 12*(a^2*d*f*x + a^2*c*f)*sinh(m*log(d*x + c
)))/(d*f*m + d*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cosh\left(fx+e\right)+a\right)^{2}(dx+c)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cosh(f*x + e) + a)^2*(d*x + c)^m, x)
```

3.153 $\int (c+dx)^m(a+a\cosh(e+fx))\,dx$

Optimal. Leaf size=131

$$\frac{ae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{ae^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c+}{d(m}$$

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + (a*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
+ m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (a*E^(-e + (c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.147174, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3307, 2181}

$$\frac{ae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{ae^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c+}{d(m}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + a*Cosh[e + f*x]),x]
```


```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + (a*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
+ m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (a*E^(-e + (c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m(a+a\cosh(e+fx))\,dx = \int \left(a(c+dx)^m + a(c+dx)^m\cosh(e+fx)\right)dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)} + a\int (c+dx)^m\cosh(e+fx)\,dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}a\int e^{-i(ie+ifx)}(c+dx)^m\,dx + \frac{1}{2}a\int e^{i(ie+ifx)}(c+dx)^m\,dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{ae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f} - \frac{ae^{-e+\frac{cf}{d}}(c+$$

Mathematica [A] time = 0.31201, size = 189, normalized size = 1.44

$$-\frac{ae^{-\frac{cf}{d}-e}(c+dx)^m(\cosh(e+fx)+1)\text{sech}^2\left(\frac{1}{2}(e+fx)\right)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(-de^{2e}(m+1)\left(f\left(\frac{c}{d}+x\right)\right)^m\text{Gamma}\left(m+1,-\frac{f}{}\right.\right.}{4df(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x]),x]
```

```
[Out] -(a*E^(-e - (c*f)/d)*(c + d*x)^m*(1 + Cosh[e + f*x])*(-2*E^(e + (c*f)/d)*f*
(c + d*x)*(-((f^2*(c + d*x)^2)/d^2))^m - d*E^(2*e)*(1 + m)*(f*(c/d + x))^m*
Gamma[1 + m, -((f*(c + d*x))/d)] + d*E^((2*c*f)/d)*(1 + m)*(-((f*(c + d*x))
/d))^m*Gamma[1 + m, (f*(c + d*x))/d])*Sech[(e + f*x)/2]^2)/(4*d*f*(1 + m)*(
-((f^2*(c + d*x)^2)/d^2))^m)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+a\cosh\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*cosh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+a*cosh(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.21396, size = 586, normalized size = 4.47

$$-\frac{(adm+ad)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)-(adm+ad)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)-de+cf}{d}\right)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)-(adm+a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*((a*d*m + a*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x
+ c*f)/d) - (a*d*m + a*d)*cosh((d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1,
-(d*f*x + c*f)/d) - (a*d*m + a*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*l
og(f/d) + d*e - c*f)/d) + (a*d*m + a*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh
((d*m*log(-f/d) - d*e + c*f)/d) - 2*(a*d*f*x + a*c*f)*cosh(m*log(d*x + c))
- 2*(a*d*f*x + a*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*cosh(f*x+e)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cosh\left(fx+e\right)+a\right)(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(f*x + e) + a)*(d*x + c)^m, x)
```

3.154 $\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{a\cosh(e+fx)+a},x\right)$$

```
[Out] Unintegrable[(c + d*x)^m/(a + a*Cosh[e + f*x]), x]
```

Rubi [A] time = 0.0542267, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + a*Cosh[e + f*x]),x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + a*Cosh[e + f*x]), x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx$$

Mathematica [A] time = 4.92523, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + a*Cosh[e + f*x]),x]
```

```
[Out] Integrate[(c + d*x)^m/(a + a*Cosh[e + f*x]), x]
```

Maple [A] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{a+a\cosh\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+a*cosh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m/(a+a*cosh(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{a\cosh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(a*cosh(f*x + e) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^m}{a\cosh\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m/(a*cosh(f*x + e) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(c+dx)^m}{\cosh\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*cosh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**m/(cosh(e + f*x) + 1), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{a\cosh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(a*cosh(f*x + e) + a), x)
```

3.155 $\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{(a\cosh(e+fx)+a)^2},x\right)$$

```
[Out] Unintegrable[(c + d*x)^m/(a + a*Cosh[e + f*x])^2, x]
```

Rubi [A] time = 0.0521821, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + a*Cosh[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx=\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx$$

Mathematica [A] time = 8.88377, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m/(a + a*Cosh[e + f*x])^2, x]
```

Maple [A] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(a+a\cosh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+a*cosh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m/(a+a*cosh(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(a\cosh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(a*cosh(f*x + e) + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx+c)^m}{a^2\cosh\left(fx+e\right)^2+2\,a^2\cosh\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m/(a^2*cosh(f*x + e)^2 + 2*a^2*cosh(f*x + e) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(c+dx)^m}{\cosh^2\left(e+fx\right)+2\cosh\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**m/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x)/a**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(a\cosh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(a*cosh(f*x + e) + a)^2, x)
```

3.156 $\int (c+dx)^3(a+b\cosh(e+fx))\,dx$

Optimal. Leaf size=89

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\sinh(e+fx)}{f^3}-\frac{3bd(c+dx)^2\cosh(e+fx)}{f^2}+\frac{b(c+dx)^3\sinh(e+fx)}{f}-\frac{6bd^3\cosh(e+fx)}{f^4}$$

```
[Out] (a*(c + d*x)^4)/(4*d) - (6*b*d^3*Cosh[e + f*x])/f^4 - (3*b*d*(c + d*x)^2*Co
sh[e + f*x])/f^2 + (6*b*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (b*(c + d*x)^3*S
inh[e + f*x])/f
```

Rubi [A] time = 0.131794, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3296, 2638}

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\sinh(e+fx)}{f^3}-\frac{3bd(c+dx)^2\cosh(e+fx)}{f^2}+\frac{b(c+dx)^3\sinh(e+fx)}{f}-\frac{6bd^3\cosh(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) - (6*b*d^3*Cosh[e + f*x])/f^4 - (3*b*d*(c + d*x)^2*Co
sh[e + f*x])/f^2 + (6*b*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (b*(c + d*x)^3*S
inh[e + f*x])/f
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^3(a+b\cosh(e+fx))\,dx = \int \left(a(c+dx)^3+b(c+dx)^3\cosh(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^4}{4d} + b\int (c+dx)^3\cosh(e+fx)\,dx$$
$$= \frac{a(c+dx)^4}{4d} + \frac{b(c+dx)^3\sinh(e+fx)}{f} - \frac{(3bd)\int (c+dx)^2\sinh(e+fx)\,dx}{f}$$
$$= \frac{a(c+dx)^4}{4d} - \frac{3bd(c+dx)^2\cosh(e+fx)}{f^2} + \frac{b(c+dx)^3\sinh(e+fx)}{f} + \frac{\left(6bd^2\right)\int (c+$$
$$= \frac{a(c+dx)^4}{4d} - \frac{3bd(c+dx)^2\cosh(e+fx)}{f^2} + \frac{6bd^2(c+dx)\sinh(e+fx)}{f^3} + \frac{b(c+dx)^3 s}{}$$
$$= \frac{a(c+dx)^4}{4d} - \frac{6bd^3\cosh(e+fx)}{f^4} - \frac{3bd(c+dx)^2\cosh(e+fx)}{f^2} + \frac{6bd^2(c+dx)\sinh(}{f^3}$$

Mathematica [A] time = 0.461463, size = 123, normalized size = 1.38

$$\frac{1}{4}ax\left(6c^2dx+4c^3+4cd^2x^2+d^3x^3\right) + \frac{b(c+dx)\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+6\right)\right)\sinh(e+fx)}{f^3} - \frac{3bd\left(c^2f^2+2cdf^2x+d^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (a*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (3*b*d*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x])/f^4 + (b*(c + d*x)*(c^2*f^2 +
2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Sinh[e + f*x])/f^3
```

Maple [B] time = 0.014, size = 482, normalized size = 5.4

$$\frac{1}{f}\left(\frac{d^3a\left(fx+e\right)^4}{4f^3} + \frac{d^3b\left(\left(fx+e\right)^3\sinh\left(fx+e\right)-3\left(fx+e\right)^2\cosh\left(fx+e\right)+6\left(fx+e\right)\sinh\left(fx+e\right)-6\cosh\left(fx+\right.\right.}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*cosh(f*x+e)),x)
```

```
[Out] 1/f*(1/4/f^3*d^3*a*(f*x+e)^4+1/f^3*d^3*b*((f*x+e)^3*sinh(f*x+e)-3*(f*x+e)^2
*cosh(f*x+e)+6*(f*x+e)*sinh(f*x+e)-6*cosh(f*x+e))-1/f^3*d^3*e*a*(f*x+e)^3-3
/f^3*d^3*e*b*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f*x+e)+2*sinh(f*x+e))+3/
2/f^3*d^3*e^2*a*(f*x+e)^2+3/f^3*d^3*e^2*b*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))
-d^3*e^3/f^3*a*(f*x+e)-1/f^3*d^3*e^3*b*sinh(f*x+e)+1/f^2*d^2*c*a*(f*x+e)^3+
3/f^2*c*d^2*b*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f*x+e)+2*sinh(f*x+e))-3
/f^2*d^2*e*c*a*(f*x+e)^2-6/f^2*c*d^2*e*b*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))+
3*d^2*e^2/f^2*c*a*(f*x+e)+3/f^2*c*d^2*e^2*b*sinh(f*x+e)+3/2/f*d*c^2*a*(f*x+
e)^2+3/f*c^2*d*b*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))-3*d*e/f*c^2*a*(f*x+e)-3/
f*c^2*d*e*b*sinh(f*x+e)+c^3*a*(f*x+e)+b*c^3*sinh(f*x+e))
```

Maxima [B] time = 1.20251, size = 320, normalized size = 3.6

$$\frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x + \frac{3}{2}bc^2d\left(\frac{\left(fxe^e-e^e\right)e^{(fx)}}{f^2} - \frac{\left(fx+1\right)e^{(-fx-e)}}{f^2}\right) + \frac{3}{2}bcd^2\left(\frac{\left(f^2x^2e^e-2fxe^e+2e^e\right)e^{(fx)}}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*b*c^2*d*((f*x
*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*b*c*d^2*((f^2*x
^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e
)/f^3) + 1/2*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*
x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + b*c^3*sinh(f
*x + e)/f
```

Fricas [A] time = 2.12095, size = 365, normalized size = 4.1

$$\frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x - 12\left(bd^3f^2x^2 + 2\,bcd^2f^2x + bc^2df^2 + 2\,bd^3\right)\cosh\left(fx+e\right) + 4\left(bd^3f^3\right.}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
- 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2 + 2*b*d^3)*cosh(f*x + e
) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + b*c^3*f^3 + 6*b*c*d^2*f + 3*(b*c
^2*d*f^3 + 2*b*d^3*f)*x)*sinh(f*x + e))/f^4
```

Sympy [A] time = 2.23101, size = 264, normalized size = 2.97

$$\begin{cases} ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} + \frac{bc^3\sinh\left(e+fx\right)}{f} + \frac{3bc^2dx\sinh\left(e+fx\right)}{f} - \frac{3bc^2d\cosh\left(e+fx\right)}{f^2} + \frac{3bcd^2x^2\sinh\left(e+fx\right)}{f} - \frac{6bcd^2x\cosh\left(e+f\right.}{f^2} \\ \left(a + b\cosh\left(e\right)\right)\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 + b
*c**3*sinh(e + f*x)/f + 3*b*c**2*d*x*sinh(e + f*x)/f - 3*b*c**2*d*cosh(e +
f*x)/f**2 + 3*b*c*d**2*x**2*sinh(e + f*x)/f - 6*b*c*d**2*x*cosh(e + f*x)/f*
*2 + 6*b*c*d**2*sinh(e + f*x)/f**3 + b*d**3*x**3*sinh(e + f*x)/f - 3*b*d**3
*x**2*cosh(e + f*x)/f**2 + 6*b*d**3*x*sinh(e + f*x)/f**3 - 6*b*d**3*cosh(e
+ f*x)/f**4, Ne(f, 0)), ((a + b*cosh(e))*(c**3*x + 3*c**2*d*x**2/2 + c*d**2
*x**3 + d**3*x**4/4), True))
```

Giac [B] time = 1.28077, size = 351, normalized size = 3.94

$$\frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{\left(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + 3\,bc^2df^3x - 3\,bd^3f^2x^2 + bc^3f^3 - 6\,bcd^2f^2x - 3\,bc^2df\right.}{2\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 1/2*(b*d^3*f^3*x^
3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x - 3*b*d^3*f^2*x^2 + b*c^3*f^3 - 6*b
*c*d^2*f^2*x - 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*b*c*d^2*f - 6*b*d^3)*e^(f*x
+ e)/f^4 - 1/2*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*d
^3*f^2*x^2 + b*c^3*f^3 + 6*b*c*d^2*f^2*x + 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*
b*c*d^2*f + 6*b*d^3)*e^(-f*x - e)/f^4
```

3.157 $\int (c+dx)^2(a+b\cosh(e+fx))\,dx$

Optimal. Leaf size=67

$$\frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}+\frac{b(c+dx)^2\sinh(e+fx)}{f}+\frac{2bd^2\sinh(e+fx)}{f^3}$$

```
[Out] (a*(c + d*x)^3)/(3*d) - (2*b*d*(c + d*x)*Cosh[e + f*x])/f^2 + (2*b*d^2*Sinh
[e + f*x])/f^3 + (b*(c + d*x)^2*Sinh[e + f*x])/f
```

Rubi [A] time = 0.0901676, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3296, 2637}

$$\frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}+\frac{b(c+dx)^2\sinh(e+fx)}{f}+\frac{2bd^2\sinh(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) - (2*b*d*(c + d*x)*Cosh[e + f*x])/f^2 + (2*b*d^2*Sinh
[e + f*x])/f^3 + (b*(c + d*x)^2*Sinh[e + f*x])/f
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2(a+b\cosh(e+fx))\,dx &= \int \left(a(c+dx)^2+b(c+dx)^2\cosh(e+fx)\right)dx\\
&= \frac{a(c+dx)^3}{3d}+b\int (c+dx)^2\cosh(e+fx)\,dx\\
&= \frac{a(c+dx)^3}{3d}+\frac{b(c+dx)^2\sinh(e+fx)}{f}-\frac{(2bd)\int (c+dx)\sinh(e+fx)\,dx}{f}\\
&= \frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}+\frac{b(c+dx)^2\sinh(e+fx)}{f}+\frac{\left(2bd^2\right)\int \text{co}\\
&= \frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}+\frac{2bd^2\sinh(e+fx)}{f^3}+\frac{b(c+dx)^2\sinh(e}{f}
\end{aligned}$$

Mathematica [A] time = 0.317152, size = 83, normalized size = 1.24

$$\frac{1}{3}ax\left(3c^2+3cdx+d^2x^2\right)+\frac{b\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\sinh(e+fx)}{f^3}-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (a*x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 - (2*b*d*(c + d*x)*Cosh[e + f*x])/f^2 +
 (b*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x])/f^3
```

Maple [B] time = 0.01, size = 240, normalized size = 3.6

$$\frac{1}{f}\left(\frac{ad^2\left(fx+e\right)^3}{3\,f^2}+\frac{bd^2\left(\left(fx+e\right)^2\sinh\left(fx+e\right)-2\,\left(fx+e\right)\cosh\left(fx+e\right)+2\,\sinh\left(fx+e\right)\right)}{f^2}-\frac{d^2ea\left(fx+e\right)^2}{f^2}-2\,\frac{d^2e}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*cosh(f*x+e)),x)
```

```
[Out] 1/f*(1/3/f^2*d^2*a*(f*x+e)^3+1/f^2*d^2*b*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*c
osh(f*x+e)+2*sinh(f*x+e))-1/f^2*d^2*e*a*(f*x+e)^2-2/f^2*d^2*e*b*((f*x+e)*si
nh(f*x+e)-cosh(f*x+e))+d^2*e^2/f^2*a*(f*x+e)+1/f^2*d^2*e^2*b*sinh(f*x+e)+1/
f*d*c*a*(f*x+e)^2+2/f*c*d*b*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))-2*d*e/f*c*a*(
f*x+e)-2/f*c*d*e*b*sinh(f*x+e)+c^2*a*(f*x+e)+b*c^2*sinh(f*x+e))
```

Maxima [B] time = 1.19243, size = 190, normalized size = 2.84

$$\frac{1}{3}\,ad^2x^3+acdx^2+ac^2x+bcd\left(\frac{\left(fxe^e-e^e\right)e^{(fx)}}{f^2}-\frac{\left(fx+1\right)e^{(-fx-e)}}{f^2}\right)+\frac{1}{2}\,bd^2\left(\frac{\left(f^2x^2e^e-2\,fxe^e+2\,e^e\right)e^{(fx)}}{f^3}-\frac{\left(f^2x^2+2\,fx\right.}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + b*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2 -
(f*x + 1)*e^(-f*x - e)/f^2) + 1/2*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*
e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + b*c^2*sinh(f*x + e)
/f
```

Fricas [A] time = 2.01065, size = 231, normalized size = 3.45

$$\frac{ad^2f^3x^3+3\,acdf^3x^2+3\,ac^2f^3x-6\left(bd^2fx+bcdf\right)\cosh\left(fx+e\right)+3\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2+2\,bd^2\right)\sinh\left(fx+\right.}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x - 6*(b*d^2*f*x + b*c*d
*f)*cosh(f*x + e) + 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2)
*sinh(f*x + e))/f^3
```

Sympy [A] time = 1.1568, size = 151, normalized size = 2.25

$$\begin{cases} ac^2x + acdx^2 + \frac{ad^2x^3}{3} + \frac{bc^2\sinh\left(e+fx\right)}{f} + \frac{2bcdx\sinh\left(e+fx\right)}{f} - \frac{2bcd\cosh\left(e+fx\right)}{f^2} + \frac{bd^2x^2\sinh\left(e+fx\right)}{f} - \frac{2bd^2x\cosh\left(e+fx\right)}{f^2} + \frac{2bd^2\sinh\left(e\right.}{f^3} \\ \left(a + b\cosh\left(e\right)\right)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + b*c**2*sinh(e + f*x)/f +
 2*b*c*d*x*sinh(e + f*x)/f - 2*b*c*d*cosh(e + f*x)/f**2 + b*d**2*x**2*sinh(
e + f*x)/f - 2*b*d**2*x*cosh(e + f*x)/f**2 + 2*b*d**2*sinh(e + f*x)/f**3, N
e(f, 0)), ((a + b*cosh(e))*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [B] time = 1.23721, size = 200, normalized size = 2.99

$$\frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + \frac{\left(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 - 2\,bd^2fx - 2\,bcdf + 2\,bd^2\right)e^{\left(fx+e\right)}}{2\,f^3} - \frac{\left(bd^2f^2x^2 + 2\,bcdf^2x + b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 1/2*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x +
b*c^2*f^2 - 2*b*d^2*f*x - 2*b*c*d*f + 2*b*d^2)*e^(f*x + e)/f^3 - 1/2*(b*d^2
*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2*f*x + 2*b*c*d*f + 2*b*d^2)*e
^(-f*x - e)/f^3
```

3.158 $\int (c + dx)(a + b\cosh(e + fx))\,dx$

Optimal. Leaf size=45

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\sinh(e+fx)}{f}-\frac{bd\cosh(e+fx)}{f^2}$$

```
[Out] (a*(c + d*x)^2)/(2*d) - (b*d*Cosh[e + f*x])/f^2 + (b*(c + d*x)*Sinh[e + f*x
])/f
```

Rubi [A] time = 0.0455859, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3317, 3296, 2638}

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\sinh(e+fx)}{f}-\frac{bd\cosh(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) - (b*d*Cosh[e + f*x])/f^2 + (b*(c + d*x)*Sinh[e + f*x
])/f
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)(a+b\cosh(e+fx))\,dx &= \int (a(c+dx)+b(c+dx)\cosh(e+fx))\,dx\\
&= \frac{a(c+dx)^2}{2d}+b\int (c+dx)\cosh(e+fx)\,dx\\
&= \frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\sinh(e+fx)}{f}-\frac{(bd)\int \sinh(e+fx)\,dx}{f}\\
&= \frac{a(c+dx)^2}{2d}-\frac{bd\cosh(e+fx)}{f^2}+\frac{b(c+dx)\sinh(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.0962886, size = 46, normalized size = 1.02

$$\frac{f(afx(2c+dx)+2b(c+dx)\sinh(e+fx))-2bd\cosh(e+fx)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (-2*b*d*Cosh[e + f*x] + f*(a*f*x*(2*c + d*x) + 2*b*(c + d*x)*Sinh[e + f*x])
)/(2*f^2)
```

Maple [B] time = 0.012, size = 91, normalized size = 2.

$$\frac{1}{f}\left(\frac{da\left(fx+e\right)^2}{2f}+\frac{bd\left(\left(fx+e\right)\sinh\left(fx+e\right)-\cosh\left(fx+e\right)\right)}{f}-\frac{dea\left(fx+e\right)}{f}-\frac{bde\sinh\left(fx+e\right)}{f}+ac\left(fx+e\right)+cb\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*cosh(f*x+e)),x)
```

```
[Out] 1/f*(1/2/f*d*a*(f*x+e)^2+1/f*d*b*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))-d*e/f*a*
(f*x+e)-d*e/f*b*sinh(f*x+e)+a*c*(f*x+e)+c*b*sinh(f*x+e))
```

Maxima [A] time = 1.14151, size = 89, normalized size = 1.98

$$\frac{1}{2}adx^2+acx+\frac{1}{2}bd\left(\frac{\left(fxe^e-e^e\right)e^{(fx)}}{f^2}-\frac{\left(fx+1\right)e^{(-fx-e)}}{f^2}\right)+\frac{bc\sinh\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-
f*x - e)/f^2) + b*c*sinh(f*x + e)/f
```

Fricas [A] time = 2.00576, size = 128, normalized size = 2.84

$$\frac{adf^2x^2+2\,acf^2x-2\,bd\cosh\left(fx+e\right)+2\left(bdfx+bcf\right)\sinh\left(fx+e\right)}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x - 2*b*d*cosh(f*x + e) + 2*(b*d*f*x + b*c*f)*
sinh(f*x + e))/f^2
```

Sympy [A] time = 0.71541, size = 68, normalized size = 1.51

$$\begin{cases} acx + \frac{adx^2}{2} + \frac{bc\sinh(e+fx)}{f} + \frac{bdx\sinh(e+fx)}{f} - \frac{bd\cosh(e+fx)}{f^2} & \text{for } f \neq 0 \\ (a + b\cosh(e))\left(cx + \frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*d*x**2/2 + b*c*sinh(e + f*x)/f + b*d*x*sinh(e + f*x)/f
 - b*d*cosh(e + f*x)/f**2, Ne(f, 0)), ((a + b*cosh(e))*(c*x + d*x**2/2), Tr
ue))
```

Giac [A] time = 1.17115, size = 89, normalized size = 1.98

$$\frac{1}{2}adx^2 + acx + \frac{(bdfx + bcf - bd)e^{(fx+e)}}{2f^2} - \frac{(bdfx + bcf + bd)e^{(-fx-e)}}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*(b*d*f*x + b*c*f - b*d)*e^(f*x + e)/f^2 - 1/2*(b*
d*f*x + b*c*f + b*d)*e^(-f*x - e)/f^2
```

3.159 $\int \frac{a+b\cosh(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=64

$$\frac{a\log(c+dx)}{d}+\frac{b\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}$$

```
[Out] (b*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d +
(b*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Rubi [A] time = 0.120665, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3317, 3303, 3298, 3301}

$$\frac{a\log(c+dx)}{d}+\frac{b\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[e + f*x])/(c + d*x),x]
```

```
[Out] (b*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d +
(b*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{a+b\cosh(e+fx)}{c+dx}\,dx = \int \left(\frac{a}{c+dx} + \frac{b\cosh(e+fx)}{c+dx}\right) dx$$
$$= \frac{a\log(c+dx)}{d} + b\int \frac{\cosh(e+fx)}{c+dx}\,dx$$
$$= \frac{a\log(c+dx)}{d} + \left(b\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx + \left(b\sinh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{cf}{d}+f\right.}{c+dx}$$
$$= \frac{b\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d} + \frac{a\log(c+dx)}{d} + \frac{b\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

Mathematica [A] time = 0.136922, size = 57, normalized size = 0.89

$$\frac{a\log(c+dx) + b\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\cosh\left(e-\frac{cf}{d}\right) + b\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])/(c + d*x),x]
```

```
[Out] (b*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + a*Log[c + d*x] + b*Sinh[e
- (c*f)/d]*SinhIntegral[f*(c/d + x)])/d
```

Maple [A] time = 0.028, size = 94, normalized size = 1.5

$$\frac{a\ln(dx+c)}{d} - \frac{b}{2d}\mathrm{e}^{\frac{cf-de}{d}}\mathrm{Ei}\left(1, fx+e+\frac{cf-de}{d}\right) - \frac{b}{2d}\mathrm{e}^{-\frac{cf-de}{d}}\mathrm{Ei}\left(1, -fx-e-\frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(f*x+e))/(d*x+c),x)
```

```
[Out] a*ln(d*x+c)/d-1/2*b/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*b/d*exp(
-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A] time = 1.30706, size = 95, normalized size = 1.48

$$-\frac{1}{2}b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right) + \frac{a\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*b*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d + e^(e - c*f/d)*e
xp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

Fricas [A] time = 2.16901, size = 230, normalized size = 3.59

$$\frac{\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+2\,a\log\left(dx+c\right)-\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{de-cf}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((b*Ei((d*f*x + c*f)/d) + b*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d)
+ 2*a*log(d*x + c) - (b*Ei((d*f*x + c*f)/d) - b*Ei(-(d*f*x + c*f)/d))*sinh(
-(d*e - c*f)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\cosh\left(e+fx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c),x)
```

```
[Out] Integral((a + b*cosh(e + f*x))/(c + d*x), x)
```

Giac [A] time = 1.18747, size = 93, normalized size = 1.45

$$\frac{b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+2\,a\log\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + b*Ei((d*f*x + c*f)/d)*e^(-c*f/d
 + e) + 2*a*log(d*x + c))/d
```

3.160 $\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{a}{d(c+dx)}+\frac{bf\,\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{bf\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\cosh(e+fx)}{d(c+dx)}$$

```
[Out] -(a/(d*(c + d*x))) - (b*Cosh[e + f*x])/(d*(c + d*x)) + (b*f*CoshIntegral[(c
*f)/d + f*x]*Sinh[e - (c*f)/d])/d^2 + (b*f*Cosh[e - (c*f)/d]*SinhIntegral[(
c*f)/d + f*x])/d^2
```

Rubi [A] time = 0.151882, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3317, 3297, 3303, 3298, 3301}

$$-\frac{a}{d(c+dx)}+\frac{bf\,\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{bf\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\cosh(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) - (b*Cosh[e + f*x])/(d*(c + d*x)) + (b*f*CoshIntegral[(c
*f)/d + f*x]*Sinh[e - (c*f)/d])/d^2 + (b*f*Cosh[e - (c*f)/d]*SinhIntegral[(
c*f)/d + f*x])/d^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx &= \int \left(\frac{a}{(c+dx)^2}+\frac{b\cosh(e+fx)}{(c+dx)^2}\right)dx\\ &= -\frac{a}{d(c+dx)}+b\int \frac{\cosh(e+fx)}{(c+dx)^2}\,dx\\ &= -\frac{a}{d(c+dx)}-\frac{b\cosh(e+fx)}{d(c+dx)}+\frac{(bf)\int \frac{\sinh(e+fx)}{c+dx}\,dx}{d}\\ &= -\frac{a}{d(c+dx)}-\frac{b\cosh(e+fx)}{d(c+dx)}+\frac{\left(bf\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}+\frac{\left(bf\sinh\left(e-\frac{cf}{d}\right)\right)\int}{d}\\ &= -\frac{a}{d(c+dx)}-\frac{b\cosh(e+fx)}{d(c+dx)}+\frac{bf\,\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{bf\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}\right.}{d^2}\end{aligned}$$

Mathematica [A] time = 0.400004, size = 71, normalized size = 0.82

$$\frac{-\frac{d(a+b\cosh(e+fx))}{c+dx}+bf\,\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+bf\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] (-((d*(a + b*Cosh[e + f*x]))/(c + d*x)) + b*f*CoshIntegral[f*(c/d + x)]*Sin
h[e - (c*f)/d] + b*f*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d^2
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(f*x+e))/(d*x+c)^2,x)
```

```
[Out] int((a+b*cosh(f*x+e))/(d*x+c)^2,x)
```

Maxima [A] time = 1.39857, size = 117, normalized size = 1.34

$$-\frac{1}{2}b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) + e^(
e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x + c*
d)
```

Fricas [A] time = 2.35, size = 351, normalized size = 4.03

$$-\frac{2\,bd\cosh\left(fx+e\right)+2\,ad-\left(\left(bdfx+bcf\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-\left(bdfx+bcf\right)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left(\left(bdfx+bcf\right)\mathrm{Ei}\left(\right.\right.}{2\left(d^{3}x+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*cosh(f*x + e) + 2*a*d - ((b*d*f*x + b*c*f)*Ei((d*f*x + c*f)/d)
- (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + ((b*d*f*x
+ b*c*f)*Ei((d*f*x + c*f)/d) + (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh
(-(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20195, size = 227, normalized size = 2.61

$$-\frac{\left(dfx\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}-dfx\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+cf\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}-cf\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+de^{\left(fx+e\right)}+de^{\left(-fx\right.}\right.}{2\left(d^{3}x+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(d*f*x*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) - d*f*x*Ei((d*f*x + c*f)/d)*
e^(-c*f/d + e) + c*f*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) - c*f*Ei((d*f*x + c
*f)/d)*e^(-c*f/d + e) + d*e^(f*x + e) + d*e^(-f*x - e))*b/(d^3*x + c*d^2) -
 a/((d*x + c)*d)
```

3.161 $\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=123

$$-\frac{a}{2d(c+dx)^2}+\frac{bf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{bf^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\sinh(e+fx)}{2d^2(c+dx)}-\frac{b\cosh(e+f}{2d(c+dx)^2}$$

```
[Out] -a/(2*d*(c + d*x)^2) - (b*Cosh[e + f*x])/(2*d*(c + d*x)^2) + (b*f^2*Cosh[e
- (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/(2*d^3) - (b*f*Sinh[e + f*x])/(2*d^
2*(c + d*x)) + (b*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(2*d^3
)
```

Rubi [A] time = 0.196539, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3317, 3297, 3303, 3298, 3301}

$$-\frac{a}{2d(c+dx)^2}+\frac{bf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{bf^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\sinh(e+fx)}{2d^2(c+dx)}-\frac{b\cosh(e+f}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[e + f*x])/(c + d*x)^3,x]
```


```
[Out] -a/(2*d*(c + d*x)^2) - (b*Cosh[e + f*x])/(2*d*(c + d*x)^2) + (b*f^2*Cosh[e
- (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/(2*d^3) - (b*f*Sinh[e + f*x])/(2*d^
2*(c + d*x)) + (b*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(2*d^3
)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx &= \int \left(\frac{a}{(c+dx)^3}+\frac{b\cosh(e+fx)}{(c+dx)^3}\right) dx\\
&= -\frac{a}{2d(c+dx)^2}+b\int \frac{\cosh(e+fx)}{(c+dx)^3}\,dx\\
&= -\frac{a}{2d(c+dx)^2}-\frac{b\cosh(e+fx)}{2d(c+dx)^2}+\frac{(bf)\int \frac{\sinh(e+fx)}{(c+dx)^2}\,dx}{2d}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{b\cosh(e+fx)}{2d(c+dx)^2}-\frac{bf\sinh(e+fx)}{2d^2(c+dx)}+\frac{\left(bf^2\right)\int \frac{\cosh(e+fx)}{c+dx}\,dx}{2d^2}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{b\cosh(e+fx)}{2d(c+dx)^2}-\frac{bf\sinh(e+fx)}{2d^2(c+dx)}+\frac{\left(bf^2\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{2d^2}+\\
&= -\frac{a}{2d(c+dx)^2}-\frac{b\cosh(e+fx)}{2d(c+dx)^2}+\frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{2d^3}-\frac{bf\sinh(e+fx)}{2d^2(c+dx)}+\end{aligned}$$

Mathematica [A] time = 0.602372, size = 95, normalized size = 0.77

$$\frac{-\frac{d(ad+bf(c+dx)\sinh(e+fx)+bd\cosh(e+fx))}{(c+dx)^2}+bf^2\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\cosh\left(e-\frac{cf}{d}\right)+bf^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] (b*f^2*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - (d*(a*d + b*d*Cosh[e +
 f*x] + b*f*(c + d*x)*Sinh[e + f*x]))/(c + d*x)^2 + b*f^2*Sinh[e - (c*f)/d]
*SinhIntegral[f*(c/d + x)])/(2*d^3)
```

Maple [B] time = 0.029, size = 296, normalized size = 2.4

$$-\frac{a}{2d(dx+c)^2}+\frac{bf^3e^{-fx-e}x}{4d\left(d^2f^2x^2+2cdf^2x+c^2f^2\right)}+\frac{bf^3e^{-fx-e}c}{4d^2\left(d^2f^2x^2+2cdf^2x+c^2f^2\right)}-\frac{f^2be^{-fx-e}}{4d\left(d^2f^2x^2+2cdf^2x+c^2f^2\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(f*x+e))/(d*x+c)^3,x)
```

```
[Out] -1/2*a/d/(d*x+c)^2+1/4*b*f^3*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2
)*x+1/4*b*f^3*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/4*b*f^2
*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/4*b*f^2/d^3*exp((c*f-d*e
)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*f^2*b/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/4*f^2
*b/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*f^2*b/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(
c*f-d*e)/d)
```

Maxima [A] time = 1.33117, size = 132, normalized size = 1.07

$$-\frac{1}{2}\,b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-\frac{a}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*(e^(-e + c*f/d)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) + e
^(e - c*f/d)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/(d^
3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] time = 2.36779, size = 572, normalized size = 4.65

$$-\frac{2\,bd^2\cosh\left(fx+e\right)+2\,ad^2-\left(\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2\right)\mathrm{Ei}\left(-\frac{dfx+c}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*d^2*cosh(f*x + e) + 2*a*d^2 - ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b
*c^2*f^2)*Ei((d*f*x + c*f)/d) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)
*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + 2*(b*d^2*f*x + b*c*d*f)*sinh(
f*x + e) + ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei((d*f*x + c*f)/d)
 - (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*sinh(-
(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14867, size = 443, normalized size = 3.6

$$\frac{bd^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+bd^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+2\,bcdf^2x\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+2\,bcdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b*d^2*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + b*d^2*f^2*x^2*Ei((d
*f*x + c*f)/d)*e^(-c*f/d + e) + 2*b*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(c*f/d
 - e) + 2*b*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) + b*c^2*f^2*Ei(-(d
*f*x + c*f)/d)*e^(c*f/d - e) + b*c^2*f^2*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e)
 - b*d^2*f*x*e^(f*x + e) + b*d^2*f*x*e^(-f*x - e) - b*c*d*f*e^(f*x + e) + b
*c*d*f*e^(-f*x - e) - b*d^2*e^(f*x + e) - b*d^2*e^(-f*x - e) - 2*a*d^2)/(d^
5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.162 $\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx$

Optimal. Leaf size=250

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\sinh(e+fx)}{f^3}-\frac{6abd(c+dx)^2\cosh(e+fx)}{f^2}+\frac{2ab(c+dx)^3\sinh(e+fx)}{f}-\frac{12abd^3\cosh}{f^4}$$

```
[Out] (3*b^2*c*d^2*x)/(4*f^2) + (3*b^2*d^3*x^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d)
 + (b^2*(c + d*x)^4)/(8*d) - (12*a*b*d^3*Cosh[e + f*x])/f^4 - (6*a*b*d*(c +
 d*x)^2*Cosh[e + f*x])/f^2 - (3*b^2*d^3*Cosh[e + f*x]^2)/(8*f^4) - (3*b^2*d
*(c + d*x)^2*Cosh[e + f*x]^2)/(4*f^2) + (12*a*b*d^2*(c + d*x)*Sinh[e + f*x]
)/f^3 + (2*a*b*(c + d*x)^3*Sinh[e + f*x])/f + (3*b^2*d^2*(c + d*x)*Cosh[e +
 f*x]*Sinh[e + f*x])/(4*f^3) + (b^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x]
)/(2*f)
```

Rubi [A] time = 0.284548, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3317, 3296, 2638, 3311, 32, 3310}

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\sinh(e+fx)}{f^3}-\frac{6abd(c+dx)^2\cosh(e+fx)}{f^2}+\frac{2ab(c+dx)^3\sinh(e+fx)}{f}-\frac{12abd^3\cosh}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (3*b^2*c*d^2*x)/(4*f^2) + (3*b^2*d^3*x^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d)
 + (b^2*(c + d*x)^4)/(8*d) - (12*a*b*d^3*Cosh[e + f*x])/f^4 - (6*a*b*d*(c +
 d*x)^2*Cosh[e + f*x])/f^2 - (3*b^2*d^3*Cosh[e + f*x]^2)/(8*f^4) - (3*b^2*d
*(c + d*x)^2*Cosh[e + f*x]^2)/(4*f^2) + (12*a*b*d^2*(c + d*x)*Sinh[e + f*x]
)/f^3 + (2*a*b*(c + d*x)^3*Sinh[e + f*x])/f + (3*b^2*d^2*(c + d*x)*Cosh[e +
 f*x]*Sinh[e + f*x])/(4*f^3) + (b^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x]
)/(2*f)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
```

```
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx &= \int \left(a^2(c+dx)^3+2ab(c+dx)^3\cosh(e+fx)+b^2(c+dx)^3\cosh^2(e+fx)\right)dx\\ &= \frac{a^2(c+dx)^4}{4d}+(2ab)\int (c+dx)^3\cosh(e+fx)\,dx+b^2\int (c+dx)^3\cosh^2(e+fx)\,dx\\ &= \frac{a^2(c+dx)^4}{4d}-\frac{3b^2d(c+dx)^2\cosh^2(e+fx)}{4f^2}+\frac{2ab(c+dx)^3\sinh(e+fx)}{f}+\frac{b^2(c+\ldots}{}\\ &= \frac{a^2(c+dx)^4}{4d}+\frac{b^2(c+dx)^4}{8d}-\frac{6abd(c+dx)^2\cosh(e+fx)}{f^2}-\frac{3b^2d^3\cosh^2(e+fx)}{8f^4}-\ldots\\ &= \frac{3b^2cd^2x}{4f^2}+\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}+\frac{b^2(c+dx)^4}{8d}-\frac{6abd(c+dx)^2\cosh(e+fx)}{f^2}-\ldots\\ &= \frac{3b^2cd^2x}{4f^2}+\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}+\frac{b^2(c+dx)^4}{8d}-\frac{12abd^3\cosh(e+fx)}{f^4}-\frac{6abd(c\ldots}{}\end{aligned}$$

Mathematica [A] time = 1.39422, size = 232, normalized size = 0.93

$$\frac{2f\left(f^3x\left(2a^2+b^2\right)\left(6c^2dx+4c^3+4cd^2x^2+d^3x^3\right)+16ab(c+dx)\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+6\right)\right)\sinh(e+fx)+b^2(c+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (-96*a*b*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x] - 3*b^
2*d*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e + f*x)] + 2*f
*((2*a^2 + b^2)*f^3*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 16*a*b*
(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Sinh[e + f*x] + b^2*(
c + d*x)*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e + f*x)])
)/(16*f^4)
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*cosh(f*x+e))^2,x)

[Out] int((d*x+c)^3*(a+b*cosh(f*x+e))^2,x)
```

Maxima [B] time = 1.25178, size = 706, normalized size = 2.82

$$\frac{1}{4}a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + \frac{3}{16}\left(4x^2 + \frac{\left(2fxe^{(2e)} - e^{(2e)}\right)e^{(2fx)}}{f^2} - \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)b^2c^2d + \frac{1}{16}\left(8x^3 + \frac{3(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")

[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + 3/16*(4*x^2 + (2*f*x*
e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 - (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*c^
2*d + 1/16*(8*x^3 + 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*
x)/f^3 - 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*c*d^2 + 1/32*(
4*x^4 + (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(2*e))
*e^(2*f*x)/f^4 - (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)/f^4)*
b^2*d^3 + 1/8*b^2*c^3*(4*x + e^(2*f*x + 2*e)/f - e^(-2*f*x - 2*e)/f) + a^2*
c^3*x + 3*a*b*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f
^2) + 3*a*b*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2
 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6
*f*x*e^e - 6*e^e)*e^(f*x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x -
 e)/f^4) + 2*a*b*c^3*sinh(f*x + e)/f
```

Fricas [A] time = 2.11445, size = 879, normalized size = 3.52

$$\frac{2\left(2a^2+b^2\right)d^3f^4x^4 + 8\left(2a^2+b^2\right)cd^2f^4x^3 + 12\left(2a^2+b^2\right)c^2df^4x^2 + 8\left(2a^2+b^2\right)c^3f^4x - 3\left(2b^2d^3f^2x^2 + 4b^2cd^2f^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cosh(f*x+e))^2,x, algorithm="fricas")

[Out] 1/16*(2*(2*a^2 + b^2)*d^3*f^4*x^4 + 8*(2*a^2 + b^2)*c*d^2*f^4*x^3 + 12*(2*a
^2 + b^2)*c^2*d*f^4*x^2 + 8*(2*a^2 + b^2)*c^3*f^4*x - 3*(2*b^2*d^3*f^2*x^2
+ 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*cosh(f*x + e)^2 - 3*(2*b^2
*d^3*f^2*x^2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*sinh(f*x + e)
^2 - 96*(a*b*d^3*f^2*x^2 + 2*a*b*c*d^2*f^2*x + a*b*c^2*d*f^2 + 2*a*b*d^3)*c
osh(f*x + e) + 4*(8*a*b*d^3*f^3*x^3 + 24*a*b*c*d^2*f^3*x^2 + 8*a*b*c^3*f^3
+ 48*a*b*c*d^2*f + 24*(a*b*c^2*d*f^3 + 2*a*b*d^3*f)*x + (2*b^2*d^3*f^3*x^3
+ 6*b^2*c*d^2*f^3*x^2 + 2*b^2*c^3*f^3 + 3*b^2*c*d^2*f + 3*(2*b^2*c^2*d*f^3
+ b^2*d^3*f)*x)*cosh(f*x + e))*sinh(f*x + e))/f^4
```

Sympy [A] time = 5.54047, size = 779, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**3*x + 3*a**2*c**2*d*x**2/2 + a**2*c*d**2*x**3 + a**2*d**
3*x**4/4 + 2*a*b*c**3*sinh(e + f*x)/f + 6*a*b*c**2*d*x*sinh(e + f*x)/f - 6*
a*b*c**2*d*cosh(e + f*x)/f**2 + 6*a*b*c*d**2*x**2*sinh(e + f*x)/f - 12*a*b*
c*d**2*x*cosh(e + f*x)/f**2 + 12*a*b*c*d**2*sinh(e + f*x)/f**3 + 2*a*b*d**3
*x**3*sinh(e + f*x)/f - 6*a*b*d**3*x**2*cosh(e + f*x)/f**2 + 12*a*b*d**3*x*
sinh(e + f*x)/f**3 - 12*a*b*d**3*cosh(e + f*x)/f**4 - b**2*c**3*x*sinh(e +
f*x)**2/2 + b**2*c**3*x*cosh(e + f*x)**2/2 + b**2*c**3*sinh(e + f*x)*cosh(e
 + f*x)/(2*f) - 3*b**2*c**2*d*x**2*sinh(e + f*x)**2/4 + 3*b**2*c**2*d*x**2*
cosh(e + f*x)**2/4 + 3*b**2*c**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) - 3*
b**2*c**2*d*sinh(e + f*x)**2/(4*f**2) - b**2*c*d**2*x**3*sinh(e + f*x)**2/2
 + b**2*c*d**2*x**3*cosh(e + f*x)**2/2 + 3*b**2*c*d**2*x**2*sinh(e + f*x)*c
osh(e + f*x)/(2*f) - 3*b**2*c*d**2*x*sinh(e + f*x)**2/(4*f**2) - 3*b**2*c*d
**2*x*cosh(e + f*x)**2/(4*f**2) + 3*b**2*c*d**2*sinh(e + f*x)*cosh(e + f*x)
/(4*f**3) - b**2*d**3*x**4*sinh(e + f*x)**2/8 + b**2*d**3*x**4*cosh(e + f*x
)**2/8 + b**2*d**3*x**3*sinh(e + f*x)*cosh(e + f*x)/(2*f) - 3*b**2*d**3*x**
2*sinh(e + f*x)**2/(8*f**2) - 3*b**2*d**3*x**2*cosh(e + f*x)**2/(8*f**2) +
3*b**2*d**3*x*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) - 3*b**2*d**3*sinh(e + f
*x)**2/(8*f**4), Ne(f, 0)), ((a + b*cosh(e))**2*(c**3*x + 3*c**2*d*x**2/2 +
 c*d**2*x**3 + d**3*x**4/4), True))
```

Giac [B] time = 1.22437, size = 814, normalized size = 3.26

$$\frac{1}{4}a^2d^3x^4 + \frac{1}{8}b^2d^3x^4 + a^2cd^2x^3 + \frac{1}{2}b^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + \frac{3}{4}b^2c^2dx^2 + a^2c^3x + \frac{1}{2}b^2c^3x + \frac{\left(4\,b^2d^3f^3x^3 + 12\,b^2cd^2f^3x^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/4*a^2*d^3*x^4 + 1/8*b^2*d^3*x^4 + a^2*c*d^2*x^3 + 1/2*b^2*c*d^2*x^3 + 3/2
*a^2*c^2*d*x^2 + 3/4*b^2*c^2*d*x^2 + a^2*c^3*x + 1/2*b^2*c^3*x + 1/32*(4*b^
2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x - 6*b^2*d^3*f^2*x
^2 + 4*b^2*c^3*f^3 - 12*b^2*c*d^2*f^2*x - 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x +
 6*b^2*c*d^2*f - 3*b^2*d^3)*e^(2*f*x + 2*e)/f^4 + (a*b*d^3*f^3*x^3 + 3*a*b*
c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x - 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 - 6*a*b
*c*d^2*f^2*x - 3*a*b*c^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f - 6*a*b*d^3)
*e^(f*x + e)/f^4 - (a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3
*x + 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*f^2*x + 3*a*b*c^2*d*f^2
+ 6*a*b*d^3*f*x + 6*a*b*c*d^2*f + 6*a*b*d^3)*e^(-f*x - e)/f^4 - 1/32*(4*b^2
*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 6*b^2*d^3*f^2*x^
2 + 4*b^2*c^3*f^3 + 12*b^2*c*d^2*f^2*x + 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x +
6*b^2*c*d^2*f + 3*b^2*d^3)*e^(-2*f*x - 2*e)/f^4
```

3.163 $\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx$

Optimal. Leaf size=182

$$\frac{a^2(c+dx)^3}{3d}-\frac{4abd(c+dx)\cosh(e+fx)}{f^2}+\frac{2ab(c+dx)^2\sinh(e+fx)}{f}+\frac{4abd^2\sinh(e+fx)}{f^3}-\frac{b^2d(c+dx)\cosh^2(e+}{2f^2}$$

```
[Out] (b^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(3*d) + (b^2*(c + d*x)^3)/(6*d) - (
4*a*b*d*(c + d*x)*Cosh[e + f*x])/f^2 - (b^2*d*(c + d*x)*Cosh[e + f*x]^2)/(2
*f^2) + (4*a*b*d^2*Sinh[e + f*x])/f^3 + (2*a*b*(c + d*x)^2*Sinh[e + f*x])/f
 + (b^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) + (b^2*(c + d*x)^2*Cosh[e
+ f*x]*Sinh[e + f*x])/(2*f)
```

Rubi [A] time = 0.194222, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3317, 3296, 2637, 3311, 32, 2635, 8}

$$\frac{a^2(c+dx)^3}{3d}-\frac{4abd(c+dx)\cosh(e+fx)}{f^2}+\frac{2ab(c+dx)^2\sinh(e+fx)}{f}+\frac{4abd^2\sinh(e+fx)}{f^3}-\frac{b^2d(c+dx)\cosh^2(e+}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (b^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(3*d) + (b^2*(c + d*x)^3)/(6*d) - (
4*a*b*d*(c + d*x)*Cosh[e + f*x])/f^2 - (b^2*d*(c + d*x)*Cosh[e + f*x]^2)/(2
*f^2) + (4*a*b*d^2*Sinh[e + f*x])/f^3 + (2*a*b*(c + d*x)^2*Sinh[e + f*x])/f
 + (b^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) + (b^2*(c + d*x)^2*Cosh[e
+ f*x]*Sinh[e + f*x])/(2*f)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx = \int \left(a^2(c+dx)^2 + 2ab(c+dx)^2\cosh(e+fx) + b^2(c+dx)^2\cosh^2(e+fx)\right)dx$$
$$= \frac{a^2(c+dx)^3}{3d} + (2ab)\int (c+dx)^2\cosh(e+fx)\,dx + b^2\int (c+dx)^2\cosh^2(e+fx)\,dx$$
$$= \frac{a^2(c+dx)^3}{3d} - \frac{b^2d(c+dx)\cosh^2(e+fx)}{2f^2} + \frac{2ab(c+dx)^2\sinh(e+fx)}{f} + \frac{b^2(c+dx}{}$$
$$= \frac{a^2(c+dx)^3}{3d} + \frac{b^2(c+dx)^3}{6d} - \frac{4abd(c+dx)\cosh(e+fx)}{f^2} - \frac{b^2d(c+dx)\cosh^2(e+f}{2f^2}$$
$$= \frac{b^2d^2x}{4f^2} + \frac{a^2(c+dx)^3}{3d} + \frac{b^2(c+dx)^3}{6d} - \frac{4abd(c+dx)\cosh(e+fx)}{f^2} - \frac{b^2d(c+dx)\cos}{2f^2}$$

Mathematica [A] time = 0.949753, size = 252, normalized size = 1.38

$$\frac{1}{24}\left(24a^2c^2x + 24a^2cdx^2 + 8a^2d^2x^3 + \frac{48abc^2\sinh(e+fx)}{f} - \frac{96abd(c+dx)\cosh(e+fx)}{f^2} + \frac{96abcdx\sinh(e+fx)}{f} + \frac{96a}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (24*a^2*c^2*x + 12*b^2*c^2*x + 24*a^2*c*d*x^2 + 12*b^2*c*d*x^2 + 8*a^2*d^2*
x^3 + 4*b^2*d^2*x^3 - (96*a*b*d*(c + d*x)*Cosh[e + f*x])/f^2 - (6*b^2*d*(c
+ d*x)*Cosh[2*(e + f*x)])/f^2 + (96*a*b*d^2*Sinh[e + f*x])/f^3 + (48*a*b*c^
2*Sinh[e + f*x])/f + (96*a*b*c*d*x*Sinh[e + f*x])/f + (48*a*b*d^2*x^2*Sinh[
e + f*x])/f + (3*b^2*d^2*Sinh[2*(e + f*x)])/f^3 + (6*b^2*c^2*Sinh[2*(e + f*
x)])/f + (12*b^2*c*d*x*Sinh[2*(e + f*x)])/f + (6*b^2*d^2*x^2*Sinh[2*(e + f*
x)])/f)/24
```

Maple [B] time = 0.015, size = 535, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*cosh(f*x+e))^2,x)
```

```
[Out] 1/f*(1/3/f^2*d^2*a^2*(f*x+e)^3+2/f^2*d^2*a*b*((f*x+e)^2*sinh(f*x+e)-2*(f*x+
e)*cosh(f*x+e)+2*sinh(f*x+e))+1/f^2*d^2*b^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh
(f*x+e)+1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*sinh(f*x+e)*cosh(f*x+e)
+1/4*f*x+1/4*e)-1/f^2*d^2*e*a^2*(f*x+e)^2-4/f^2*d^2*e*a*b*((f*x+e)*sinh(f*x
+e)-cosh(f*x+e))-2/f^2*d^2*e*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(
f*x+e)^2-1/4*cosh(f*x+e)^2)+1/f^2*d^2*e^2*a^2*(f*x+e)+2/f^2*d^2*e^2*a*b*sin
h(f*x+e)+1/f^2*d^2*e^2*b^2*(1/2*sinh(f*x+e)*cosh(f*x+e)+1/2*f*x+1/2*e)+1/f*
c*d*a^2*(f*x+e)^2+4/f*c*d*a*b*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))+2/f*c*d*b^2
*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-2/f*
c*d*e*a^2*(f*x+e)-4/f*c*d*e*a*b*sinh(f*x+e)-2/f*c*d*e*b^2*(1/2*sinh(f*x+e)*
cosh(f*x+e)+1/2*f*x+1/2*e)+a^2*c^2*(f*x+e)+2*c^2*a*b*sinh(f*x+e)+c^2*b^2*(1
/2*sinh(f*x+e)*cosh(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A] time = 1.2212, size = 437, normalized size = 2.4

$$\frac{1}{3}a^2d^2x^3 + a^2cdx^2 + \frac{1}{8}\left(4x^2 + \frac{\left(2fxe^{(2e)} - e^{(2e)}\right)e^{(2fx)}}{f^2} - \frac{\left(2fx+1\right)e^{(-2fx-2e)}}{f^2}\right)b^2cd + \frac{1}{48}\left(8x^3 + \frac{3\left(2f^2x^2e^{(2e)} - 2fx\right.}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + 1/8*(4*x^2 + (2*f*x*e^(2*e) - e^(2*e))*e^(2
*f*x)/f^2 - (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*c*d + 1/48*(8*x^3 + 3*(2*
f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 - 3*(2*f^2*x^2 + 2
*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*d^2 + 1/8*b^2*c^2*(4*x + e^(2*f*x + 2*e
)/f - e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*a*b*c*d*((f*x*e^e - e^e)*e^(f*x)/
f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + a*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e
^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*a*b*c^2*sinh(
f*x + e)/f
```

Fricas [A] time = 2.03493, size = 541, normalized size = 2.97

$$\frac{2\left(2a^2+b^2\right)d^2f^3x^3 + 6\left(2a^2+b^2\right)cdf^3x^2 + 6\left(2a^2+b^2\right)c^2f^3x - 3\left(b^2d^2fx + b^2cdf\right)\cosh\left(fx+e\right)^2 - 3\left(b^2d^2fx + b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(2*a^2 + b^2)*d^2*f^3*x^3 + 6*(2*a^2 + b^2)*c*d*f^3*x^2 + 6*(2*a^2
+ b^2)*c^2*f^3*x - 3*(b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x + e)^2 - 3*(b^2*d^2
*f*x + b^2*c*d*f)*sinh(f*x + e)^2 - 48*(a*b*d^2*f*x + a*b*c*d*f)*cosh(f*x +
 e) + 3*(8*a*b*d^2*f^2*x^2 + 16*a*b*c*d*f^2*x + 8*a*b*c^2*f^2 + 16*a*b*d^2
+ (2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + b^2*d^2)*cosh(f*x
+ e))*sinh(f*x + e))/f^3
```

Sympy [A] time = 2.35164, size = 456, normalized size = 2.51

$$\begin{cases} a^2c^2x + a^2cdx^2 + \frac{a^2d^2x^3}{3} + \frac{2abc^2\sinh\left(e+fx\right)}{f} + \frac{4abcdx\sinh\left(e+fx\right)}{f} - \frac{4abcd\cosh\left(e+fx\right)}{f^2} + \frac{2abd^2x^2\sinh\left(e+fx\right)}{f} - \frac{4abd^2x\cosh\left(e+fx\right)}{f^2} + \\ \left(a+b\cosh\left(e\right)\right)^2\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x**3/3 + 2*a*b*c**2*sinh
(e + f*x)/f + 4*a*b*c*d*x*sinh(e + f*x)/f - 4*a*b*c*d*cosh(e + f*x)/f**2 +
2*a*b*d**2*x**2*sinh(e + f*x)/f - 4*a*b*d**2*x*cosh(e + f*x)/f**2 + 4*a*b*d
**2*sinh(e + f*x)/f**3 - b**2*c**2*x*sinh(e + f*x)**2/2 + b**2*c**2*x*cosh(
e + f*x)**2/2 + b**2*c**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*c*d*x**2
*sinh(e + f*x)**2/2 + b**2*c*d*x**2*cosh(e + f*x)**2/2 + b**2*c*d*x*sinh(e
+ f*x)*cosh(e + f*x)/f - b**2*c*d*sinh(e + f*x)**2/(2*f**2) - b**2*d**2*x**
3*sinh(e + f*x)**2/6 + b**2*d**2*x**3*cosh(e + f*x)**2/6 + b**2*d**2*x**2*s
inh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d**2*x*sinh(e + f*x)**2/(4*f**2) -
b**2*d**2*x*cosh(e + f*x)**2/(4*f**2) + b**2*d**2*sinh(e + f*x)*cosh(e + f*
x)/(4*f**3), Ne(f, 0)), ((a + b*cosh(e))**2*(c**2*x + c*d*x**2 + d**2*x**3/
3), True))
```

<u>Giac</u> [B] time = 1.19398, size = 471, normalized size = 2.59

$$\frac{1}{3}a^2d^2x^3 + \frac{1}{6}b^2d^2x^3 + a^2cdx^2 + \frac{1}{2}b^2cdx^2 + a^2c^2x + \frac{1}{2}b^2c^2x + \frac{\left(2b^2d^2f^2x^2 + 4b^2cdf^2x + 2b^2c^2f^2 - 2b^2d^2fx - 2b^2cdf\right.}{16f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*d^2*x^3 + 1/6*b^2*d^2*x^3 + a^2*c*d*x^2 + 1/2*b^2*c*d*x^2 + a^2*c^2
*x + 1/2*b^2*c^2*x + 1/16*(2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*
f^2 - 2*b^2*d^2*f*x - 2*b^2*c*d*f + b^2*d^2)*e^(2*f*x + 2*e)/f^3 + (a*b*d^2
*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2*f*x - 2*a*b*c*d*f + 2*
a*b*d^2)*e^(f*x + e)/f^3 - (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2
 + 2*a*b*d^2*f*x + 2*a*b*c*d*f + 2*a*b*d^2)*e^(-f*x - e)/f^3 - 1/16*(2*b^2*
d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + 2*b^2*d^2*f*x + 2*b^2*c*d*f
 + b^2*d^2)*e^(-2*f*x - 2*e)/f^3
```

3.164 $\int (c+dx)(a+b\cosh (e+fx))^2\,dx$

Optimal. Leaf size=116

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\sinh(e+fx)}{f}-\frac{2abd\cosh(e+fx)}{f^2}+\frac{b^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{1}{2}b^2cx-\frac{b^2d\,c}{}$$

```
[Out] (b^2*c*x)/2 + (b^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a*b*d*Cosh[e + f
*x])/f^2 - (b^2*d*Cosh[e + f*x]^2)/(4*f^2) + (2*a*b*(c + d*x)*Sinh[e + f*x]
)/f + (b^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)
```

Rubi [A] time = 0.10104, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3317, 3296, 2638, 3310}

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\sinh(e+fx)}{f}-\frac{2abd\cosh(e+fx)}{f^2}+\frac{b^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{1}{2}b^2cx-\frac{b^2d\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Cosh[e + f*x])^2,x]
```


```
[Out] (b^2*c*x)/2 + (b^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a*b*d*Cosh[e + f
*x])/f^2 - (b^2*d*Cosh[e + f*x]^2)/(4*f^2) + (2*a*b*(c + d*x)*Sinh[e + f*x]
)/f + (b^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)(a+b\cosh(e+fx))^2\,dx = \int \left(a^2(c+dx)+2ab(c+dx)\cosh(e+fx)+b^2(c+dx)\cosh^2(e+fx)\right)dx$$
$$= \frac{a^2(c+dx)^2}{2d}+(2ab)\int (c+dx)\cosh(e+fx)\,dx+b^2\int (c+dx)\cosh^2(e+fx)\,dx$$
$$= \frac{a^2(c+dx)^2}{2d}-\frac{b^2d\cosh^2(e+fx)}{4f^2}+\frac{2ab(c+dx)\sinh(e+fx)}{f}+\frac{b^2(c+dx)\cosh(e+}{2f}$$
$$= \frac{1}{2}b^2cx+\frac{1}{4}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{2abd\cosh(e+fx)}{f^2}-\frac{b^2d\cosh^2(e+fx)}{4f^2}+\frac{2ab(c+}{}$$

Mathematica [A] time = 0.846393, size = 96, normalized size = 0.83

$$-\frac{2\left(2a^2+b^2\right)(e+fx)(d(e-fx)-2cf)-16abf(c+dx)\sinh(e+fx)+16abd\cosh(e+fx)-2b^2f(c+dx)\sinh(2(e+fx}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] -(2*(2*a^2 + b^2)*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*a*b*d*Cosh[e + f*x]
 + b^2*d*Cosh[2*(e + f*x)] - 16*a*b*f*(c + d*x)*Sinh[e + f*x] - 2*b^2*f*(c
+ d*x)*Sinh[2*(e + f*x)])/(8*f^2)
```

Maple [A] time = 0.013, size = 208, normalized size = 1.8

$$\frac{1}{f}\left(\frac{da^2\left(fx+e\right)^2}{2f}+2\,\frac{bda\left(\left(fx+e\right)\sinh\left(fx+e\right)-\cosh\left(fx+e\right)\right)}{f}+\frac{db^2}{f}\left(\frac{\left(fx+e\right)\cosh\left(fx+e\right)\sinh\left(fx+e\right)}{2}+\frac{\left(fx+\right.}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*cosh(f*x+e))^2,x)
```

```
[Out] 1/f*(1/2/f*d*a^2*(f*x+e)^2+2/f*d*a*b*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))+1/f*
d*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)
-d*e/f*a^2*(f*x+e)-2*d*e/f*a*b*sinh(f*x+e)-d*e/f*b^2*(1/2*sinh(f*x+e)*cosh(
f*x+e)+1/2*f*x+1/2*e)+c*a^2*(f*x+e)+2*c*a*b*sinh(f*x+e)+c*b^2*(1/2*sinh(f*x
+e)*cosh(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A] time = 1.17923, size = 223, normalized size = 1.92

$$\frac{1}{2}a^2dx^2+\frac{1}{16}\left(4x^2+\frac{\left(2fxe^{(2e)}-e^{(2e)}\right)e^{(2fx)}}{f^2}-\frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)b^2d+\frac{1}{8}b^2c\left(4x+\frac{e^{(2fx+2e)}}{f}-\frac{e^{(-2fx-2e)}}{f}\right)+a^2cx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2 + 1/16*(4*x^2 + (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 - (2*
f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*d + 1/8*b^2*c*(4*x + e^(2*f*x + 2*e)/f -
 e^(-2*f*x - 2*e)/f) + a^2*c*x + a*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x
```

```
+ 1)*e^(-f*x - e)/f^2) + 2*a*b*c*sinh(f*x + e)/f
```

Fricas [A] time = 2.04278, size = 294, normalized size = 2.53

$$\frac{2\left(2a^2+b^2\right)df^2x^2+4\left(2a^2+b^2\right)cf^2x-b^2d\cosh\left(fx+e\right)^2-b^2d\sinh\left(fx+e\right)^2-16\,abd\cosh\left(fx+e\right)+4\left(4\,abdfx\right.}{8f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2 + b^2)*d*f^2*x^2 + 4*(2*a^2 + b^2)*c*f^2*x - b^2*d*cosh(f*x +
 e)^2 - b^2*d*sinh(f*x + e)^2 - 16*a*b*d*cosh(f*x + e) + 4*(4*a*b*d*f*x + 4
*a*b*c*f + (b^2*d*f*x + b^2*c*f)*cosh(f*x + e))*sinh(f*x + e))/f^2
```

Sympy [A] time = 0.928946, size = 219, normalized size = 1.89

$$\begin{cases} a^2cx+\frac{a^2dx^2}{2}+\frac{2abc\sinh\left(e+fx\right)}{f}+\frac{2abdx\sinh\left(e+fx\right)}{f}-\frac{2abd\cosh\left(e+fx\right)}{f^2}-\frac{b^2cx\sinh^2\left(e+fx\right)}{2}+\frac{b^2cx\cosh^2\left(e+fx\right)}{2}+\frac{b^2c\sinh\left(e+fx\right)\cosh\left(\right.}{2f} \\ \left(a+b\cosh\left(e\right)\right)^2\left(cx+\frac{dx^2}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c*x + a**2*d*x**2/2 + 2*a*b*c*sinh(e + f*x)/f + 2*a*b*d*x*s
inh(e + f*x)/f - 2*a*b*d*cosh(e + f*x)/f**2 - b**2*c*x*sinh(e + f*x)**2/2 +
 b**2*c*x*cosh(e + f*x)**2/2 + b**2*c*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b
**2*d*x**2*sinh(e + f*x)**2/4 + b**2*d*x**2*cosh(e + f*x)**2/4 + b**2*d*x*s
inh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d*sinh(e + f*x)**2/(4*f**2), Ne(f,
0)), ((a + b*cosh(e))**2*(c*x + d*x**2/2), True))
```

Giac [A] time = 1.20244, size = 221, normalized size = 1.91

$$\frac{1}{2}a^2dx^2+\frac{1}{4}b^2dx^2+a^2cx+\frac{1}{2}b^2cx+\frac{\left(2b^2dfx+2b^2cf-b^2d\right)e^{\left(2fx+2e\right)}}{16f^2}+\frac{\left(abdfx+abcf-abd\right)e^{\left(fx+e\right)}}{f^2}-\frac{\left(abdfx+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d*x^2 + 1/4*b^2*d*x^2 + a^2*c*x + 1/2*b^2*c*x + 1/16*(2*b^2*d*f*x +
 2*b^2*c*f - b^2*d)*e^(2*f*x + 2*e)/f^2 + (a*b*d*f*x + a*b*c*f - a*b*d)*e^(
f*x + e)/f^2 - (a*b*d*f*x + a*b*c*f + a*b*d)*e^(-f*x - e)/f^2 - 1/16*(2*b^2
*d*f*x + 2*b^2*c*f + b^2*d)*e^(-2*f*x - 2*e)/f^2
```

3.165 $\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=156

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{b^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}$$

```
[Out] (2*a*b*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (b^2*Cosh[2*e - (
2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (a^2*Log[c + d*x])/d + (
b^2*Log[c + d*x])/(2*d) + (2*a*b*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f
*x])/d + (b^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rubi [A] time = 0.310177, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3317, 3303, 3298, 3301, 3312}

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{b^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[e + f*x])^2/(c + d*x),x]
```

```
[Out] (2*a*b*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (b^2*Cosh[2*e - (
2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (a^2*Log[c + d*x])/d + (
b^2*Log[c + d*x])/(2*d) + (2*a*b*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f
*x])/d + (b^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx = \int\left(\frac{a^2}{c+dx}+\frac{2ab\cosh(e+fx)}{c+dx}+\frac{b^2\cosh^2(e+fx)}{c+dx}\right)dx$$
$$= \frac{a^2\log(c+dx)}{d}+(2ab)\int\frac{\cosh(e+fx)}{c+dx}\,dx+b^2\int\frac{\cosh^2(e+fx)}{c+dx}\,dx$$
$$= \frac{a^2\log(c+dx)}{d}+b^2\int\left(\frac{1}{2(c+dx)}+\frac{\cosh(2e+2fx)}{2(c+dx)}\right)dx+\left(2ab\cosh\left(e-\frac{cf}{d}\right)\right)\int\frac{\cos}{}$$
$$= \frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a^2\log(c+dx)}{d}+\frac{b^2\log(c+dx)}{2d}+\frac{2ab\sinh\left(e-\frac{cf}{d}\right)}{d}$$
$$= \frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a^2\log(c+dx)}{d}+\frac{b^2\log(c+dx)}{2d}+\frac{2ab\sinh\left(e-\frac{cf}{d}\right)}{d}$$
$$= \frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{b^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{a^2\log(c+dx}{d}$$

Mathematica [A] time = 0.282409, size = 133, normalized size = 0.85

$$\frac{2a^2\log(c+dx)+4ab\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\cosh\left(e-\frac{cf}{d}\right)+4ab\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+b^2\text{Chi}\left(\frac{2f(c+dx)}{d}\right)\cosh\left(2e-\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])^2/(c + d*x),x]
```

```
[Out] (4*a*b*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + b^2*Cosh[2*e - (2*c*f)
/d]*CoshIntegral[(2*f*(c + d*x))/d] + 2*a^2*Log[c + d*x] + b^2*Log[c + d*x]
 + 4*a*b*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + b^2*Sinh[2*e - (2*c*
f)/d]*SinhIntegral[(2*f*(c + d*x))/d])/(2*d)
```

Maple [A] time = 0.102, size = 202, normalized size = 1.3

$$-\frac{ab}{d}e^{\frac{cf-de}{d}}\text{Ei}\left(1,fx+e+\frac{cf-de}{d}\right)-\frac{ab}{d}e^{-\frac{cf-de}{d}}\text{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)+\frac{a^2\ln(dx+c)}{d}+\frac{b^2\ln(dx+c)}{2d}-\frac{b^2}{4d}e^{2\frac{cf-de}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(f*x+e))^2/(d*x+c),x)
```

```
[Out] -a*b/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-a*b/d*exp(-(c*f-d*e)/d)*Ei(
1,-f*x-e-(c*f-d*e)/d)+a^2*ln(d*x+c)/d+1/2*b^2*ln(d*x+c)/d-1/4*b^2/d*exp(2*(
c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4*b^2/d*exp(-2*(c*f-d*e)/d)*Ei(
1,-2*f*x-2*e-2*(c*f-d*e)/d)
```

Maxima [A] time = 1.44658, size = 200, normalized size = 1.28

$$-\frac{1}{4}b^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_1\left(-\frac{2(dx+c)f}{d}\right)}{d}-\frac{2\log(dx+c)}{d}\right)-ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(2*e
- 2*c*f/d)*exp_integral_e(1, -2*(d*x + c)*f/d)/d - 2*log(d*x + c)/d) - a*b*
(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d + e^(e - c*f/d)*exp_inte
gral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

Fricas [A] time = 2.13337, size = 483, normalized size = 3.1

$$\frac{4\left(ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left(b^2\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)+b^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\right)\cosh\left(-\frac{2(de-cf)}{d}\right)+2\left(2a^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(a*b*Ei((d*f*x + c*f)/d) + a*b*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*
f)/d) + (b^2*Ei(2*(d*f*x + c*f)/d) + b^2*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(d
*e - c*f)/d) + 2*(2*a^2 + b^2)*log(d*x + c) - 4*(a*b*Ei((d*f*x + c*f)/d) -
a*b*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) - (b^2*Ei(2*(d*f*x + c*f)/d)
 - b^2*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\cosh\left(e+fx\right)\right)^2}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))**2/(d*x+c),x)
```

```
[Out] Integral((a + b*cosh(e + f*x))**2/(c + d*x), x)
```

Giac [A] time = 1.25399, size = 200, normalized size = 1.28

$$\frac{b^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(\frac{2cf}{d}-2e\right)}+4ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+4ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+b^2\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)e^{\left(-\frac{2cf}{d}+2e\right)}+4a^2\log(}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(b^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*a*b*Ei(-(d*f*x + c*f)
/d)*e^(c*f/d - e) + 4*a*b*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) + b^2*Ei(2*(d*
f*x + c*f)/d)*e^(-2*c*f/d + 2*e) + 4*a^2*log(d*x + c) + 2*b^2*log(d*x + c))
/d
```

3.166 $\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=183

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\,\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2abf\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\cosh(e+fx)}{d(c+dx)}+\frac{b^2f\,\mathrm{Chi}\left(2xf\right.}{}$$

```
[Out] -(a^2/(d*(c + d*x))) - (2*a*b*Cosh[e + f*x])/(d*(c + d*x)) - (b^2*Cosh[e +
f*x]^2)/(d*(c + d*x)) + (b^2*f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (
2*c*f)/d])/d^2 + (2*a*b*f*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^
2 + (2*a*b*f*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*Co
sh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Rubi [A] time = 0.344979, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3317, 3297, 3303, 3298, 3301, 3313, 12}

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\,\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2abf\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\cosh(e+fx)}{d(c+dx)}+\frac{b^2f\,\mathrm{Chi}\left(2xf\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[e + f*x])^2/(c + d*x)^2,x]
```


```
[Out] -(a^2/(d*(c + d*x))) - (2*a*b*Cosh[e + f*x])/(d*(c + d*x)) - (b^2*Cosh[e +
f*x]^2)/(d*(c + d*x)) + (b^2*f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (
2*c*f)/d])/d^2 + (2*a*b*f*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^
2 + (2*a*b*f*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*Co
sh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^2
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx = \int \left(\frac{a^2}{(c+dx)^2}+\frac{2ab\cosh(e+fx)}{(c+dx)^2}+\frac{b^2\cosh^2(e+fx)}{(c+dx)^2}\right)dx$$

$$= -\frac{a^2}{d(c+dx)}+(2ab)\int \frac{\cosh(e+fx)}{(c+dx)^2}\,dx+b^2\int \frac{\cosh^2(e+fx)}{(c+dx)^2}\,dx$$

$$= -\frac{a^2}{d(c+dx)}-\frac{2ab\cosh(e+fx)}{d(c+dx)}-\frac{b^2\cosh^2(e+fx)}{d(c+dx)}+\frac{(2abf)\int \frac{\sinh(e+fx)}{c+dx}\,dx}{d}+\frac{\left(2ib^2f\right)\ldots}{\ldots}$$

$$= -\frac{a^2}{d(c+dx)}-\frac{2ab\cosh(e+fx)}{d(c+dx)}-\frac{b^2\cosh^2(e+fx)}{d(c+dx)}+\frac{\left(b^2f\right)\int \frac{\sinh(2e+2fx)}{c+dx}\,dx}{d}+\frac{\left(2abf\,c\ldots\right.}{\ldots}$$

$$= -\frac{a^2}{d(c+dx)}-\frac{2ab\cosh(e+fx)}{d(c+dx)}-\frac{b^2\cosh^2(e+fx)}{d(c+dx)}+\frac{2abf\,\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)\ldots}{d^2}$$

$$= -\frac{a^2}{d(c+dx)}-\frac{2ab\cosh(e+fx)}{d(c+dx)}-\frac{b^2\cosh^2(e+fx)}{d(c+dx)}+\frac{b^2f\,\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\ldots\right.}{d^2}$$

Mathematica [A] time = 0.760369, size = 233, normalized size = 1.27

$$\frac{-2a^2d+4abf(c+dx)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+4abcf\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+4abdfx\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (-2*a^2*d - b^2*d - 4*a*b*d*Cosh[e + f*x] - b^2*d*Cosh[2*(e + f*x)] + 2*b^2
*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*Sinh[2*e - (2*c*f)/d] + 4*a*b*
f*(c + d*x)*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + 4*a*b*c*f*Cosh[e
- (c*f)/d]*SinhIntegral[f*(c/d + x)] + 4*a*b*d*f*x*Cosh[e - (c*f)/d]*SinhIn
tegral[f*(c/d + x)] + 2*b^2*c*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c
+ d*x))/d] + 2*b^2*d*f*x*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))
```

```
/d])/(2*d^2*(c + d*x))
```

Maple [A] time = 0.12, size = 319, normalized size = 1.7

$$-\frac{abf\mathrm{e}^{-fx-e}}{d\left(dfx+cf\right)}+\frac{abf}{d^2}\mathrm{e}^{\frac{cf-de}{d}}\mathrm{Ei}\left(1,fx+e+\frac{cf-de}{d}\right)-\frac{abf\mathrm{e}^{fx+e}}{d^2}\left(\frac{cf}{d}+fx\right)^{-1}-\frac{abf}{d^2}\mathrm{e}^{-\frac{cf-de}{d}}\mathrm{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)-\frac{}{d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(f*x+e))^2/(d*x+c)^2,x)
```

```
[Out] -a*b*f*exp(-f*x-e)/d/(d*f*x+c*f)+a*b*f/d^2*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f
-d*e)/d)-a*b*f/d^2*exp(f*x+e)/(c*f/d+f*x)-a*b*f/d^2*exp(-(c*f-d*e)/d)*Ei(1,
-f*x-e-(c*f-d*e)/d)-a^2/d/(d*x+c)-1/2*b^2/d/(d*x+c)-1/4*b^2*f*exp(-2*f*x-2*
e)/d/(d*f*x+c*f)+1/2*b^2*f/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e
)/d)-1/4*f*b^2/d^2*exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*f*b^2/d^2*exp(-2*(c*f-d*e
)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)
```

Maxima [A] time = 1.3857, size = 244, normalized size = 1.33

$$-\frac{1}{4}b^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_2\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)d}+\frac{2}{d^2x+cd}\right)-ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*
d) + e^(2*e - 2*c*f/d)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) +
2/(d^2*x + c*d)) - a*b*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d
*x + c)*d) + e^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d))
 - a^2/(d^2*x + c*d)
```

Fricas [A] time = 2.15579, size = 778, normalized size = 4.25

$$-\frac{b^2d\cosh\left(fx+e\right)^2+b^2d\sinh\left(fx+e\right)^2+4\,abd\cosh\left(fx+e\right)+\left(2\,a^2+b^2\right)d-2\left(\left(abdfx+abcf\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-\left(abdfx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d*cosh(f*x + e)^2 + b^2*d*sinh(f*x + e)^2 + 4*a*b*d*cosh(f*x + e)
 + (2*a^2 + b^2)*d - 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) - (a*b*d*
f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - ((b^2*d*f*x + b
^2*c*f)*Ei(2*(d*f*x + c*f)/d) - (b^2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x + c*f)/d
))*cosh(-2*(d*e - c*f)/d) + 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) +
(a*b*d*f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + ((b^2*d*
```

```
f*x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d) + (b^2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x +
 c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\cosh\left(e + fx\right)\right)^2}{\left(c + dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*cosh(e + f*x))**2/(c + d*x)**2, x)
```

Giac [A] time = 1.92411, size = 485, normalized size = 2.65

$$-\frac{2\,b^2 dfx\text{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,cf}{d}-2\,e\right)} + 4\,abdfx\text{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)} - 4\,abdfx\text{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)} - 2\,b^2 dfx\text{Ei}\left(\frac{2\,(dfx+cf)}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*b^2*d*f*x*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*a*b*d*f*x*Ei
(-(d*f*x + c*f)/d)*e^(c*f/d - e) - 4*a*b*d*f*x*Ei((d*f*x + c*f)/d)*e^(-c*f/
d + e) - 2*b^2*d*f*x*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e) + 2*b^2*c*f*E
i(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*a*b*c*f*Ei(-(d*f*x + c*f)/d)*e^
(c*f/d - e) - 4*a*b*c*f*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) - 2*b^2*c*f*Ei(2
*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e) + b^2*d*e^(2*f*x + 2*e) + 4*a*b*d*e^(f
*x + e) + 4*a*b*d*e^(-f*x - e) + b^2*d*e^(-2*f*x - 2*e))/(d^3*x + c*d^2) -
1/2*(2*a^2 + b^2)/((d*x + c)*d)
```

3.167 $\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{a^2}{2d(c+dx)^2}+\frac{abf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^3}+\frac{abf^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{abf\sinh(e+fx)}{d^2(c+dx)}-\frac{ab\cosh(e+}{d(c+dx)^2}$$

```
[Out] -a^2/(2*d*(c + d*x)^2) - (a*b*Cosh[e + f*x])/(d*(c + d*x)^2) - (b^2*Cosh[e
+ f*x]^2)/(2*d*(c + d*x)^2) + (a*b*f^2*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)
/d + f*x])/d^3 + (b^2*f^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*
f*x])/d^3 - (a*b*f*Sinh[e + f*x])/(d^2*(c + d*x)) - (b^2*f*Cosh[e + f*x]*Si
nh[e + f*x])/(d^2*(c + d*x)) + (a*b*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f
)/d + f*x])/d^3 + (b^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2
*f*x])/d^3
```

Rubi [A] time = 0.42844, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3317, 3297, 3303, 3298, 3301, 3314, 31, 3312}

$$-\frac{a^2}{2d(c+dx)^2}+\frac{abf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^3}+\frac{abf^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{abf\sinh(e+fx)}{d^2(c+dx)}-\frac{ab\cosh(e+}{d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[e + f*x])^2/(c + d*x)^3,x]
```


```
[Out] -a^2/(2*d*(c + d*x)^2) - (a*b*Cosh[e + f*x])/(d*(c + d*x)^2) - (b^2*Cosh[e
+ f*x]^2)/(2*d*(c + d*x)^2) + (a*b*f^2*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)
/d + f*x])/d^3 + (b^2*f^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*
f*x])/d^3 - (a*b*f*Sinh[e + f*x])/(d^2*(c + d*x)) - (b^2*f*Cosh[e + f*x]*Si
nh[e + f*x])/(d^2*(c + d*x)) + (a*b*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f
)/d + f*x])/d^3 + (b^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2
*f*x])/d^3
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx = \int \left(\frac{a^2}{(c+dx)^3}+\frac{2ab\cosh(e+fx)}{(c+dx)^3}+\frac{b^2\cosh^2(e+fx)}{(c+dx)^3}\right)dx$$

$$= -\frac{a^2}{2d(c+dx)^2}+(2ab)\int \frac{\cosh(e+fx)}{(c+dx)^3}\,dx+b^2\int \frac{\cosh^2(e+fx)}{(c+dx)^3}\,dx$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}-\frac{b^2 f\cosh(e+fx)\sinh(e+fx)}{d^2(c+dx)}+$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}-\frac{b^2f^2\log(c+dx)}{d^3}-\frac{abf\sinh(e+}{d^2(c+dx}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}-\frac{abf\sinh(e+fx)}{d^2(c+dx)}-\frac{b^2f\cosh(e}{d^2}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}+\frac{abf^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right.}{d^3}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}+\frac{abf^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right.}{d^3}$$

Mathematica [A] time = 1.23414, size = 394, normalized size = 1.63

$$-\frac{2a^2d^2 - 4abc^2f^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right) - 4abf^2(c+dx)^2\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\cosh\left(e-\frac{cf}{d}\right) - 4abd^2f^2x^2\sinh\left(e-\frac{cf}{d}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] -(2*a^2*d^2 + b^2*d^2 + 4*a*b*d^2*Cosh[e + f*x] + b^2*d^2*Cosh[2*(e + f*x)]
 - 4*a*b*f^2*(c + d*x)^2*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - 4*b^
2*f^2*(c + d*x)^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] + 4
*a*b*c*d*f*Sinh[e + f*x] + 4*a*b*d^2*f*x*Sinh[e + f*x] + 2*b^2*c*d*f*Sinh[2
*(e + f*x)] + 2*b^2*d^2*f*x*Sinh[2*(e + f*x)] - 4*a*b*c^2*f^2*Sinh[e - (c*f
)/d]*SinhIntegral[f*(c/d + x)] - 8*a*b*c*d*f^2*x*Sinh[e - (c*f)/d]*SinhInte
gral[f*(c/d + x)] - 4*a*b*d^2*f^2*x^2*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d
 + x)] - 4*b^2*c^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d
] - 8*b^2*c*d*f^2*x*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] -
 4*b^2*d^2*f^2*x^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d])/(
4*d^3*(c + d*x)^2)
```

Maple [B] time = 0.133, size = 626, normalized size = 2.6

$$\frac{abf^3\mathrm{e}^{-fx-e}x}{2\,d\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)} + \frac{abf^3\mathrm{e}^{-fx-e}c}{2\,d^2\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)} - \frac{abf^2\mathrm{e}^{-fx-e}}{2\,d\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)} - \frac{abf^2}{2\,d^3}\mathrm{e}^{\frac{cf-de}{d}}\mathrm{Ei}\left(1, fx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(f*x+e))^2/(d*x+c)^3,x)
```

```
[Out] 1/2*a*b*f^3*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x+1/2*a*b*f^3*e
xp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/2*a*b*f^2*exp(-f*x-e)/
d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/2*a*b*f^2/d^3*exp((c*f-d*e)/d)*Ei(1,f
*x+e+(c*f-d*e)/d)-1/2*a*b*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/2*a*b*f^2/d^3*
exp(f*x+e)/(c*f/d+f*x)-1/2*a*b*f^2/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d
*e)/d)-1/2*a^2/d/(d*x+c)^2-1/4*b^2/d/(d*x+c)^2+1/4*b^2*f^3*exp(-2*f*x-2*e)/
d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x+1/4*b^2*f^3*exp(-2*f*x-2*e)/d^2/(d^2*
f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/8*b^2*f^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2
*c*d*f^2*x+c^2*f^2)-1/2*b^2*f^2/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*
f-d*e)/d)-1/8*b^2*f^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)^2-1/4*b^2*f^2/d^3*exp(
2*f*x+2*e)/(c*f/d+f*x)-1/2*b^2*f^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-
2*(c*f-d*e)/d)
```

Maxima [A] time = 1.48984, size = 271, normalized size = 1.12

$$-\frac{1}{4}b^2\left(\frac{1}{d^3x^2+2\,cd^2x+c^2d} + \frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_3\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d} + \frac{e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_3\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}\right) - ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d} + \frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\right.}{(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) + e^(-2*e + 2*c*f/d)*exp_integral
_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) + e^(2*e - 2*c*f/d)*exp_integral_e(3
, -2*(d*x + c)*f/d)/((d*x + c)^2*d)) - a*b*(e^(-e + c*f/d)*exp_integral_e(3
, (d*x + c)*f/d)/((d*x + c)^2*d) + e^(e - c*f/d)*exp_integral_e(3, -(d*x +
c)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] time = 2.24205, size = 1234, normalized size = 5.1

$$-\frac{b^2d^2\cosh\left(fx+e\right)^2+b^2d^2\sinh\left(fx+e\right)^2+4\,abd^2\cosh\left(fx+e\right)+\left(2\,a^2+b^2\right)d^2-2\left(\left(abd^2f^2x^2+2\,abcdf^2x+abc^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*cosh(f*x + e)^2 + b^2*d^2*sinh(f*x + e)^2 + 4*a*b*d^2*cosh(f*
x + e) + (2*a^2 + b^2)*d^2 - 2*((a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^
2*f^2)*Ei((d*f*x + c*f)/d) + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f
^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - 2*((b^2*d^2*f^2*x^2 + 2*b^
2*c*d*f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) + (b^2*d^2*f^2*x^2 + 2*b^2
*c*d*f^2*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(d*e - c*f)/d) +
4*(a*b*d^2*f*x + a*b*c*d*f + (b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x + e))*sinh(
f*x + e) + 2*((a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei((d*f*x +
 c*f)/d) - (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei(-(d*f*x + c
*f)/d))*sinh(-(d*e - c*f)/d) + 2*((b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*
c^2*f^2)*Ei(2*(d*f*x + c*f)/d) - (b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c
^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*
x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\cosh\left(e+fx\right)\right)^2}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))**2/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*cosh(e + f*x))**2/(c + d*x)**3, x)
```

Giac [B] time = 1.26454, size = 948, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*a*b*d^2
*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + 4*a*b*d^2*f^2*x^2*Ei((d*f*x +
```

```
 c*f)/d)*e^(-c*f/d + e) + 4*b^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f
/d + 2*e) + 8*b^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 8*a*
b*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + 8*a*b*c*d*f^2*x*Ei((d*f*x
+ c*f)/d)*e^(-c*f/d + e) + 8*b^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f/
d + 2*e) + 4*b^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*a*b*c
^2*f^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + 4*a*b*c^2*f^2*Ei((d*f*x + c*f)/
d)*e^(-c*f/d + e) + 4*b^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e)
- 2*b^2*d^2*f*x*e^(2*f*x + 2*e) - 4*a*b*d^2*f*x*e^(f*x + e) + 4*a*b*d^2*f*x
*e^(-f*x - e) + 2*b^2*d^2*f*x*e^(-2*f*x - 2*e) - 2*b^2*c*d*f*e^(2*f*x + 2*e
) - 4*a*b*c*d*f*e^(f*x + e) + 4*a*b*c*d*f*e^(-f*x - e) + 2*b^2*c*d*f*e^(-2*
f*x - 2*e) - b^2*d^2*e^(2*f*x + 2*e) - 4*a*b*d^2*e^(f*x + e) - 4*a*b*d^2*e^
(-f*x - e) - b^2*d^2*e^(-2*f*x - 2*e) - 4*a^2*d^2 - 2*b^2*d^2)/(d^5*x^2 + 2
*c*d^4*x + c^2*d^3)
```

3.168 $\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx$

Optimal. Leaf size=436

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^3\sqrt{a^2-b^2}}+\frac{3d(c+dx)^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}-$$

```
[Out] ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^
2]*f) - ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/(Sqrt[
a^2 - b^2]*f) + (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2
 - b^2]))])/(Sqrt[a^2 - b^2]*f^2) - (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e +
 f*x))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^2) - (6*d^2*(c + d*x)*Po
lyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^3) +
 (6*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/(Sq
rt[a^2 - b^2]*f^3) + (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^
2]))])/(Sqrt[a^2 - b^2]*f^4) - (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a + Sqr
t[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^4)
```

Rubi [A] time = 0.820881, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3320, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^3\sqrt{a^2-b^2}}+\frac{3d(c+dx)^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Cosh[e + f*x]),x]
```

```
[Out] ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^
2]*f) - ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/(Sqrt[
a^2 - b^2]*f) + (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2
 - b^2]))])/(Sqrt[a^2 - b^2]*f^2) - (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e +
 f*x))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^2) - (6*d^2*(c + d*x)*Po
lyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^3) +
 (6*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/(Sq
rt[a^2 - b^2]*f^3) + (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^
2]))])/(Sqrt[a^2 - b^2]*f^4) - (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a + Sqr
t[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^4)
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx = 2\int \frac{e^{e+fx}(c+dx)^3}{b+2ae^{e+fx}+be^{2(e+fx)}}\,dx$$
$$= \frac{(2b)\int \frac{e^{e+fx}(c+dx)^3}{2a-2\sqrt{a^2-b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{e+fx}(c+dx)^3}{2a+2\sqrt{a^2-b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2-b^2}}$$
$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(3d)\int(c+dx)^2\log\left(1+\frac{2b}{2a-2}\right.}{\sqrt{a^2-b^2}f}$$
$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2}$$
$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2}$$
$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2}$$
$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2}$$

Mathematica [A] time = 1.46927, size = 384, normalized size = 0.88

$$\frac{\frac{3d\left(f^2(c+dx)^2\text{PolyLog}\left(2,\frac{b(\sinh(e+fx)+\cosh(e+fx))}{\sqrt{a^2-b^2}-a}\right)-2df(c+dx)\text{PolyLog}\left(3,\frac{b(\sinh(e+fx)+\cosh(e+fx))}{\sqrt{a^2-b^2}-a}\right)+2d^2\text{PolyLog}\left(4,\frac{b(\sinh(e+fx)+\cosh(e+fx))}{\sqrt{a^2-b^2}-a}\right)\right)}{f^3} - 3d\left(f^2(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Cosh[e + f*x]),x]
```

```
[Out] ((c + d*x)^3*Log[1 + (b*(Cosh[e + f*x] + Sinh[e + f*x]))/(a - Sqrt[a^2 - b^
2])] - (c + d*x)^3*Log[1 + (b*(Cosh[e + f*x] + Sinh[e + f*x]))/(a + Sqrt[a^
2 - b^2])] + (3*d*(f^2*(c + d*x)^2*PolyLog[2, (b*(Cosh[e + f*x] + Sinh[e +
f*x]))/(-a + Sqrt[a^2 - b^2])] - 2*d*f*(c + d*x)*PolyLog[3, (b*(Cosh[e + f*
x] + Sinh[e + f*x]))/(-a + Sqrt[a^2 - b^2])] + 2*d^2*PolyLog[4, (b*(Cosh[e
+ f*x] + Sinh[e + f*x]))/(-a + Sqrt[a^2 - b^2])]))/f^3 - (3*d*(f^2*(c + d*x
)^2*PolyLog[2, -((b*(Cosh[e + f*x] + Sinh[e + f*x]))/(a + Sqrt[a^2 - b^2]))
] - 2*d*f*(c + d*x)*PolyLog[3, -((b*(Cosh[e + f*x] + Sinh[e + f*x]))/(a + S
qrt[a^2 - b^2]))] + 2*d^2*PolyLog[4, -((b*(Cosh[e + f*x] + Sinh[e + f*x]))/
(a + Sqrt[a^2 - b^2]))]))/f^3)/(Sqrt[a^2 - b^2]*f)
```

Maple [F] time = 0.226, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{a+b\cosh(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*cosh(f*x+e)),x)
```

```
[Out] int((d*x+c)^3/(a+b*cosh(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.31073, size = 2418, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (6*b*d^3*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(f*x + e) + a*sinh(f*x +
e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) - 6*b*d^
3*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(f*x + e) + a*sinh(f*x + e) - (b
*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) + 3*(b*d^3*f^2*
x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f
*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 -
 b^2)/b^2) + b)/b + 1) - 3*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*
sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f
*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (b*d^3*e^3 -
 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 - b^2)/b^2)*log(2
*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - (
b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 - b^2)
/b^2)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 - b^2)/b^2)
 + 2*a) + (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3
- 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x
+ e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^
2)/b^2) + b)/b) - (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*
d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 - b^2)/b^2)*log((a*c
osh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((
a^2 - b^2)/b^2) + b)/b) - 6*(b*d^3*f*x + b*c*d^2*f)*sqrt((a^2 - b^2)/b^2)*p
olylog(3, -(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f
*x + e))*sqrt((a^2 - b^2)/b^2))/b) + 6*(b*d^3*f*x + b*c*d^2*f)*sqrt((a^2 -
b^2)/b^2)*polylog(3, -(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e)
 + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b))/((a^2 - b^2)*f^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + dx)^3}{a + b \cosh(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*cosh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**3/(a + b*cosh(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{b\cosh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*cosh(f*x + e) + a), x)
```

3.169 $\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx$

Optimal. Leaf size=320

$$\frac{2d(c+dx)\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}-\frac{2d(c+dx)\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}}-\frac{2d^2\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{2d^2\mathrm{PolyLog}\left(\right.}{f^3\sqrt{a^2}}$$

```
[Out] ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^
2]*f) - ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/(Sqrt[
a^2 - b^2]*f) + (2*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 -
 b^2]))])/(Sqrt[a^2 - b^2]*f^2) - (2*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x
))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^2) - (2*d^2*PolyLog[3, -((b*
E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^3) + (2*d^2*PolyLo
g[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^3)
```

Rubi [A] time = 0.669973, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3320, 2264, 2190, 2531, 2282, 6589}

$$\frac{2d(c+dx)\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}-\frac{2d(c+dx)\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}}-\frac{2d^2\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{2d^2\mathrm{PolyLog}\left(\right.}{f^3\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Cosh[e + f*x]),x]
```


```
[Out] ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^
2]*f) - ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/(Sqrt[
a^2 - b^2]*f) + (2*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 -
 b^2]))])/(Sqrt[a^2 - b^2]*f^2) - (2*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x
))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^2) - (2*d^2*PolyLog[3, -((b*
E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^3) + (2*d^2*PolyLo
g[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^3)
```


Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx = 2\int \frac{e^{e+fx}(c+dx)^2}{b+2ae^{e+fx}+be^{2(e+fx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{e+fx}(c+dx)^2}{2a-2\sqrt{a^2-b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{e+fx}(c+dx)^2}{2a+2\sqrt{a^2-b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2-b^2}}$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(2d)\int(c+dx)\log\left(1+\frac{2be\ldots}{2a-2\ldots}\right.}{\sqrt{a^2-b^2}f}$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{2d(c+dx)\mathrm{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2} - \ldots$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{2d(c+dx)\mathrm{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2} - \ldots$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{2d(c+dx)\mathrm{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2} - \ldots$$

Mathematica [A] time = 0.963388, size = 247, normalized size = 0.77

$$\frac{\frac{2d\left(f(c+dx)\mathrm{PolyLog}\left(2,\frac{be^{e+fx}}{\sqrt{a^2-b^2}-a}\right)-d\mathrm{PolyLog}\left(3,\frac{be^{e+fx}}{\sqrt{a^2-b^2}-a}\right)\right)}{f^2} - \frac{2d\left(f(c+dx)\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)-d\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)\right)}{f^2} + (c+dx)^2\log\left(\frac{be\ldots}{a-\sqrt{\ldots}}\right.\ldots}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Cosh[e + f*x]),x]
```

```
[Out] ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])] - (c + d*x)^2*L
og[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])] + (2*d*(f*(c + d*x)*PolyLog[2
, (b*E^(e + f*x))/(-a + Sqrt[a^2 - b^2])] - d*PolyLog[3, (b*E^(e + f*x))/(-
a + Sqrt[a^2 - b^2])]))/f^2 - (2*d*(f*(c + d*x)*PolyLog[2, -((b*E^(e + f*x)
)/(a + Sqrt[a^2 - b^2]))] - d*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 -
b^2]))]))/f^2)/(Sqrt[a^2 - b^2]*f)
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{a+b\cosh\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

```
[Out] int((d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.26104, size = 1736, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(2*b*d^2*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(f*x + e) + a*sinh(f*x +
 e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) - 2*b*d
^2*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(f*x + e) + a*sinh(f*x + e) - (
b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) - 2*(b*d^2*f*x
 + b*c*d*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x + e)
 + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) +
2*(b*d^2*f*x + b*c*d*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*s
inh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) +
b)/b + 1) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 - b^2)/b^2)*log
(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) -
 (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(f
*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - (b*d^2*f^2
*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*sqrt((a^2 - b^2)/b^2)*log((
```

```
a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqr
t((a^2 - b^2)/b^2) + b)/b) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2
*b*c*d*e*f)*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x + e) + a*sinh(f*x + e) -
(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b))/((a^2 -
b^2)*f^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^2}{a+b\cosh\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*cosh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**2/(a + b*cosh(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{b\cosh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*cosh(f*x + e) + a), x)
```

3.170 $\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx$

Optimal. Leaf size=203

$$\frac{d\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}-\frac{d\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f\sqrt{a^2-b^2}}-\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f\sqrt{a^2-b^2}}$$

```
[Out] ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]
*f) - ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2
- b^2]*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[
a^2 - b^2]*f^2) - (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/
(Sqrt[a^2 - b^2]*f^2)
```

Rubi [A] time = 0.376068, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3320, 2264, 2190, 2279, 2391}

$$\frac{d\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}-\frac{d\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f\sqrt{a^2-b^2}}-\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Cosh[e + f*x]),x]
```

```
[Out] ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]
*f) - ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2
- b^2]*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[
a^2 - b^2]*f^2) - (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/
(Sqrt[a^2 - b^2]*f^2)
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx = 2\int \frac{e^{e+fx}(c+dx)}{b+2ae^{e+fx}+be^{2(e+fx)}}\,dx$$
$$= \frac{(2b)\int \frac{e^{e+fx}(c+dx)}{2a-2\sqrt{a^2-b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{e+fx}(c+dx)}{2a+2\sqrt{a^2-b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2-b^2}}$$
$$= \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{d\int \log\left(1+\frac{2be^{e+fx}}{2a-2\sqrt{a^2-b^2}}\right)dx}{\sqrt{a^2-b^2}f} +$$
$$= \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{d\,\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{2bx}{2a-2\sqrt{a^2-b^2}}\right)}{x}\,dx\right.}{\sqrt{a^2-b^2}f^2}$$
$$= \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f} + \frac{d\mathrm{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}f^2} - \frac{d\mathrm{Li}_2\left(-\frac{}{a+}\right.}{\sqrt{a^2-}}$$

Mathematica [A] time = 0.886612, size = 152, normalized size = 0.75

$$\frac{d\mathrm{PolyLog}\left(2,\frac{be^{e+fx}}{\sqrt{a^2-b^2}-a}\right)-d\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)+f(c+dx)\left(\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)-\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)\right)}{f^2\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Cosh[e + f*x]),x]
```

```
[Out] (f*(c + d*x)*(Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])] - Log[1 + (b*E
^(e + f*x))/(a + Sqrt[a^2 - b^2])]) + d*PolyLog[2, (b*E^(e + f*x))/(-a + Sq
rt[a^2 - b^2])] - d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/(
Sqrt[a^2 - b^2]*f^2)
```

Maple [B] time = 0.063, size = 437, normalized size = 2.2

$$2\frac{c}{f\sqrt{-a^2+b^2}}\arctan\left(1/2\frac{2\,b\mathrm{e}^{fx+e}+2\,a}{\sqrt{-a^2+b^2}}\right)+\frac{dx}{f}\ln\left(\left(-b\mathrm{e}^{fx+e}+\sqrt{a^2-b^2}-a\right)\left(-a+\sqrt{a^2-b^2}\right)^{-1}\right)\frac{1}{\sqrt{a^2-b^2}}+\frac{de}{f^2}\ln\Bigg(\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*cosh(f*x+e)),x)
```

```
[Out] 2/f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*exp(f*x+e)+2*a)/(-a^2+b^2)^(1/2))+1/
f*d/(a^2-b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2
)))*x+1/f^2*d/(a^2-b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2-b^2)^(1/2)-a)/(-a+(a^2
-b^2)^(1/2)))*e-1/f*d/(a^2-b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2-b^2)^(1/2)+a)/(
a+(a^2-b^2)^(1/2)))*x-1/f^2*d/(a^2-b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2-b^2)^(1
/2)+a)/(a+(a^2-b^2)^(1/2)))*e+1/f^2*d/(a^2-b^2)^(1/2)*dilog((-b*exp(f*x+e)+
(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/f^2*d/(a^2-b^2)^(1/2)*dilog((b*e
xp(f*x+e)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2/f^2*d*e/(-a^2+b^2)^(1/2
)*arctan(1/2*(2*b*exp(f*x+e)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12879, size = 1127, normalized size = 5.55

$$bd\sqrt{\frac{a^2-b^2}{b^2}}\,\mathrm{Li}_2\left(-\frac{a\cosh(fx+e)+a\sinh(fx+e)+\left(b\cosh(fx+e)+b\sinh(fx+e)\right)\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right)-bd\sqrt{\frac{a^2-b^2}{b^2}}\,\mathrm{Li}_2\left(-\frac{a\cosh(fx+e)+a\sinh(fx+e)-(b\,\mathrm{co}}{}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (b*d*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*c
osh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - b*d*sqr
t((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x
+ e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (b*d*e - b*c*f)
*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt
((a^2 - b^2)/b^2) + 2*a) - (b*d*e - b*c*f)*sqrt((a^2 - b^2)/b^2)*log(2*b*co
sh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + (b*d*f
*x + b*d*e)*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x + e) + a*sinh(f*x + e) +
(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) - (b*d*f*
x + b*d*e)*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x + e) + a*sinh(f*x + e) - (
b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b))/((a^2 - b
^2)*f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c+dx}{a+b\cosh\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)/(a + b*cosh(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{b\cosh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*cosh(f*x + e) + a), x)
```

3.171 $\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+b\cosh(e+fx))},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*Cosh[e + f*x])), x]
```

Rubi [A] time = 0.0613819, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Cosh[e + f*x])),x]

[Out] Defer[Int][1/((c + d*x)*(a + b*Cosh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx$$

Mathematica [A] time = 0.892867, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Cosh[e + f*x])),x]

[Out] Integrate[1/((c + d*x)*(a + b*Cosh[e + f*x])), x]
```

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+b\cosh\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*cosh(f*x+e)),x)

[Out] int(1/(d*x+c)/(a+b*cosh(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(b\cosh\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)*(b*cosh(f*x + e) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{adx+ac+(bdx+bc)\cosh\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (b*d*x + b*c)*cosh(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(b\cosh\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*cosh(f*x + e) + a)), x)
```

3.172 $\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+b\cosh(e+fx))},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + b*Cosh[e + f*x])), x]
```

Rubi [A] time = 0.0590852, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Cosh[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Cosh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx=\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx$$

Mathematica [A] time = 0.961235, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Cosh[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Cosh[e + f*x])), x]
```

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a+b\cosh\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2 \left(b\cosh\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^2*(b*cosh(f*x + e) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ad^2x^2+2\,acdx+ac^2+\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*cosh(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*cosh(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2 \left(b\cosh\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*cosh(f*x + e) + a)), x)
```

3.173 $\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=823

$$-\frac{6\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)f^4}-\frac{6\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)f^4}+\frac{6a\text{PolyLog}\left(4,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)^{3/2}f^4}-\frac{6a\text{PolyLog}\left(4,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f^4}$$

```
[Out] -((c + d*x)^3/((a^2 - b^2)*f)) + (3*d*(c + d*x)^2*Log[1 + (b*E^(e + f*x))/(
a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) + (a*(c + d*x)^3*Log[1 + (b*E^(e +
 f*x))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) + (3*d*(c + d*x)^2*Log
[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) - (a*(c + d*
x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f)
+ (6*d^2*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((
a^2 - b^2)*f^3) + (3*a*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt
[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^2) + (6*d^2*(c + d*x)*PolyLog[2, -((b*
E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)*f^3) - (3*a*d*(c + d*x)^
2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*
f^2) - (6*d^3*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 -
 b^2)*f^4) - (6*a*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2
- b^2]))])/((a^2 - b^2)^(3/2)*f^3) - (6*d^3*PolyLog[3, -((b*E^(e + f*x))/(a
 + Sqrt[a^2 - b^2]))])/((a^2 - b^2)*f^4) + (6*a*d^2*(c + d*x)*PolyLog[3, -(
(b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^3) + (6*a*d^3
*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f
^4) - (6*a*d^3*PolyLog[4, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2
- b^2)^(3/2)*f^4) - (b*(c + d*x)^3*Sinh[e + f*x])/((a^2 - b^2)*f*(a + b*Cos
h[e + f*x]))
```

Rubi [A] time = 1.35345, antiderivative size = 823, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3324, 3320, 2264, 2190, 2531, 6609, 2282, 6589, 5562}

$$-\frac{6\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)f^4}-\frac{6\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)f^4}+\frac{6a\text{PolyLog}\left(4,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d^3}{\left(a^2-b^2\right)^{3/2}f^4}-\frac{6a\text{PolyLog}\left(4,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] -((c + d*x)^3/((a^2 - b^2)*f)) + (3*d*(c + d*x)^2*Log[1 + (b*E^(e + f*x))/(
a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) + (a*(c + d*x)^3*Log[1 + (b*E^(e +
 f*x))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) + (3*d*(c + d*x)^2*Log
[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) - (a*(c + d*
x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f)
+ (6*d^2*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((
a^2 - b^2)*f^3) + (3*a*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt
[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^2) + (6*d^2*(c + d*x)*PolyLog[2, -((b*
E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)*f^3) - (3*a*d*(c + d*x)^
2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*
f^2) - (6*d^3*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 -
 b^2)*f^4) - (6*a*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2
- b^2]))])/((a^2 - b^2)^(3/2)*f^3) - (6*d^3*PolyLog[3, -((b*E^(e + f*x))/(a
 + Sqrt[a^2 - b^2]))])/((a^2 - b^2)*f^4) + (6*a*d^2*(c + d*x)*PolyLog[3, -(
(b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^3) + (6*a*d^3
*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f
```

```
^4) - (6*a*d^3*PolyLog[4, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2
- b^2)^(3/2)*f^4) - (b*(c + d*x)^3*Sinh[e + f*x])/((a^2 - b^2)*f*(a + b*Cos
h[e + f*x]))
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}\,dx = -\frac{b(c+dx)^3\sinh(e+fx)}{\left(a^2-b^2\right)f(a+b\cosh(e+fx))}+\frac{a\int\frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx}{a^2-b^2}+\frac{(3bd)\int\frac{(c+dx)^2\sinh(e+fx)}{a+b\cosh(e+fx)}\,dx}{\left(a^2-b^2\right)f}$$

$$= -\frac{(c+dx)^3}{\left(a^2-b^2\right)f}-\frac{b(c+dx)^3\sinh(e+fx)}{\left(a^2-b^2\right)f(a+b\cosh(e+fx))}+\frac{(2a)\int\frac{e^{e+fx}(c+dx)^3}{b+2ae^{e+fx}+be^{2(e+fx)}}\,dx}{a^2-b^2}+\frac{(3bd)\int \cdots}{\left(a^2 \cdots\right.}$$

$$= -\frac{(c+dx)^3}{\left(a^2-b^2\right)f}+\frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2}+\frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2}-\frac{b(c \cdots}{\left(a^2-b^2 \cdots\right.}$$

$$= -\frac{(c+dx)^3}{\left(a^2-b^2\right)f}+\frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2}+\frac{a(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f}+\frac{3d(c+dx \cdots}{\cdots}$$

$$= -\frac{(c+dx)^3}{\left(a^2-b^2\right)f}+\frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2}+\frac{a(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f}+\frac{3d(c+dx \cdots}{\cdots}$$

$$= -\frac{(c+dx)^3}{\left(a^2-b^2\right)f}+\frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2}+\frac{a(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f}+\frac{3d(c+dx \cdots}{\cdots}$$

$$= -\frac{(c+dx)^3}{\left(a^2-b^2\right)f}+\frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2}+\frac{a(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f}+\frac{3d(c+dx \cdots}{\cdots}$$

$$= -\frac{(c+dx)^3}{\left(a^2-b^2\right)f}+\frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2}+\frac{a(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f}+\frac{3d(c+dx \cdots}{\cdots}$$

Mathematica [B] time = 26.4629, size = 11178, normalized size = 13.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.289, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{\left(a+b\cosh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*cosh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^3/(a+b*cosh(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.7147, size = 14923, normalized size = 18.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -(2*(a^2*b - b^3)*d^3*e^3 - 6*(a^2*b - b^3)*c*d^2*e^2*f + 6*(a^2*b - b^3)*c
^2*d*e*f^2 - 2*(a^2*b - b^3)*c^3*f^3 + 2*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^
2*b - b^3)*c*d^2*f^3*x^2 + 3*(a^2*b - b^3)*c^2*d*f^3*x + (a^2*b - b^3)*d^3*
e^3 - 3*(a^2*b - b^3)*c*d^2*e^2*f + 3*(a^2*b - b^3)*c^2*d*e*f^2)*cosh(f*x +
 e)^2 + 2*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b - b^3)*c*d^2*f^3*x^2 + 3*(a
^2*b - b^3)*c^2*d*f^3*x + (a^2*b - b^3)*d^3*e^3 - 3*(a^2*b - b^3)*c*d^2*e^2
*f + 3*(a^2*b - b^3)*c^2*d*e*f^2)*sinh(f*x + e)^2 - 6*(a*b^2*d^3*cosh(f*x +
 e)^2 + a*b^2*d^3*sinh(f*x + e)^2 + 2*a^2*b*d^3*cosh(f*x + e) + a*b^2*d^3 +
 2*(a*b^2*d^3*cosh(f*x + e) + a^2*b*d^3)*sinh(f*x + e))*sqrt((a^2 - b^2)/b^
2)*polylog(4, -(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*si
nh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) + 6*(a*b^2*d^3*cosh(f*x + e)^2 + a*b
^2*d^3*sinh(f*x + e)^2 + 2*a^2*b*d^3*cosh(f*x + e) + a*b^2*d^3 + 2*(a*b^2*d
^3*cosh(f*x + e) + a^2*b*d^3)*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2)*polylog(
4, -(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e)
)*sqrt((a^2 - b^2)/b^2))/b) + 2*((a^3 - a*b^2)*d^3*f^3*x^3 + 3*(a^3 - a*b^2
)*c*d^2*f^3*x^2 + 3*(a^3 - a*b^2)*c^2*d*f^3*x + 2*(a^3 - a*b^2)*d^3*e^3 - 6
*(a^3 - a*b^2)*c*d^2*e^2*f + 6*(a^3 - a*b^2)*c^2*d*e*f^2 - (a^3 - a*b^2)*c^
3*f^3)*cosh(f*x + e) - 3*(2*(a^2*b - b^3)*d^3*f*x + 2*(a^2*b - b^3)*c*d^2*f
 + 2*((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*cosh(f*x + e)^2 + 2*((
a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*sinh(f*x + e)^2 + 4*((a^3 - a
```

```
*b^2)*d^3*f*x + (a^3 - a*b^2)*c*d^2*f)*cosh(f*x + e) + 4*((a^3 - a*b^2)*d^3
*f*x + (a^3 - a*b^2)*c*d^2*f + ((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2
*f)*cosh(f*x + e))*sinh(f*x + e) + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x
 + a*b^2*c^2*d*f^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d
*f^2)*cosh(f*x + e)^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^
2*d*f^2)*sinh(f*x + e)^2 + 2*(a^2*b*d^3*f^2*x^2 + 2*a^2*b*c*d^2*f^2*x + a^2
*b*c^2*d*f^2)*cosh(f*x + e) + 2*(a^2*b*d^3*f^2*x^2 + 2*a^2*b*c*d^2*f^2*x +
a^2*b*c^2*d*f^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^
2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))*dilog(-(a*cosh(f*x
+ e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^
2)/b^2) + b)/b + 1) - 3*(2*(a^2*b - b^3)*d^3*f*x + 2*(a^2*b - b^3)*c*d^2*f
+ 2*((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*cosh(f*x + e)^2 + 2*((a
^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*sinh(f*x + e)^2 + 4*((a^3 - a*
b^2)*d^3*f*x + (a^3 - a*b^2)*c*d^2*f)*cosh(f*x + e) + 4*((a^3 - a*b^2)*d^3*
f*x + (a^3 - a*b^2)*c*d^2*f + ((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*
f)*cosh(f*x + e))*sinh(f*x + e) - (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x
+ a*b^2*c^2*d*f^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*
f^2)*cosh(f*x + e)^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2
*d*f^2)*sinh(f*x + e)^2 + 2*(a^2*b*d^3*f^2*x^2 + 2*a^2*b*c*d^2*f^2*x + a^2*
b*c^2*d*f^2)*cosh(f*x + e) + 2*(a^2*b*d^3*f^2*x^2 + 2*a^2*b*c*d^2*f^2*x + a
^2*b*c^2*d*f^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2
)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))*dilog(-(a*cosh(f*x +
 e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2
)/b^2) + b)/b + 1) - (3*(a^2*b - b^3)*d^3*e^2 - 6*(a^2*b - b^3)*c*d^2*e*f +
 3*(a^2*b - b^3)*c^2*d*f^2 + 3*((a^2*b - b^3)*d^3*e^2 - 2*(a^2*b - b^3)*c*d
^2*e*f + (a^2*b - b^3)*c^2*d*f^2)*cosh(f*x + e)^2 + 3*((a^2*b - b^3)*d^3*e^
2 - 2*(a^2*b - b^3)*c*d^2*e*f + (a^2*b - b^3)*c^2*d*f^2)*sinh(f*x + e)^2 +
6*((a^3 - a*b^2)*d^3*e^2 - 2*(a^3 - a*b^2)*c*d^2*e*f + (a^3 - a*b^2)*c^2*d*
f^2)*cosh(f*x + e) + 6*((a^3 - a*b^2)*d^3*e^2 - 2*(a^3 - a*b^2)*c*d^2*e*f +
 (a^3 - a*b^2)*c^2*d*f^2 + ((a^2*b - b^3)*d^3*e^2 - 2*(a^2*b - b^3)*c*d^2*e
*f + (a^2*b - b^3)*c^2*d*f^2)*cosh(f*x + e))*sinh(f*x + e) + (a*b^2*d^3*e^3
 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3 + (a*b^2*d^3*e
^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3)*cosh(f*x +
e)^2 + (a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c
^3*f^3)*sinh(f*x + e)^2 + 2*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*
c^2*d*e*f^2 - a^2*b*c^3*f^3)*cosh(f*x + e) + 2*(a^2*b*d^3*e^3 - 3*a^2*b*c*d
^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3 + (a*b^2*d^3*e^3 - 3*a*b^2*c
*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt((a^2 - b^2)/b^2))*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b
*sqrt((a^2 - b^2)/b^2) + 2*a) - (3*(a^2*b - b^3)*d^3*e^2 - 6*(a^2*b - b^3)*
c*d^2*e*f + 3*(a^2*b - b^3)*c^2*d*f^2 + 3*((a^2*b - b^3)*d^3*e^2 - 2*(a^2*b
 - b^3)*c*d^2*e*f + (a^2*b - b^3)*c^2*d*f^2)*cosh(f*x + e)^2 + 3*((a^2*b -
b^3)*d^3*e^2 - 2*(a^2*b - b^3)*c*d^2*e*f + (a^2*b - b^3)*c^2*d*f^2)*sinh(f*
x + e)^2 + 6*((a^3 - a*b^2)*d^3*e^2 - 2*(a^3 - a*b^2)*c*d^2*e*f + (a^3 - a*
b^2)*c^2*d*f^2)*cosh(f*x + e) + 6*((a^3 - a*b^2)*d^3*e^2 - 2*(a^3 - a*b^2)*
c*d^2*e*f + (a^3 - a*b^2)*c^2*d*f^2 + ((a^2*b - b^3)*d^3*e^2 - 2*(a^2*b - b
^3)*c*d^2*e*f + (a^2*b - b^3)*c^2*d*f^2)*cosh(f*x + e))*sinh(f*x + e) - (a*
b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3 + (
a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3)*
cosh(f*x + e)^2 + (a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^
2 - a*b^2*c^3*f^3)*sinh(f*x + e)^2 + 2*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f
 + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*cosh(f*x + e) + 2*(a^2*b*d^3*e^3 -
3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3 + (a*b^2*d^3*e^3
- 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3)*cosh(f*x + e))
*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x
 + e) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - (3*(a^2*b - b^3)*d^3*f^2*x^2 + 6
*(a^2*b - b^3)*c*d^2*f^2*x - 3*(a^2*b - b^3)*d^3*e^2 + 6*(a^2*b - b^3)*c*d^
2*e*f + 3*((a^2*b - b^3)*d^3*f^2*x^2 + 2*(a^2*b - b^3)*c*d^2*f^2*x - (a^2*b
 - b^3)*d^3*e^2 + 2*(a^2*b - b^3)*c*d^2*e*f)*cosh(f*x + e)^2 + 3*((a^2*b -
```

```
b^3)*d^3*f^2*x^2 + 2*(a^2*b - b^3)*c*d^2*f^2*x - (a^2*b - b^3)*d^3*e^2 + 2*
(a^2*b - b^3)*c*d^2*e*f)*sinh(f*x + e)^2 + 6*((a^3 - a*b^2)*d^3*f^2*x^2 + 2
*(a^3 - a*b^2)*c*d^2*f^2*x - (a^3 - a*b^2)*d^3*e^2 + 2*(a^3 - a*b^2)*c*d^2*
e*f)*cosh(f*x + e) + 6*((a^3 - a*b^2)*d^3*f^2*x^2 + 2*(a^3 - a*b^2)*c*d^2*f
^2*x - (a^3 - a*b^2)*d^3*e^2 + 2*(a^3 - a*b^2)*c*d^2*e*f + ((a^2*b - b^3)*d
^3*f^2*x^2 + 2*(a^2*b - b^3)*c*d^2*f^2*x - (a^2*b - b^3)*d^3*e^2 + 2*(a^2*b
 - b^3)*c*d^2*e*f)*cosh(f*x + e))*sinh(f*x + e) + (a*b^2*d^3*f^3*x^3 + 3*a*
b^2*c*d^2*f^3*x^2 + 3*a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2
*f + 3*a*b^2*c^2*d*e*f^2 + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a
*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^
2)*cosh(f*x + e)^2 + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a*b^2*c
^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2)*sin
h(f*x + e)^2 + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*c*d^2*f^3*x^2 + 3*a^2*b*c^2*d
*f^3*x + a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2)*cosh(f*
x + e) + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*c*d^2*f^3*x^2 + 3*a^2*b*c^2*d*f^3*x
 + a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 + (a*b^2*d^3*f
^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*
b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((
a^2 - b^2)/b^2))*log((a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e)
+ b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) - (3*(a^2*b - b^3)*d^3*f^2
*x^2 + 6*(a^2*b - b^3)*c*d^2*f^2*x - 3*(a^2*b - b^3)*d^3*e^2 + 6*(a^2*b - b
^3)*c*d^2*e*f + 3*((a^2*b - b^3)*d^3*f^2*x^2 + 2*(a^2*b - b^3)*c*d^2*f^2*x
- (a^2*b - b^3)*d^3*e^2 + 2*(a^2*b - b^3)*c*d^2*e*f)*cosh(f*x + e)^2 + 3*((
a^2*b - b^3)*d^3*f^2*x^2 + 2*(a^2*b - b^3)*c*d^2*f^2*x - (a^2*b - b^3)*d^3*
e^2 + 2*(a^2*b - b^3)*c*d^2*e*f)*sinh(f*x + e)^2 + 6*((a^3 - a*b^2)*d^3*f^2
*x^2 + 2*(a^3 - a*b^2)*c*d^2*f^2*x - (a^3 - a*b^2)*d^3*e^2 + 2*(a^3 - a*b^2
)*c*d^2*e*f)*cosh(f*x + e) + 6*((a^3 - a*b^2)*d^3*f^2*x^2 + 2*(a^3 - a*b^2)
*c*d^2*f^2*x - (a^3 - a*b^2)*d^3*e^2 + 2*(a^3 - a*b^2)*c*d^2*e*f + ((a^2*b
- b^3)*d^3*f^2*x^2 + 2*(a^2*b - b^3)*c*d^2*f^2*x - (a^2*b - b^3)*d^3*e^2 +
2*(a^2*b - b^3)*c*d^2*e*f)*cosh(f*x + e))*sinh(f*x + e) - (a*b^2*d^3*f^3*x^
3 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c
*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f^3*x
^2 + 3*a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^
2*d*e*f^2)*cosh(f*x + e)^2 + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3
*a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*
f^2)*sinh(f*x + e)^2 + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*c*d^2*f^3*x^2 + 3*a^2
*b*c^2*d*f^3*x + a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2)
*cosh(f*x + e) + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*c*d^2*f^3*x^2 + 3*a^2*b*c^2
*d*f^3*x + a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 + (a*b
^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^
3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2)*cosh(f*x + e))*sinh(f*x + e)
)*sqrt((a^2 - b^2)/b^2))*log((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f
*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) + 6*((a^2*b - b^3)
*d^3*cosh(f*x + e)^2 + (a^2*b - b^3)*d^3*sinh(f*x + e)^2 + 2*(a^3 - a*b^2)*
d^3*cosh(f*x + e) + (a^2*b - b^3)*d^3 + 2*((a^2*b - b^3)*d^3*cosh(f*x + e)
+ (a^3 - a*b^2)*d^3)*sinh(f*x + e) + (a*b^2*d^3*f*x + a*b^2*c*d^2*f + (a*b^
2*d^3*f*x + a*b^2*c*d^2*f)*cosh(f*x + e)^2 + (a*b^2*d^3*f*x + a*b^2*c*d^2*f
)*sinh(f*x + e)^2 + 2*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*cosh(f*x + e) + 2*(a^
2*b*d^3*f*x + a^2*b*c*d^2*f + (a*b^2*d^3*f*x + a*b^2*c*d^2*f)*cosh(f*x + e)
)*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))*polylog(3, -(a*cosh(f*x + e) + a*si
nh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b)
 + 6*((a^2*b - b^3)*d^3*cosh(f*x + e)^2 + (a^2*b - b^3)*d^3*sinh(f*x + e)^2
 + 2*(a^3 - a*b^2)*d^3*cosh(f*x + e) + (a^2*b - b^3)*d^3 + 2*((a^2*b - b^3)
*d^3*cosh(f*x + e) + (a^3 - a*b^2)*d^3)*sinh(f*x + e) - (a*b^2*d^3*f*x + a*
b^2*c*d^2*f + (a*b^2*d^3*f*x + a*b^2*c*d^2*f)*cosh(f*x + e)^2 + (a*b^2*d^3*
f*x + a*b^2*c*d^2*f)*sinh(f*x + e)^2 + 2*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*co
sh(f*x + e) + 2*(a^2*b*d^3*f*x + a^2*b*c*d^2*f + (a*b^2*d^3*f*x + a*b^2*c*d
^2*f)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))*polylog(3, -(a*c
osh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((
```

```
a^2 - b^2)/b^2))/b) + 2*((a^3 - a*b^2)*d^3*f^3*x^3 + 3*(a^3 - a*b^2)*c*d^2*
f^3*x^2 + 3*(a^3 - a*b^2)*c^2*d*f^3*x + 2*(a^3 - a*b^2)*d^3*e^3 - 6*(a^3 -
a*b^2)*c*d^2*e^2*f + 6*(a^3 - a*b^2)*c^2*d*e*f^2 - (a^3 - a*b^2)*c^3*f^3 +
2*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b - b^3)*c*d^2*f^3*x^2 + 3*(a^2*b - b
^3)*c^2*d*f^3*x + (a^2*b - b^3)*d^3*e^3 - 3*(a^2*b - b^3)*c*d^2*e^2*f + 3*(
a^2*b - b^3)*c^2*d*e*f^2)*cosh(f*x + e))*sinh(f*x + e))/((a^4*b - 2*a^2*b^3
 + b^5)*f^4*cosh(f*x + e)^2 + (a^4*b - 2*a^2*b^3 + b^5)*f^4*sinh(f*x + e)^2
 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*f^4*cosh(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5
)*f^4 + 2*((a^4*b - 2*a^2*b^3 + b^5)*f^4*cosh(f*x + e) + (a^5 - 2*a^3*b^2 +
 a*b^4)*f^4)*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{\left(b\cosh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*cosh(f*x + e) + a)^2, x)
```

3.174 $\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=593

$$\frac{2ad(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\left(a^2-b^2\right)^{3/2}}-\frac{2ad(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^2\left(a^2-b^2\right)^{3/2}}+\frac{2d^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3\left(a^2-b^2\right)}+\frac{2d^2\text{Poly}}{f}$$

```
[Out] -((c + d*x)^2/((a^2 - b^2)*f)) + (2*d*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a
- Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) + (a*(c + d*x)^2*Log[1 + (b*E^(e + f
*x))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) + (2*d*(c + d*x)*Log[1 +
 (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) - (a*(c + d*x)^2
*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) + (2
*d^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 - b^2)*f^3
) + (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/
((a^2 - b^2)^(3/2)*f^2) + (2*d^2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2
 - b^2]))])/((a^2 - b^2)*f^3) - (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x
))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^2) - (2*a*d^2*PolyLog[3, -
((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^3) + (2*a*d^
2*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*
f^3) - (b*(c + d*x)^2*Sinh[e + f*x])/((a^2 - b^2)*f*(a + b*Cosh[e + f*x]))
```

Rubi [A] time = 1.02505, antiderivative size = 593, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3324, 3320, 2264, 2190, 2531, 2282, 6589, 5562, 2279, 2391}

$$\frac{2ad(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\left(a^2-b^2\right)^{3/2}}-\frac{2ad(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^2\left(a^2-b^2\right)^{3/2}}+\frac{2d^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3\left(a^2-b^2\right)}+\frac{2d^2\text{Poly}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] -((c + d*x)^2/((a^2 - b^2)*f)) + (2*d*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a
- Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) + (a*(c + d*x)^2*Log[1 + (b*E^(e + f
*x))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) + (2*d*(c + d*x)*Log[1 +
 (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) - (a*(c + d*x)^2
*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) + (2
*d^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 - b^2)*f^3
) + (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/
((a^2 - b^2)^(3/2)*f^2) + (2*d^2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2
 - b^2]))])/((a^2 - b^2)*f^3) - (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x
))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^2) - (2*a*d^2*PolyLog[3, -
((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^3) + (2*a*d^
2*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*
f^3) - (b*(c + d*x)^2*Sinh[e + f*x])/((a^2 - b^2)*f*(a + b*Cosh[e + f*x]))
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
```

```
2, 0] && IGtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
```

```
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx = -\frac{b(c+dx)^2\sinh(e+fx)}{\left(a^2-b^2\right)f(a+b\cosh(e+fx))} + \frac{a\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx}{a^2-b^2} + \frac{(2bd)\int \frac{(c+dx)\sinh(e+fx)}{a+b\cosh(e+fx)}\,dx}{\left(a^2-b^2\right)f}$$

$$= -\frac{(c+dx)^2}{\left(a^2-b^2\right)f} - \frac{b(c+dx)^2\sinh(e+fx)}{\left(a^2-b^2\right)f(a+b\cosh(e+fx))} + \frac{(2a)\int \frac{e^{e+fx}(c+dx)^2}{b+2ae^{e+fx}+be^{2(e+fx)}}\,dx}{a^2-b^2} + \frac{(2bd)\int}{(}$$

$$= -\frac{(c+dx)^2}{\left(a^2-b^2\right)f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} - \frac{b(c}{(a^2-b}$$

$$= -\frac{(c+dx)^2}{\left(a^2-b^2\right)f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} + \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} + \frac{2d(c+}{}$$

$$= -\frac{(c+dx)^2}{\left(a^2-b^2\right)f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} + \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} + \frac{2d(c+}{}$$

$$= -\frac{(c+dx)^2}{\left(a^2-b^2\right)f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} + \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} + \frac{2d(c+}{}$$

$$= -\frac{(c+dx)^2}{\left(a^2-b^2\right)f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)f^2} + \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} + \frac{2d(c+}{}$$

Mathematica [B] time = 22.2663, size = 6018, normalized size = 10.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.243, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{\left(a+b\cosh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*cosh(f*x+e))^2,x)

[Out] int((d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 3.0103, size = 8911, normalized size = 15.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")

[Out] (2*(a^2*b - b^3)*d^2*e^2 - 4*(a^2*b - b^3)*c*d*e*f + 2*(a^2*b - b^3)*c^2*f^
2 - 2*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*c*d*f^2*x - (a^2*b - b^3
)*d^2*e^2 + 2*(a^2*b - b^3)*c*d*e*f)*cosh(f*x + e)^2 - 2*((a^2*b - b^3)*d^2
*f^2*x^2 + 2*(a^2*b - b^3)*c*d*f^2*x - (a^2*b - b^3)*d^2*e^2 + 2*(a^2*b - b
^3)*c*d*e*f)*sinh(f*x + e)^2 - 2*(a*b^2*d^2*cosh(f*x + e)^2 + a*b^2*d^2*sin
h(f*x + e)^2 + 2*a^2*b*d^2*cosh(f*x + e) + a*b^2*d^2 + 2*(a*b^2*d^2*cosh(f*
x + e) + a^2*b*d^2)*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cos
h(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^
2 - b^2)/b^2))/b) + 2*(a*b^2*d^2*cosh(f*x + e)^2 + a*b^2*d^2*sinh(f*x + e)^
2 + 2*a^2*b*d^2*cosh(f*x + e) + a*b^2*d^2 + 2*(a*b^2*d^2*cosh(f*x + e) + a^
2*b*d^2)*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(f*x + e)
+ a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^
2))/b) - 2*((a^3 - a*b^2)*d^2*f^2*x^2 + 2*(a^3 - a*b^2)*c*d*f^2*x - 2*(a^3
- a*b^2)*d^2*e^2 + 4*(a^3 - a*b^2)*c*d*e*f - (a^3 - a*b^2)*c^2*f^2)*cosh(f*
x + e) + 2*((a^2*b - b^3)*d^2*cosh(f*x + e)^2 + (a^2*b - b^3)*d^2*sinh(f*x
+ e)^2 + 2*(a^3 - a*b^2)*d^2*cosh(f*x + e) + (a^2*b - b^3)*d^2 + 2*((a^2*b
- b^3)*d^2*cosh(f*x + e) + (a^3 - a*b^2)*d^2)*sinh(f*x + e) + (a*b^2*d^2*f*
x + a*b^2*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh(f*x + e)^2 + (a*b^2*d^
2*f*x + a*b^2*c*d*f)*sinh(f*x + e)^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh
(f*x + e) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d*f)*
cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))*dilog(-(a*cosh(f*x + e
) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/
b^2) + b)/b + 1) + 2*((a^2*b - b^3)*d^2*cosh(f*x + e)^2 + (a^2*b - b^3)*d^2
*sinh(f*x + e)^2 + 2*(a^3 - a*b^2)*d^2*cosh(f*x + e) + (a^2*b - b^3)*d^2 +
2*((a^2*b - b^3)*d^2*cosh(f*x + e) + (a^3 - a*b^2)*d^2)*sinh(f*x + e) - (a*
b^2*d^2*f*x + a*b^2*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh(f*x + e)^2 +
 (a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(f*x + e)^2 + 2*(a^2*b*d^2*f*x + a^2*b*c
*d*f)*cosh(f*x + e) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d^2*f*x + a*b
^2*c*d*f)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))*dilog(-(a*co
sh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a
^2 - b^2)/b^2) + b)/b + 1) - (2*(a^2*b - b^3)*d^2*e - 2*(a^2*b - b^3)*c*d*f
 + 2*((a^2*b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*cosh(f*x + e)^2 + 2*((a^2*
```

```
b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*sinh(f*x + e)^2 + 4*((a^3 - a*b^2)*d^
2*e - (a^3 - a*b^2)*c*d*f)*cosh(f*x + e) + 4*((a^3 - a*b^2)*d^2*e - (a^3 -
a*b^2)*c*d*f + ((a^2*b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*cosh(f*x + e))*s
inh(f*x + e) + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 + (a*b^2*d^
2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(f*x + e)^2 + (a*b^2*d^2*e^2 -
 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sinh(f*x + e)^2 + 2*(a^2*b*d^2*e^2 - 2*a^
2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(f*x + e) + 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d
*e*f + a^2*b*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*co
sh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))*log(2*b*cosh(f*x + e) +
2*b*sinh(f*x + e) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - (2*(a^2*b - b^3)*d^2
*e - 2*(a^2*b - b^3)*c*d*f + 2*((a^2*b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*
cosh(f*x + e)^2 + 2*((a^2*b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*sinh(f*x +
e)^2 + 4*((a^3 - a*b^2)*d^2*e - (a^3 - a*b^2)*c*d*f)*cosh(f*x + e) + 4*((a^
3 - a*b^2)*d^2*e - (a^3 - a*b^2)*c*d*f + ((a^2*b - b^3)*d^2*e - (a^2*b - b^
3)*c*d*f)*cosh(f*x + e))*sinh(f*x + e) - (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f +
 a*b^2*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(f*x
 + e)^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sinh(f*x + e)^2
 + 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(f*x + e) + 2*(a
^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c
*d*e*f + a*b^2*c^2*f^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2)
)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 - b^2)/b^2) + 2
*a) + (2*(a^2*b - b^3)*d^2*f*x + 2*(a^2*b - b^3)*d^2*e + 2*((a^2*b - b^3)*d
^2*f*x + (a^2*b - b^3)*d^2*e)*cosh(f*x + e)^2 + 2*((a^2*b - b^3)*d^2*f*x +
(a^2*b - b^3)*d^2*e)*sinh(f*x + e)^2 + 4*((a^3 - a*b^2)*d^2*f*x + (a^3 - a*
b^2)*d^2*e)*cosh(f*x + e) + 4*((a^3 - a*b^2)*d^2*f*x + (a^3 - a*b^2)*d^2*e
+ ((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*cosh(f*x + e))*sinh(f*x + e
) + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*
f + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*
f)*cosh(f*x + e)^2 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2
 + 2*a*b^2*c*d*e*f)*sinh(f*x + e)^2 + 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^
2*x - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f)*cosh(f*x + e) + 2*(a^2*b*d^2*f^2*x^2
 + 2*a^2*b*c*d*f^2*x - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f + (a*b^2*d^2*f^2*x^2
 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*cosh(f*x + e))*sinh
(f*x + e))*sqrt((a^2 - b^2)/b^2))*log((a*cosh(f*x + e) + a*sinh(f*x + e) +
(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) + (2*(a^2
*b - b^3)*d^2*f*x + 2*(a^2*b - b^3)*d^2*e + 2*((a^2*b - b^3)*d^2*f*x + (a^2
*b - b^3)*d^2*e)*cosh(f*x + e)^2 + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)
*d^2*e)*sinh(f*x + e)^2 + 4*((a^3 - a*b^2)*d^2*f*x + (a^3 - a*b^2)*d^2*e)*c
osh(f*x + e) + 4*((a^3 - a*b^2)*d^2*f*x + (a^3 - a*b^2)*d^2*e + ((a^2*b - b
^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*cosh(f*x + e))*sinh(f*x + e) - (a*b^2*d^
2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f + (a*b^2*d^
2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*cosh(f*x +
 e)^2 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*
d*e*f)*sinh(f*x + e)^2 + 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x - a^2*b*d
^2*e^2 + 2*a^2*b*c*d*e*f)*cosh(f*x + e) + 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*
d*f^2*x - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*
d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*cosh(f*x + e))*sinh(f*x + e))*sq
rt((a^2 - b^2)/b^2))*log((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x +
 e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) - 2*((a^3 - a*b^2)*d^2
*f^2*x^2 + 2*(a^3 - a*b^2)*c*d*f^2*x - 2*(a^3 - a*b^2)*d^2*e^2 + 4*(a^3 - a
*b^2)*c*d*e*f - (a^3 - a*b^2)*c^2*f^2 + 2*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a
^2*b - b^3)*c*d*f^2*x - (a^2*b - b^3)*d^2*e^2 + 2*(a^2*b - b^3)*c*d*e*f)*co
sh(f*x + e))*sinh(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f^3*cosh(f*x + e)^2
+ (a^4*b - 2*a^2*b^3 + b^5)*f^3*sinh(f*x + e)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^
4)*f^3*cosh(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*f^3 + 2*((a^4*b - 2*a^2*b^
3 + b^5)*f^3*cosh(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f^3)*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{\left(b\cosh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*cosh(f*x + e) + a)^2, x)
```

3.175 $\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=274

$$\frac{ad\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\left(a^2-b^2\right)^{3/2}}-\frac{ad\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^2\left(a^2-b^2\right)^{3/2}}+\frac{a(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f\left(a^2-b^2\right)^{3/2}}-\frac{a(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f\left(a^2-b^2\right)^{3/2}}$$

```
[Out] (a*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(
3/2)*f) - (a*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^
2 - b^2)^(3/2)*f) + (d*Log[a + b*Cosh[e + f*x]])/((a^2 - b^2)*f^2) + (a*d*P
olyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^2
) - (a*d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)
^(3/2)*f^2) - (b*(c + d*x)*Sinh[e + f*x])/((a^2 - b^2)*f*(a + b*Cosh[e + f*
x]))
```

Rubi [A] time = 0.456887, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{ad\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\left(a^2-b^2\right)^{3/2}}-\frac{ad\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^2\left(a^2-b^2\right)^{3/2}}+\frac{a(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f\left(a^2-b^2\right)^{3/2}}-\frac{a(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Cosh[e + f*x])^2,x]
```


```
[Out] (a*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(
3/2)*f) - (a*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^
2 - b^2)^(3/2)*f) + (d*Log[a + b*Cosh[e + f*x]])/((a^2 - b^2)*f^2) + (a*d*P
olyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^2
) - (a*d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)
^(3/2)*f^2) - (b*(c + d*x)*Sinh[e + f*x])/((a^2 - b^2)*f*(a + b*Cosh[e + f*
x]))
```


Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx = -\frac{b(c+dx)\sinh(e+fx)}{\left(a^2-b^2\right)f(a+b\cosh(e+fx))} + \frac{a\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx}{a^2-b^2} + \frac{(bd)\int \frac{\sinh(e+fx)}{a+b\cosh(e+fx)}\,dx}{\left(a^2-b^2\right)f}$$

$$= -\frac{b(c+dx)\sinh(e+fx)}{\left(a^2-b^2\right)f(a+b\cosh(e+fx))} + \frac{(2a)\int \frac{e^{e+fx}(c+dx)}{b+2ae^{e+fx}+be^{2(e+fx)}}\,dx}{a^2-b^2} + \frac{d\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\right.}{\left(a^2-b^2\right)}$$

$$= \frac{d\log(a+b\cosh(e+fx))}{\left(a^2-b^2\right)f^2} - \frac{b(c+dx)\sinh(e+fx)}{\left(a^2-b^2\right)f(a+b\cosh(e+fx))} + \frac{(2ab)\int \frac{e^{e+fx}(c+dx)}{2a-2\sqrt{a^2-b^2}+2be^{e+f}}}{\left(a^2-b^2\right)^{3/2}}$$

$$= \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} + \frac{d\log(a+b\cosh(e+fx)}{\left(a^2-b^2\right)f^2}$$

$$= \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} + \frac{d\log(a+b\cosh(e+fx)}{\left(a^2-b^2\right)f^2}$$

$$= \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f} + \frac{d\log(a+b\cosh(e+fx)}{\left(a^2-b^2\right)f^2}$$

Mathematica [A] time = 4.9781, size = 509, normalized size = 1.86

$$\frac{\left(a^2-b^2\right)\left(-ad\sqrt{b^2-a^2}\text{PolyLog}\left(2,\frac{be^{e+fx}}{\sqrt{a^2-b^2}-a}\right)+ad\sqrt{b^2-a^2}\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)+2acf\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{a+be^{e+fx}}{\sqrt{a^2-b^2}}\right)+d\sqrt{-\left(a^2-b^2\right)^2}(e+fx)-ad\sqrt{b^2-a^2}(e+fx)\log\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (((a^2 - b^2)*(Sqrt[-(a^2 - b^2)^2]*d*(e + f*x) - 2*a*Sqrt[a^2 - b^2]*d*Arc
Tan[(a + b*E^(e + f*x))/Sqrt[-a^2 + b^2]] - 2*a*Sqrt[-a^2 + b^2]*d*ArcTanh[
(a + b*E^(e + f*x))/Sqrt[a^2 - b^2]] - 2*a*Sqrt[-a^2 + b^2]*d*e*ArcTanh[(a
+ b*E^(e + f*x))/Sqrt[a^2 - b^2]] + 2*a*Sqrt[-a^2 + b^2]*c*f*ArcTanh[(a + b
*E^(e + f*x))/Sqrt[a^2 - b^2]] - a*Sqrt[-a^2 + b^2]*d*(e + f*x)*Log[1 + (b*
E^(e + f*x))/(a - Sqrt[a^2 - b^2])] + a*Sqrt[-a^2 + b^2]*d*(e + f*x)*Log[1
+ (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])] - Sqrt[-(a^2 - b^2)^2]*d*Log[b + 2
*a*E^(e + f*x) + b*E^(2*(e + f*x))] - a*Sqrt[-a^2 + b^2]*d*PolyLog[2, (b*E^
(e + f*x))/(-a + Sqrt[a^2 - b^2])] + a*Sqrt[-a^2 + b^2]*d*PolyLog[2, -((b*E
^(e + f*x))/(a + Sqrt[a^2 - b^2]))]))/(-(a^2 - b^2)^2)^(3/2) - (b*f*(c + d*
x)*Sinh[e + f*x])/((a - b)*(a + b)*(a + b*Cosh[e + f*x])))/f^2
```

Maple [B] time = 0.096, size = 585, normalized size = 2.1

$$2\,\frac{(dx+c)\left(a{\rm e}^{fx+e}+b\right)}{f\left(a^2-b^2\right)\left(b{\rm e}^{2\,fx+2\,e}+2\,a{\rm e}^{fx+e}+b\right)}+2\,\frac{ac}{f\left(a^2-b^2\right)\sqrt{-a^2+b^2}}\arctan\left(1/2\,\frac{2\,b{\rm e}^{fx+e}+2\,a}{\sqrt{-a^2+b^2}}\right)+\frac{adx}{f}\ln\left(\left(-b{\rm e}^{fx+e}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*cosh(f*x+e))^2,x)
```

```
[Out] 2*(d*x+c)*(a*exp(f*x+e)+b)/f/(a^2-b^2)/(b*exp(2*f*x+2*e)+2*a*exp(f*x+e)+b)+
2/(a^2-b^2)/f*a*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*exp(f*x+e)+2*a)/(-a^2+b^
2)^(1/2))+1/(a^2-b^2)^(3/2)/f*d*a*ln((-b*exp(f*x+e)+(a^2-b^2)^(1/2)-a)/(-a+
(a^2-b^2)^(1/2)))*x+1/(a^2-b^2)^(3/2)/f^2*d*a*ln((-b*exp(f*x+e)+(a^2-b^2)^(
1/2)-a)/(-a+(a^2-b^2)^(1/2)))*e-1/(a^2-b^2)^(3/2)/f*d*a*ln((b*exp(f*x+e)+(a
^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x-1/(a^2-b^2)^(3/2)/f^2*d*a*ln((b*exp
(f*x+e)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*e+1/(a^2-b^2)^(3/2)/f^2*d*a
*dilog((-b*exp(f*x+e)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/(a^2-b^2)^
(3/2)/f^2*d*a*dilog((b*exp(f*x+e)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2
/(a^2-b^2)/f^2*d*ln(exp(f*x+e))+1/(a^2-b^2)/f^2*d*ln(b*exp(2*f*x+2*e)+2*a*e
xp(f*x+e)+b)-2/(a^2-b^2)/f^2*a*d*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*exp(f*x
+e)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.32256, size = 4074, normalized size = 14.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -(2*(a^2*b - b^3)*d*e - 2*(a^2*b - b^3)*c*f + 2*((a^2*b - b^3)*d*f*x + (a^2
*b - b^3)*d*e)*cosh(f*x + e)^2 + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*d*e
)*sinh(f*x + e)^2 - (a*b^2*d*cosh(f*x + e)^2 + a*b^2*d*sinh(f*x + e)^2 + 2*
a^2*b*d*cosh(f*x + e) + a*b^2*d + 2*(a*b^2*d*cosh(f*x + e) + a^2*b*d)*sinh(
f*x + e))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x + e) +
 (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (a
*b^2*d*cosh(f*x + e)^2 + a*b^2*d*sinh(f*x + e)^2 + 2*a^2*b*d*cosh(f*x + e)
+ a*b^2*d + 2*(a*b^2*d*cosh(f*x + e) + a^2*b*d)*sinh(f*x + e))*sqrt((a^2 -
b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*
sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - (a*b^2*d*f*x + a*b^2*d*e
 + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e)^2 + (a*b^2*d*f*x + a*b^2*d*e)*si
nh(f*x + e)^2 + 2*(a^2*b*d*f*x + a^2*b*d*e)*cosh(f*x + e) + 2*(a^2*b*d*f*x
+ a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e))*sinh(f*x + e))*sqrt(
(a^2 - b^2)/b^2)*log((a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e)
+ b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) + (a*b^2*d*f*x + a*b^2*d*e
 + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e)^2 + (a*b^2*d*f*x + a*b^2*d*e)*si
nh(f*x + e)^2 + 2*(a^2*b*d*f*x + a^2*b*d*e)*cosh(f*x + e) + 2*(a^2*b*d*f*x
+ a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e))*sinh(f*x + e))*sqrt(
(a^2 - b^2)/b^2)*log((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e)
+ b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) + 2*((a^3 - a*b^2)*d*f*x +
 2*(a^3 - a*b^2)*d*e - (a^3 - a*b^2)*c*f)*cosh(f*x + e) - ((a^2*b - b^3)*d*
cosh(f*x + e)^2 + (a^2*b - b^3)*d*sinh(f*x + e)^2 + 2*(a^3 - a*b^2)*d*cosh(
f*x + e) + (a^2*b - b^3)*d + 2*((a^2*b - b^3)*d*cosh(f*x + e) + (a^3 - a*b^
2)*d)*sinh(f*x + e) + (a*b^2*d*e - a*b^2*c*f + (a*b^2*d*e - a*b^2*c*f)*cosh
```

```
(f*x + e)^2 + (a*b^2*d*e - a*b^2*c*f)*sinh(f*x + e)^2 + 2*(a^2*b*d*e - a^2*
b*c*f)*cosh(f*x + e) + 2*(a^2*b*d*e - a^2*b*c*f + (a*b^2*d*e - a*b^2*c*f)*c
osh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))*log(2*b*cosh(f*x + e) +
 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - ((a^2*b - b^3)*d*co
sh(f*x + e)^2 + (a^2*b - b^3)*d*sinh(f*x + e)^2 + 2*(a^3 - a*b^2)*d*cosh(f*
x + e) + (a^2*b - b^3)*d + 2*((a^2*b - b^3)*d*cosh(f*x + e) + (a^3 - a*b^2)
*d)*sinh(f*x + e) - (a*b^2*d*e - a*b^2*c*f + (a*b^2*d*e - a*b^2*c*f)*cosh(f
*x + e)^2 + (a*b^2*d*e - a*b^2*c*f)*sinh(f*x + e)^2 + 2*(a^2*b*d*e - a^2*b*
c*f)*cosh(f*x + e) + 2*(a^2*b*d*e - a^2*b*c*f + (a*b^2*d*e - a*b^2*c*f)*cos
h(f*x + e))*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))*log(2*b*cosh(f*x + e) + 2
*b*sinh(f*x + e) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 2*((a^3 - a*b^2)*d*f*
x + 2*(a^3 - a*b^2)*d*e - (a^3 - a*b^2)*c*f + 2*((a^2*b - b^3)*d*f*x + (a^2
*b - b^3)*d*e)*cosh(f*x + e))*sinh(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f^2
*cosh(f*x + e)^2 + (a^4*b - 2*a^2*b^3 + b^5)*f^2*sinh(f*x + e)^2 + 2*(a^5 -
 2*a^3*b^2 + a*b^4)*f^2*cosh(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*f^2 + 2*(
(a^4*b - 2*a^2*b^3 + b^5)*f^2*cosh(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f^2
)*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\left(b\cosh\left(fx + e\right) + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*cosh(f*x + e) + a)^2, x)
```

3.176 $\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+b\cosh(e+fx))^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*Cosh[e + f*x])^2), x]
```

Rubi [A] time = 0.0556963, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Cosh[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*Cosh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx$$

Mathematica [A] time = 49.0259, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Cosh[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*Cosh[e + f*x])^2), x]
```

Maple [A] time = 0.292, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+b\cosh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(ae^{(fx+e)}+b\right)}{a^2bcf-b^3cf+\left(a^2bdf-b^3df\right)x+\left(a^2bcfe^{(2e)}-b^3cfe^{(2e)}+\left(a^2bdfe^{(2e)}-b^3dfe^{(2e)}\right)x\right)e^{(2fx)}+2\left(a^3cfe^e-ab^2cfe^e+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2*(a*e^(f*x + e) + b)/(a^2*b*c*f - b^3*c*f + (a^2*b*d*f - b^3*d*f)*x + (a^2
*b*c*f*e^(2*e) - b^3*c*f*e^(2*e) + (a^2*b*d*f*e^(2*e) - b^3*d*f*e^(2*e))*x)
*e^(2*f*x) + 2*(a^3*c*f*e^e - a*b^2*c*f*e^e + (a^3*d*f*e^e - a*b^2*d*f*e^e)
*x)*e^(f*x)) + integrate(2*(b*d + (a*d*f*x*e^e + (c*f*e^e + d*e^e)*a)*e^(f*
x))/(a^2*b*c^2*f - b^3*c^2*f + (a^2*b*d^2*f - b^3*d^2*f)*x^2 + 2*(a^2*b*c*d
*f - b^3*c*d*f)*x + (a^2*b*c^2*f*e^(2*e) - b^3*c^2*f*e^(2*e) + (a^2*b*d^2*f
*e^(2*e) - b^3*d^2*f*e^(2*e))*x^2 + 2*(a^2*b*c*d*f*e^(2*e) - b^3*c*d*f*e^(2
*e))*x)*e^(2*f*x) + 2*(a^3*c^2*f*e^e - a*b^2*c^2*f*e^e + (a^3*d^2*f*e^e - a
*b^2*d^2*f*e^e)*x^2 + 2*(a^3*c*d*f*e^e - a*b^2*c*d*f*e^e)*x)*e^(f*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2dx+a^2c+\left(b^2dx+b^2c\right)\cosh\left(fx+e\right)^2+2\left(abdx+abc\right)\cosh\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*cosh(f*x + e)^2 + 2*(a*b*d*
x + a*b*c)*cosh(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(dx+c)\left(b\cosh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*cosh(f*x + e) + a)^2), x)
```

3.177 $\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + b*Cosh[e + f*x])^2), x]
```

Rubi [A] time = 0.0543985, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Cosh[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Cosh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx=\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx$$

Mathematica [A] time = 51.9672, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Cosh[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Cosh[e + f*x])^2), x]
```

Maple [A] time = 0.461, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a+b\cosh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(ae^{\left(fx+\right.}\right.}{a^2bc^2f - b^3c^2f + \left(a^2bd^2f - b^3d^2f\right)x^2 + 2\left(a^2bcdf - b^3cdf\right)x + \left(a^2bc^2fe^{(2e)} - b^3c^2fe^{(2e)} + \left(a^2bd^2fe^{(2e)} - b^3d^2fe^{(2e)}\right)x^2 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2*(a*e^(f*x + e) + b)/(a^2*b*c^2*f - b^3*c^2*f + (a^2*b*d^2*f - b^3*d^2*f)*
x^2 + 2*(a^2*b*c*d*f - b^3*c*d*f)*x + (a^2*b*c^2*f*e^(2*e) - b^3*c^2*f*e^(2
*e) + (a^2*b*d^2*f*e^(2*e) - b^3*d^2*f*e^(2*e))*x^2 + 2*(a^2*b*c*d*f*e^(2*e
) - b^3*c*d*f*e^(2*e))*x)*e^(2*f*x) + 2*(a^3*c^2*f*e^e - a*b^2*c^2*f*e^e +
(a^3*d^2*f*e^e - a*b^2*d^2*f*e^e)*x^2 + 2*(a^3*c*d*f*e^e - a*b^2*c*d*f*e^e)
*x)*e^(f*x)) + integrate(2*(2*b*d + (a*d*f*x*e^e + (c*f*e^e + 2*d*e^e)*a)*e
^(f*x))/(a^2*b*c^3*f - b^3*c^3*f + (a^2*b*d^3*f - b^3*d^3*f)*x^3 + 3*(a^2*b
*c*d^2*f - b^3*c*d^2*f)*x^2 + 3*(a^2*b*c^2*d*f - b^3*c^2*d*f)*x + (a^2*b*c^
3*f*e^(2*e) - b^3*c^3*f*e^(2*e) + (a^2*b*d^3*f*e^(2*e) - b^3*d^3*f*e^(2*e))
*x^3 + 3*(a^2*b*c*d^2*f*e^(2*e) - b^3*c*d^2*f*e^(2*e))*x^2 + 3*(a^2*b*c^2*d
*f*e^(2*e) - b^3*c^2*d*f*e^(2*e))*x)*e^(2*f*x) + 2*(a^3*c^3*f*e^e - a*b^2*c
^3*f*e^e + (a^3*d^3*f*e^e - a*b^2*d^3*f*e^e)*x^3 + 3*(a^3*c*d^2*f*e^e - a*b
^2*c*d^2*f*e^e)*x^2 + 3*(a^3*c^2*d*f*e^e - a*b^2*c^2*d*f*e^e)*x)*e^(f*x)),
x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2d^2x^2 + 2\,a^2cdx + a^2c^2 + \left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\cosh\left(fx+e\right)^2 + 2\left(abd^2x^2 + 2\,abcdx + abc^2\right)\cosh\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cosh(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*cosh
(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(b\cosh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*cosh(f*x + e) + a)^2), x)
```

3.178 $\int (c+dx)^m (a+b\cosh(e+fx))^n\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((c+dx)^m(a+b\cosh(e+fx))^n,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*(a + b*Cosh[e + f*x])^n, x]
```

Rubi [A] time = 0.0503341, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + b*Cosh[e + f*x])^n,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(a + b*Cosh[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx=\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx$$

Mathematica [A] time = 4.12516, size = 0, normalized size = 0.

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x])^n,x]
```

```
[Out] Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x])^n, x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+b\cosh\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*cosh(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(a+b*cosh(f*x+e))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m\left(b\cosh\left(fx+e\right)+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*(b*cosh(f*x + e) + a)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m\left(b\cosh\left(fx+e\right)+a\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*(b*cosh(f*x + e) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*cosh(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m\left(b\cosh\left(fx+e\right)+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*(b*cosh(f*x + e) + a)^n, x)
```

3.179 $\int (c+dx)^m (a+b\cosh(e+fx))^3\,dx$

Optimal. Leaf size=543

$$\frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{3a^2be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+$$

```
[Out] (a^3*(c + d*x)^(1 + m))/(d*(1 + m)) + (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1 +
 m)) + (3^(-1 - m)*b^3*E^(3*e - (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(
c + d*x))/d])/(8*f*(-((f*(c + d*x))/d))^m) + (3*2^(-3 - m)*a*b^2*E^(2*e - (
2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))
/d))^m) + (3*a^2*b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))
/d)])/(2*f*(-((f*(c + d*x))/d))^m) + (3*b^3*E^(e - (c*f)/d)*(c + d*x)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)])/(8*f*(-((f*(c + d*x))/d))^m) - (3*a^2*b*E^(-
e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))
/d)^m) - (3*b^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])
/(8*f*((f*(c + d*x))/d)^m) - (3*2^(-3 - m)*a*b^2*E^(-2*e + (2*c*f)/d)*(c +
d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - (3^(-1 -
m)*b^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/d])/(8
*f*((f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.743432, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3317, 3307, 2181, 3312}

$$\frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{3a^2be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Cosh[e + f*x])^3,x]
```


```
[Out] (a^3*(c + d*x)^(1 + m))/(d*(1 + m)) + (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1 +
 m)) + (3^(-1 - m)*b^3*E^(3*e - (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(
c + d*x))/d])/(8*f*(-((f*(c + d*x))/d))^m) + (3*2^(-3 - m)*a*b^2*E^(2*e - (
2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))
/d))^m) + (3*a^2*b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))
/d)])/(2*f*(-((f*(c + d*x))/d))^m) + (3*b^3*E^(e - (c*f)/d)*(c + d*x)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)])/(8*f*(-((f*(c + d*x))/d))^m) - (3*a^2*b*E^(-
e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))
/d)^m) - (3*b^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])
/(8*f*((f*(c + d*x))/d)^m) - (3*2^(-3 - m)*a*b^2*E^(-2*e + (2*c*f)/d)*(c +
d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - (3^(-1 -
m)*b^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/d])/(8
*f*((f*(c + d*x))/d)^m)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^m (a+b\cosh(e+fx))^3\,dx = \int \left(a^3(c+dx)^m + 3a^2b(c+dx)^m\cosh(e+fx) + 3ab^2(c+dx)^m\cosh^2(e+fx)\right.$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \left(3a^2b\right)\int (c+dx)^m\cosh(e+fx)\,dx + \left(3ab^2\right)\int (c+dx)^m \cos$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}\left(3a^2b\right)\int e^{-i(ie+ifx)}(c+dx)^m\,dx + \frac{1}{2}\left(3a^2b\right)\int e^{i(ie+ifx)}(c+$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} + \frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\right.}{2f}$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} + \frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\right.}{2f}$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} + \frac{3^{-1-m}b^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1\right.}{8f}$$

Mathematica [A] time = 1.68044, size = 447, normalized size = 0.82

$$2^{-m-3}3^{-m-1}e^{-3\left(\frac{cf}{d}+e\right)}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(-e^{\frac{3cf}{d}}\left(2^m\left(b^3d(m+1)e^{\frac{3cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^m \text{Gamma}\left(m+1,\frac{3f(c+dx)}{d}\right) - 4ae^3\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x])^3,x]
```

```
[Out] (2^(-3 - m)*3^(-1 - m)*(c + d*x)^m*(2^m*b^3*d*E^(6*e)*(1 + m)*((f*(c + d*x)
)/d)^m*Gamma[1 + m, (-3*f*(c + d*x))/d] + 3^(2 + m)*a*b^2*d*E^(5*e + (c*f)/
d)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, (-2*f*(c + d*x))/d] + 2^m*3^(2 + m)
*b*(4*a^2 + b^2)*d*E^(4*e + (2*c*f)/d)*(1 + m)*((f*(c + d*x))/d)^m*Gamma[1
+ m, -((f*(c + d*x))/d)] - E^((3*c*f)/d)*(2^m*3^(2 + m)*b*(4*a^2 + b^2)*d*E
^(2*e + (c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (f*(c + d*x))/
d] + 3^(2 + m)*a*b^2*d*E^(e + (2*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gam
ma[1 + m, (2*f*(c + d*x))/d] + 2^m*(-4*3^(1 + m)*a*(2*a^2 + 3*b^2)*E^(3*e)*
f*(c + d*x)*(-((f^2*(c + d*x)^2)/d^2))^m + b^3*d*E^((3*c*f)/d)*(1 + m)*(-((
```

```
f*(c + d*x))/d))^m*Gamma[1 + m, (3*f*(c + d*x))/d]))))/(d*E^(3*(e + (c*f)/d
))*f*(1 + m)*(-((f^2*(c + d*x)^2)/d^2))^m)
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int (dx+c)^m \left(a+b\cosh\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*cosh(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m*(a+b*cosh(f*x+e))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.45387, size = 1886, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/24*((b^3*d*m + b^3*d)*cosh((d*m*log(3*f/d) + 3*d*e - 3*c*f)/d)*gamma(m +
 1, 3*(d*f*x + c*f)/d) + 9*(a*b^2*d*m + a*b^2*d)*cosh((d*m*log(2*f/d) + 2*d
*e - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) + 9*((4*a^2*b + b^3)*d*m + (
4*a^2*b + b^3)*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x +
c*f)/d) - 9*((4*a^2*b + b^3)*d*m + (4*a^2*b + b^3)*d)*cosh((d*m*log(-f/d) -
 d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - 9*(a*b^2*d*m + a*b^2*d)*cos
h((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) - (
b^3*d*m + b^3*d)*cosh((d*m*log(-3*f/d) - 3*d*e + 3*c*f)/d)*gamma(m + 1, -3*
(d*f*x + c*f)/d) - (b^3*d*m + b^3*d)*gamma(m + 1, 3*(d*f*x + c*f)/d)*sinh((
d*m*log(3*f/d) + 3*d*e - 3*c*f)/d) - 9*(a*b^2*d*m + a*b^2*d)*gamma(m + 1, 2
*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d) - 9*((4*a^2*b +
b^3)*d*m + (4*a^2*b + b^3)*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*log(f
/d) + d*e - c*f)/d) + 9*((4*a^2*b + b^3)*d*m + (4*a^2*b + b^3)*d)*gamma(m +
 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) - d*e + c*f)/d) + 9*(a*b^2*d*m +
a*b^2*d)*gamma(m + 1, -2*(d*f*x + c*f)/d)*sinh((d*m*log(-2*f/d) - 2*d*e + 2
*c*f)/d) + (b^3*d*m + b^3*d)*gamma(m + 1, -3*(d*f*x + c*f)/d)*sinh((d*m*log
(-3*f/d) - 3*d*e + 3*c*f)/d) - 12*((2*a^3 + 3*a*b^2)*d*f*x + (2*a^3 + 3*a*b
^2)*c*f)*cosh(m*log(d*x + c)) - 12*((2*a^3 + 3*a*b^2)*d*f*x + (2*a^3 + 3*a*
b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*cosh(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cosh\left(fx+e\right)+a\right)^{3}(dx+c)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*cosh(f*x + e) + a)^3*(d*x + c)^m, x)
```

3.180 $\int (c+dx)^m (a+b\cosh(e+fx))^2\,dx$

Optimal. Leaf size=282

$$\frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{f}-\frac{abe^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{f}+\frac{b^22}{}$$

```
[Out] (a^2*(c + d*x)^(1 + m))/(d*(1 + m)) + (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m))
 + (2^(-3 - m)*b^2*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c +
d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(e - (c*f)/d)*(c + d*x)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) - (a*b*E^(-e + (c
*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) -
 (2^(-3 - m)*b^2*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*
x))/d])/(f*((f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.362635, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3317, 3307, 2181, 3312}

$$\frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{f}-\frac{abe^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{f}+\frac{b^22}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (a^2*(c + d*x)^(1 + m))/(d*(1 + m)) + (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m))
 + (2^(-3 - m)*b^2*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c +
d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(e - (c*f)/d)*(c + d*x)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) - (a*b*E^(-e + (c
*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) -
 (2^(-3 - m)*b^2*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*
x))/d])/(f*((f*(c + d*x))/d)^m)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^m(a+b\cosh(e+fx))^2\,dx = \int \left(a^2(c+dx)^m + 2ab(c+dx)^m\cosh(e+fx) + b^2(c+dx)^m\cosh^2(e+fx)\right)dx$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (2ab)\int (c+dx)^m\cosh(e+fx)\,dx + b^2\int (c+dx)^m\cosh^2(e+$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (ab)\int e^{-i(ie+ifx)}(c+dx)^m\,dx + (ab)\int e^{i(ie+ifx)}(c+dx)^m\,dx +$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} + \frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c}{}\right.}{f}$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} + \frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c}{}\right.}{f}$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}b^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+\right.}{f}$$

Mathematica [A] time = 0.720988, size = 254, normalized size = 0.9

$$\frac{(c+dx)^m\left(8abd(m+1)e^{e-\frac{cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right) - 8abd(m+1)e^{\frac{cf}{d}-e}\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] ((c + d*x)^m*(8*a^2*f*(c + d*x) + 4*b^2*f*(c + d*x) + (b^2*d*E^(2*e - (2*c*
f)/d)*(1 + m)*Gamma[1 + m, (-2*f*(c + d*x))/d])/(2^m*(-((f*(c + d*x))/d))^m
) + (8*a*b*d*E^(e - (c*f)/d)*(1 + m)*Gamma[1 + m, -((f*(c + d*x))/d)])/(-((
f*(c + d*x))/d))^m - (8*a*b*d*E^(-e + (c*f)/d)*(1 + m)*Gamma[1 + m, (f*(c +
 d*x))/d])/((f*(c + d*x))/d)^m - (b^2*d*E^(-2*e + (2*c*f)/d)*(1 + m)*Gamma[
1 + m, (2*f*(c + d*x))/d])/(2^m*((f*(c + d*x))/d)^m)))/(8*d*f*(1 + m))
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+b\cosh\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*cosh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+b*cosh(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.30203, size = 1191, normalized size = 4.22

$$-\frac{\left(b^2dm+b^2d\right)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)\Gamma\left(m+1,\frac{2\left(dfx+cf\right)}{d}\right)+8\left(abdm+abd\right)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/8*((b^2*d*m + b^2*d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m +
1, 2*(d*f*x + c*f)/d) + 8*(a*b*d*m + a*b*d)*cosh((d*m*log(f/d) + d*e - c*f)
/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8*(a*b*d*m + a*b*d)*cosh((d*m*log(-f/d)
 - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (b^2*d*m + b^2*d)*cosh((d
*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) - (b^2*
d*m + b^2*d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e -
 2*c*f)/d) - 8*(a*b*d*m + a*b*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*lo
g(f/d) + d*e - c*f)/d) + 8*(a*b*d*m + a*b*d)*gamma(m + 1, -(d*f*x + c*f)/d)
*sinh((d*m*log(-f/d) - d*e + c*f)/d) + (b^2*d*m + b^2*d)*gamma(m + 1, -2*(d
*f*x + c*f)/d)*sinh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d) - 4*((2*a^2 + b^2)
*d*f*x + (2*a^2 + b^2)*c*f)*cosh(m*log(d*x + c)) - 4*((2*a^2 + b^2)*d*f*x +
 (2*a^2 + b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cosh\left(fx+e\right)+a\right)^2(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*cosh(f*x + e) + a)^2*(d*x + c)^m, x)
```

3.181 $\int (c+dx)^m (a+b\cosh(e+fx))\,dx$

Optimal. Leaf size=131

$$\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\operatorname{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\operatorname{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c}{d}$$

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
+ m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (b*E^(-e + (c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.145442, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3307, 2181}

$$\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\operatorname{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\operatorname{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Cosh[e + f*x]),x]
```


```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
+ m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (b*E^(-e + (c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m(a+b\cosh(e+fx))\,dx = \int \left(a(c+dx)^m+b(c+dx)^m\cosh(e+fx)\right)dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)}+b\int (c+dx)^m\cosh(e+fx)\,dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{1}{2}b\int e^{-i(ie+ifx)}(c+dx)^m\,dx+\frac{1}{2}b\int e^{i(ie+ifx)}(c+dx)^m\,dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{be^{-e+\frac{cf}{d}}(c+dx)$$

Mathematica [A] time = 0.172064, size = 119, normalized size = 0.91

$$\frac{1}{2}(c+dx)^m\left(-\frac{be^{\frac{cf}{d}-e}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{f}+\frac{be^{e-\frac{cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{f}+\frac{2a(c+}{d(m+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x]),x]
```

```
[Out] ((c + d*x)^m*((2*a*(c + d*x))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*Gamma[1 + m,
 -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) - (b*E^(-e + (c*f)/d)*Gamm
a[1 + m, (f*(c + d*x))/d])/(f*(f*(c/d + x))^m)))/2
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+b\cosh\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*cosh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+b*cosh(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15879, size = 586, normalized size = 4.47

$$-\frac{(bdm+bd)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)-(bdm+bd)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)-de+cf}{d}\right)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)-(bdm+b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*((b*d*m + b*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x
+ c*f)/d) - (b*d*m + b*d)*cosh((d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1,
-(d*f*x + c*f)/d) - (b*d*m + b*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*l
og(f/d) + d*e - c*f)/d) + (b*d*m + b*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh
((d*m*log(-f/d) - d*e + c*f)/d) - 2*(a*d*f*x + a*c*f)*cosh(m*log(d*x + c))
- 2*(a*d*f*x + a*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*cosh(f*x+e)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cosh\left(fx+e\right)+a\right)(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(f*x + e) + a)*(d*x + c)^m, x)
```

3.182 $\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{a+b\cosh(e+fx)},x\right)$$

```
[Out] Unintegrable[(c + d*x)^m/(a + b*Cosh[e + f*x]), x]
```

Rubi [A] time = 0.0563509, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + b*Cosh[e + f*x]),x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + b*Cosh[e + f*x]), x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx$$

Mathematica [A] time = 1.12237, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + b*Cosh[e + f*x]),x]
```

```
[Out] Integrate[(c + d*x)^m/(a + b*Cosh[e + f*x]), x]
```

Maple [A] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{a+b\cosh\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+b*cosh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m/(a+b*cosh(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{b\cosh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(b*cosh(f*x + e) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^m}{b\cosh\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m/(b*cosh(f*x + e) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{a+b\cosh\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*cosh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**m/(a + b*cosh(e + f*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{b\cosh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(b*cosh(f*x + e) + a), x)
```

3.183 $\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{(a+b\cosh(e+fx))^2},x\right)$$

```
[Out] Unintegrable[(c + d*x)^m/(a + b*Cosh[e + f*x])^2, x]
```

Rubi [A] time = 0.0550124, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + b*Cosh[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx$$

Mathematica [A] time = 5.08517, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m/(a + b*Cosh[e + f*x])^2, x]
```

Maple [A] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(a+b\cosh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+b*cosh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m/(a+b*cosh(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(b\cosh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(b*cosh(f*x + e) + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^m}{b^2\cosh\left(fx+e\right)^2+2\,ab\cosh\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m/(b^2*cosh(f*x + e)^2 + 2*a*b*cosh(f*x + e) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{\left(a+b\cosh\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**m/(a + b*cosh(e + f*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(b\cosh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(b*cosh(f*x + e) + a)^2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
    )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
    type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```